

*Theoretical Models for
Dark Energy: A Brief Review*

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Laboratory Tests of Dark Energy
Fermilab
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Overview

- Cosmic Acceleration - Dark Energy - Modified Gravity
- Central question: How do new degrees of freedom couple to Standard Model + Dark Matter
- Categorizing Models of Dark Energy
- Example Theoretical Constructions

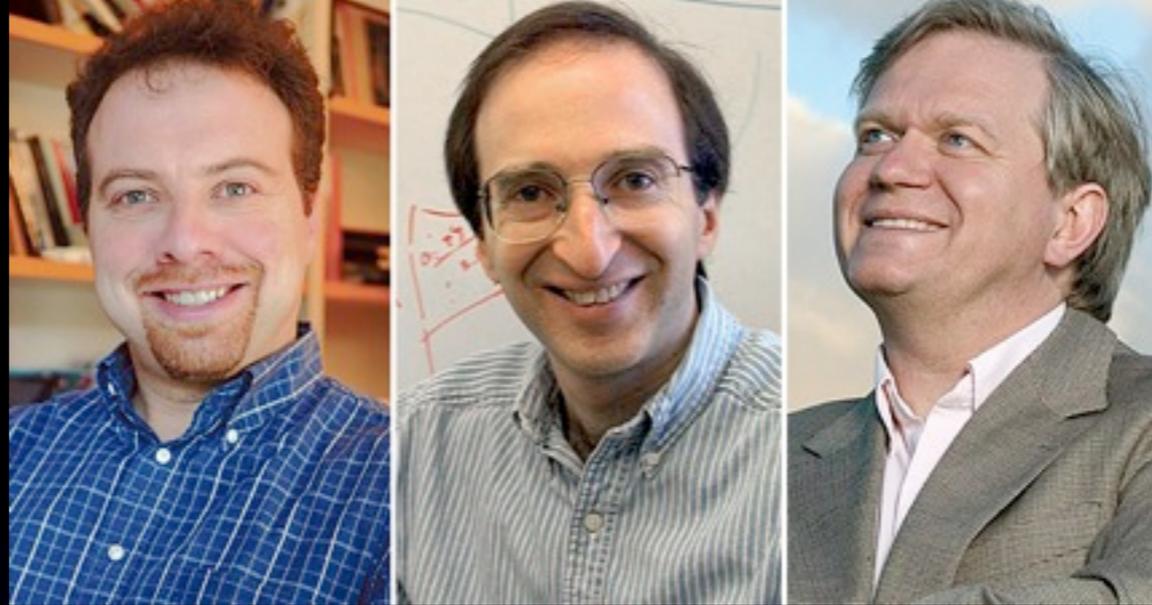
Cosmic Acceleration

Dark Energy?

Modified Gravity?

Cosmological constant?

Cosmic Acceleration



2011 Nobel Prize
in Physics

Riess, Perlmutter, Schmidt

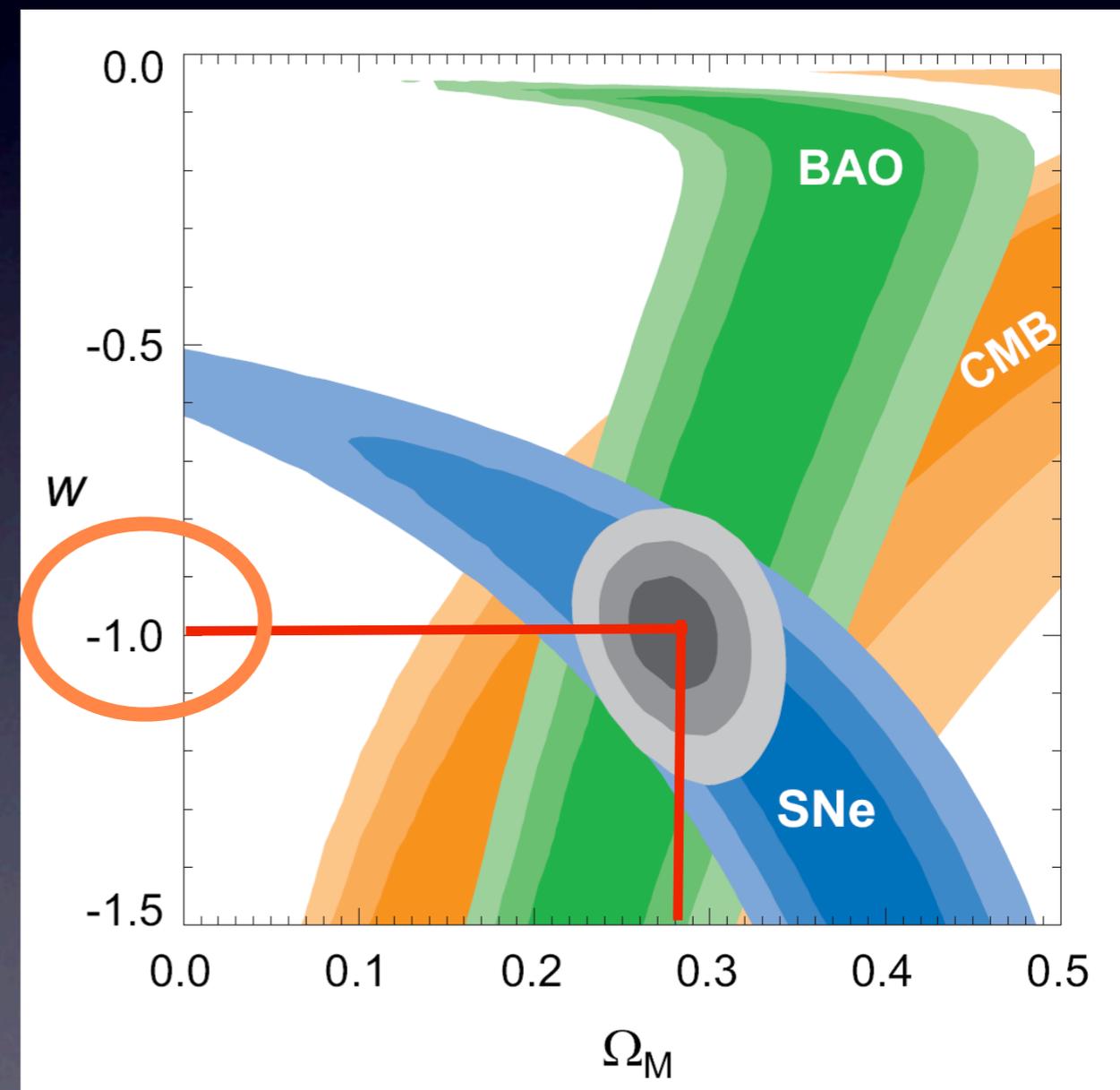
The Universe is Accelerating!
Acceleration can only occur if

$$w = \frac{p}{\rho} < -1/3$$

Data points tantalizingly
close to

$$w = -1$$

$$w = -0.94 \pm 0.1$$



Frieman et al. (2008) Ann.Rev.Astron.Astrophys

Why are we so concerned?

New physics at Hubble
scales?

New physics at a
millimeter scales?

Cosmic Coincidence
Problem

Cosmological Constant
Problem

Scales of Dark Energy

There are two natural scales associated with Dark Energy

$$H^2 = \frac{8\pi G}{3} \rho$$

Curvature scale Energy scale

$$\Lambda \sim m^2 \sim \frac{1}{R^2}$$

$$m = 10^{-33} \text{ eV}$$
$$R \sim 3800 \text{ Mpc}$$

Cosmological scale

$$\rho_\Lambda = \frac{1}{8\pi G} \Lambda \sim m^4$$

$$L = 0.1 \text{ mm}$$
$$m = 1 \text{ meV}$$

Submillimeter scale

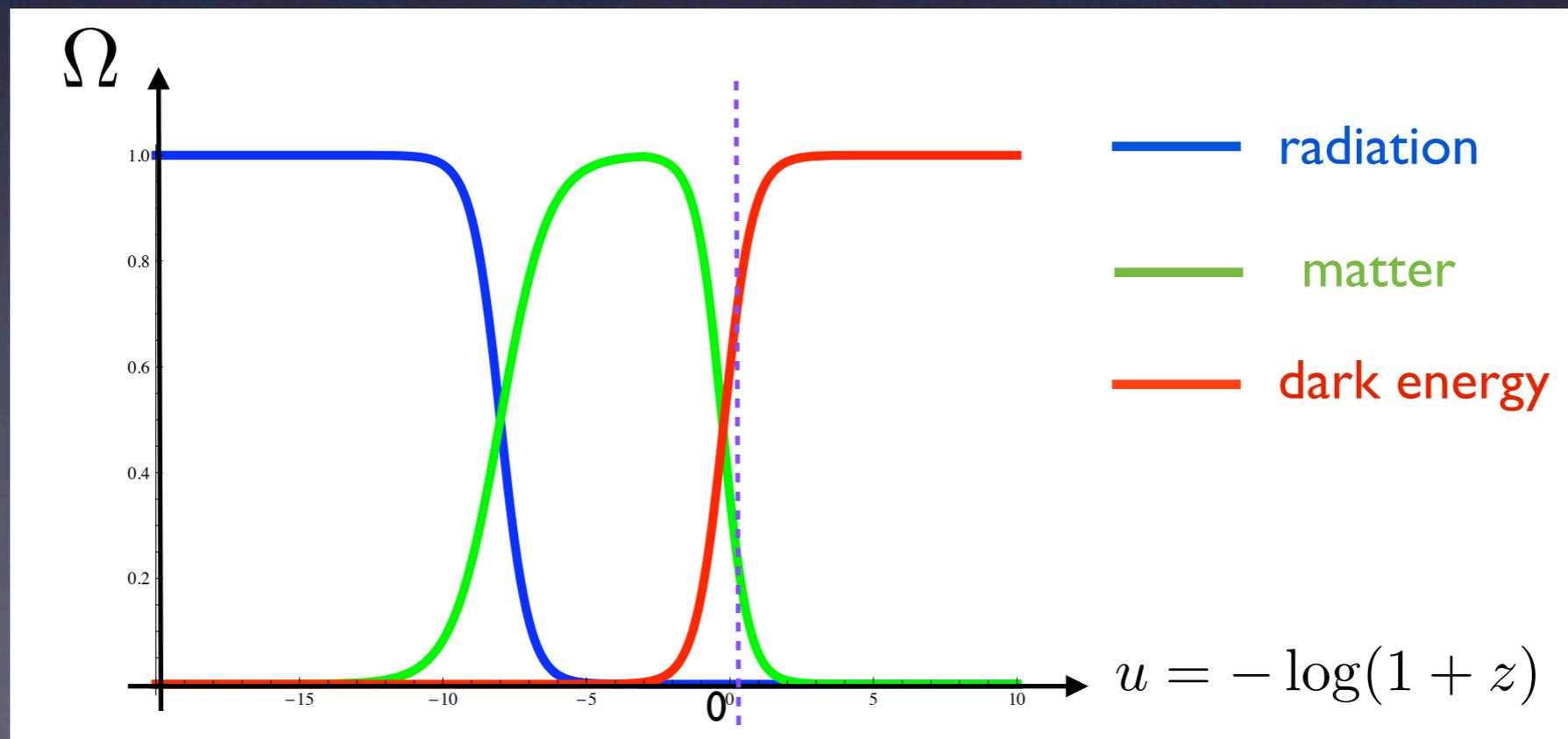
Cosmic Coincidence problem

*Why does dark energy come to dominate today?
around the time of structure formation?*

Universe began accelerating about
redshift $z \sim 0.4$ and age 10 Gyr

Also seems
coincidental that
visible and dark
matter are only a few
orders of magnitude
away from each other
today

$$\frac{\Omega_{\text{d.e.}}}{\Omega_{\text{M}}} \sim a^3$$



Cosmological constant problem

Why is Λ so un(technically) naturally small?

C.C. is leading 'relevant operator' in action for gravity

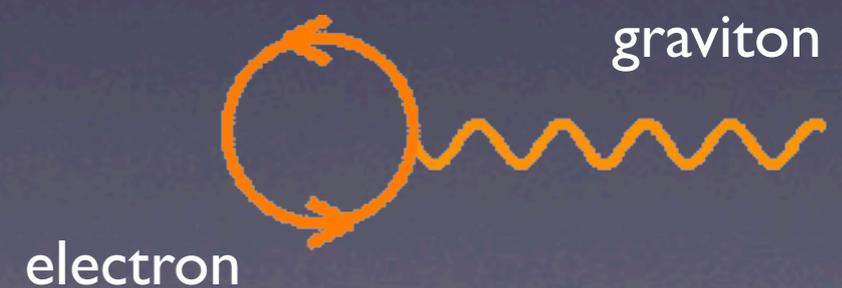
$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{16\pi G} \Lambda + \frac{1}{16\pi G} R + \mathcal{L}_M \right)$$

Despite being most relevant operator: most UV sensitive!

$$\Delta\rho_\Lambda = \frac{8\pi G}{3} \Delta\Lambda \sim \sum m_i^4 \ln(m_i/\mu)$$

$$m_e^4/\rho \sim 10^{36}$$

$$m_W^4/\rho \sim 10^{56}$$



Cosmological constant problem

Why doesn't Lambda pick up a large contribution from Phase Transitions?

Examples:

Potential energy of Higgs field

$$V \sim (100\text{GeV})^4$$

QCD condensate energy in presence of qqbar bilinears (chiral symmetry breaking)

$$V \sim (100\text{MeV})^4$$

Even if we resolve zero-point fluctuations, we have to classically tune away these large contributions

Cosmological Constant

versus

Dynamical Dark Energy

$$w = -1$$



Cosmological constant

$$T_{\mu\nu} = -\frac{1}{8\pi G}\Lambda g_{\mu\nu}$$

$$w \neq -1$$



Dynamical Dark Energy

existence of new d.o.f.
i.e. new particles
beyond gravity + SM (+DM)

Categorizing Dark Energy/ Modified Gravity Models via Screening

New Degrees of Freedom

Cosmological constant is the 'unique' large distance modification to GR that does not introduce any new degrees of freedom

Dynamical Models of Dark Energy or Modified Gravity will be distinguished by new d.o.f. (i.e. dynamical perturbations!)

New degrees of freedom must necessarily be incredibly light!

$$m_{\text{d.e.}} \leq 10^{-33} \text{eV}$$

New gravitational degrees of freedom that coupled to matter are highly constrained

Fifth Forces (solar system)

Equivalence Principle Tests etc.

Binary Pulsar Timing

Nucleosynthesis

Cosmological Moduli Problems

Need some kind of
screening mechanism to
hide extra d.o.f.

Why are new d.o.f. *nearly* always scalars?

If theory Lorentz invariance, new d.o.f characterized by spin

Must be effectively bosonic (even if fundamentally fermionic)

(=GR!)

Massive spin 2 = Massless spin 2 + Massless spin 1 + **Scalar**

Massive spin 1 = Massless spin 1 + **Scalar**

Massless spin 1 must coupled to conserved vector but $\partial_\mu T^\mu{}_\nu = 0$

~ always some range of energies for which
every D.E./modified gravity theory looks
like GR plus scalars!

Interactions of new d.o.f.

Imagine a scalar $\phi = \phi_b + \delta\phi$

coupled to the energy density $\rho = \rho_b + \delta\rho$

Generic form of effective action for perturbations:

$$S_2 = \int d^4x - \frac{1}{2} Z(\phi_b, \rho_b) (\partial\delta\phi)^2 - \frac{1}{2} m^2(\phi_b, \rho_b) \delta\phi^2 + \frac{\beta(\phi_b, \rho_b)}{M_{\text{Pl}}} \delta\rho \delta\phi$$

kinetic term

mass term

coupling to matter

Fifth force contribution

$$S_2 = \int d^4x -\frac{1}{2}Z(\phi_b, \rho_b)(\partial\delta\phi)^2 - \frac{1}{2}m^2(\phi_b, \rho_b)\delta\phi^2 + \frac{\beta(\phi_b, \rho_b)}{M_{\text{Pl}}} \delta\rho \delta\phi$$

Force between two
masses:

$$F \approx \frac{m_1 m_2}{M_{\text{Pl}}^2 r^2} \frac{\beta(\phi_b, \rho_b)}{\sqrt{Z(\phi_b, \rho_b)}} \exp(-m(\phi_b, \rho_b)r)$$

Fifth force constraints: screening

To ensure fifth forces are small

$$\frac{\beta(\phi_b, \rho_b)}{\sqrt{Z(\phi_b, \rho_b)}} \exp(-m(\phi_b, \rho_b)r) \ll 1$$

Only three independent possibilities!

(a) Coupling is small $\beta(\phi_b, \rho_b) \ll 1$

(b) Mass is large $m(\phi_b, \rho_b) \gg \frac{1}{r_{exp}}$

(c) Kinetic term is large $Z(\phi_b, \rho_b) \gg 1$

Screening Mechanisms

I. *'Screening without Screening'* - make coupling to matter universally small - e.g. Quintessence

$$\beta(\phi_b, \rho_b) \ll 1$$

$$F \approx \frac{m_1 m_2}{M_{\text{Pl}}^2 r^2} \frac{\beta(\phi_b, \rho_b)}{\sqrt{Z(\phi_b, \rho_b)}} \exp(-m(\phi_b, \rho_b)r)$$

Screening Mechanisms

I. *'Screening without Screening'* - make coupling to matter universally small - e.g. Quintessence

II. Make coupling to matter environmental (small in high density environments, large in low density environments) - e.g. Symmetron

at high densities: $\beta(\phi_b, \rho_b) \ll 1$

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at high densities: $m(\phi_b, \rho_b) \gg \frac{1}{r_{exp}}$

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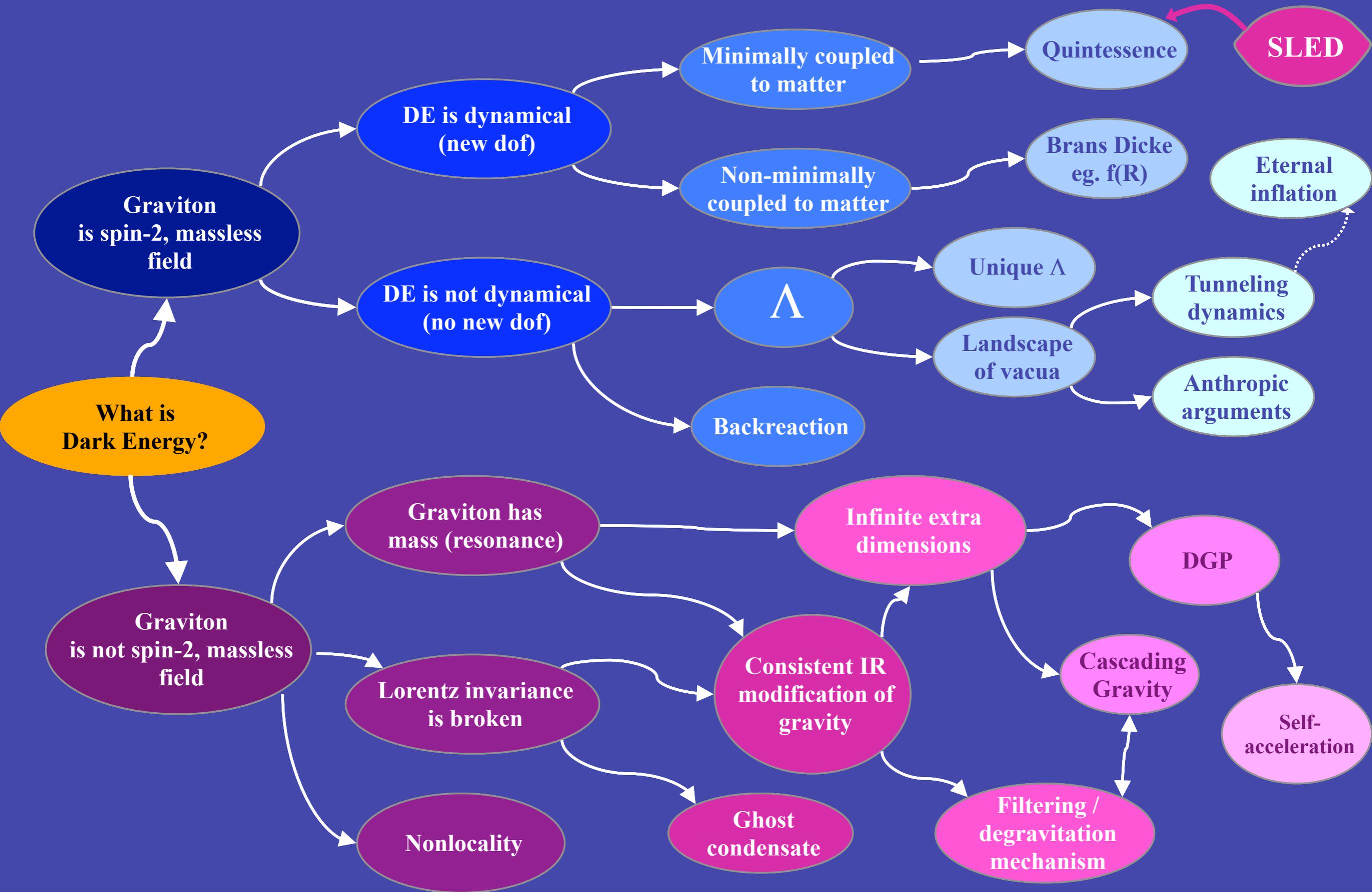
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- II. Make coupling to matter environmental (small in high density environments, large in low density environments) - e.g. Symmetron
- III. Make mass environmental (large in high density environments, small in low density environments) - e.g. Chameleon
- IV. Make kinetic term environmental (large in high density environments, small in low density environments) - e.g. Vainshtein mechanism - Massive Gravity, Galileon

$$Z(\phi_b, \rho_b) \gg 1$$

Constructing Models of Dark Energy





I. Making the coupling small universally

Theoretical Models: $\beta(\phi_b, \rho_b) \ll 1$

Quintessence and its multifarious
generalizations!!!

These are the *Vanilla* models of Dark Energy

Quintessence

Canonical Example: Scalar field with no direct coupling to matter

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{\text{pl}}^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right) + S_{\text{m}}$$

Dark energy contributes to the background evolution, and plays an indirect role in perturbations, additional isocurvature modes

Can tolerate a very small coupling (e.g. one protected by a shift symmetry)

Brax et al. 0904.3471

Tuning/Technical Naturalness

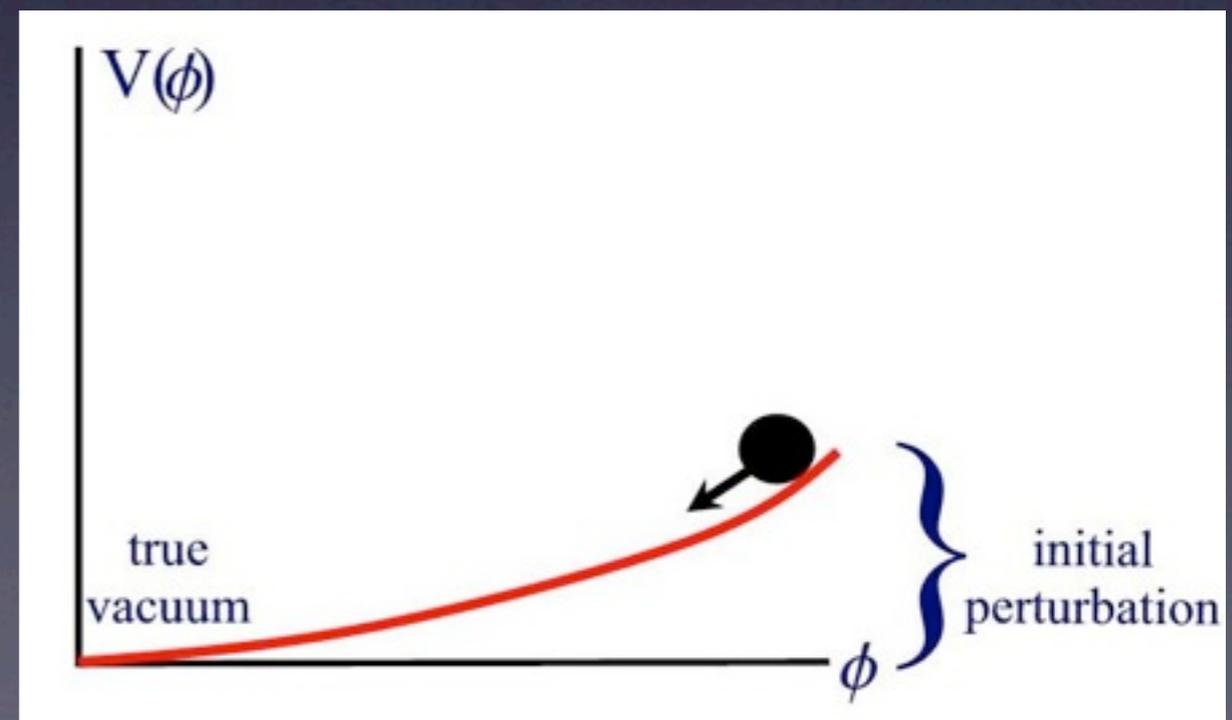
Typically not **technically natural** (Eta problem in Inflation) - significantly worse for Dark Energy

$$\Delta V \sim V(\phi) \frac{\phi^2}{M_{\text{pl}}^2}$$

dim 6 operators

mass quadratically
divergent, pick up mass
comparable to heaviest
particle

Closely akin to Higgs
mass/gauge hierarchy
problem



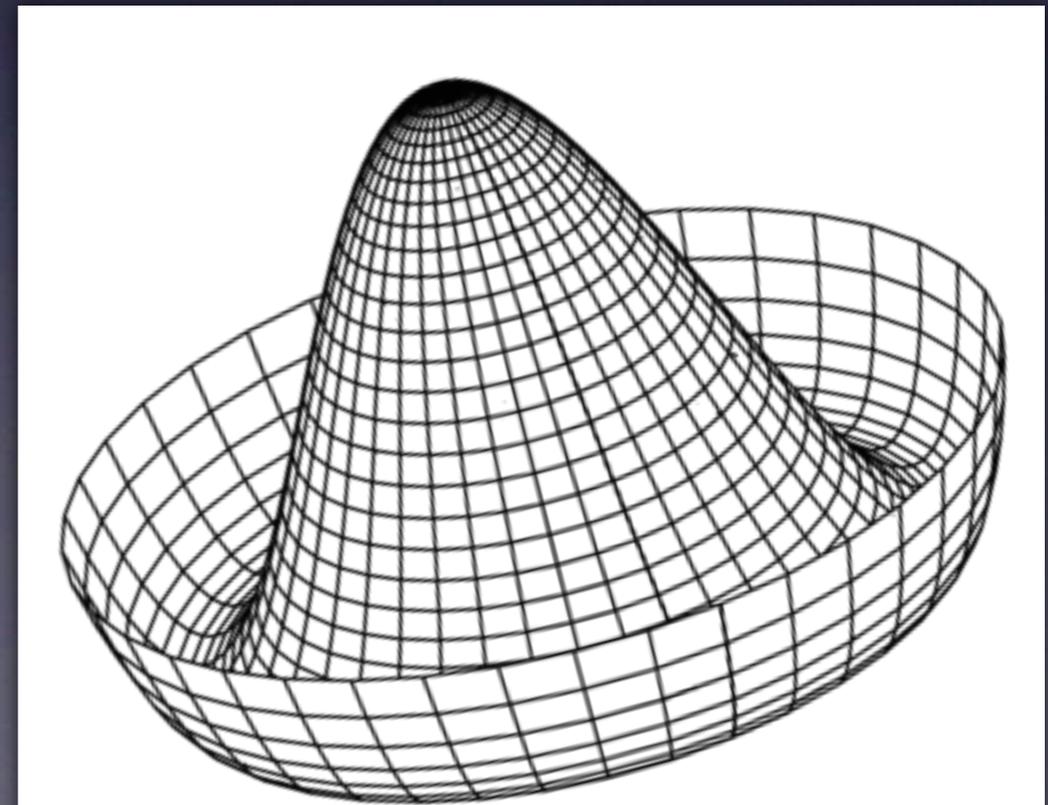
Scalars kept light by an approximate shift symmetry

- Technically natural Scalar Field arises as a *pseudo-Nambu-Goldstone* field associated with an approximately broken continuous global symmetry

$$\phi \rightarrow \phi + C$$

Explicitly broken but by a small amount

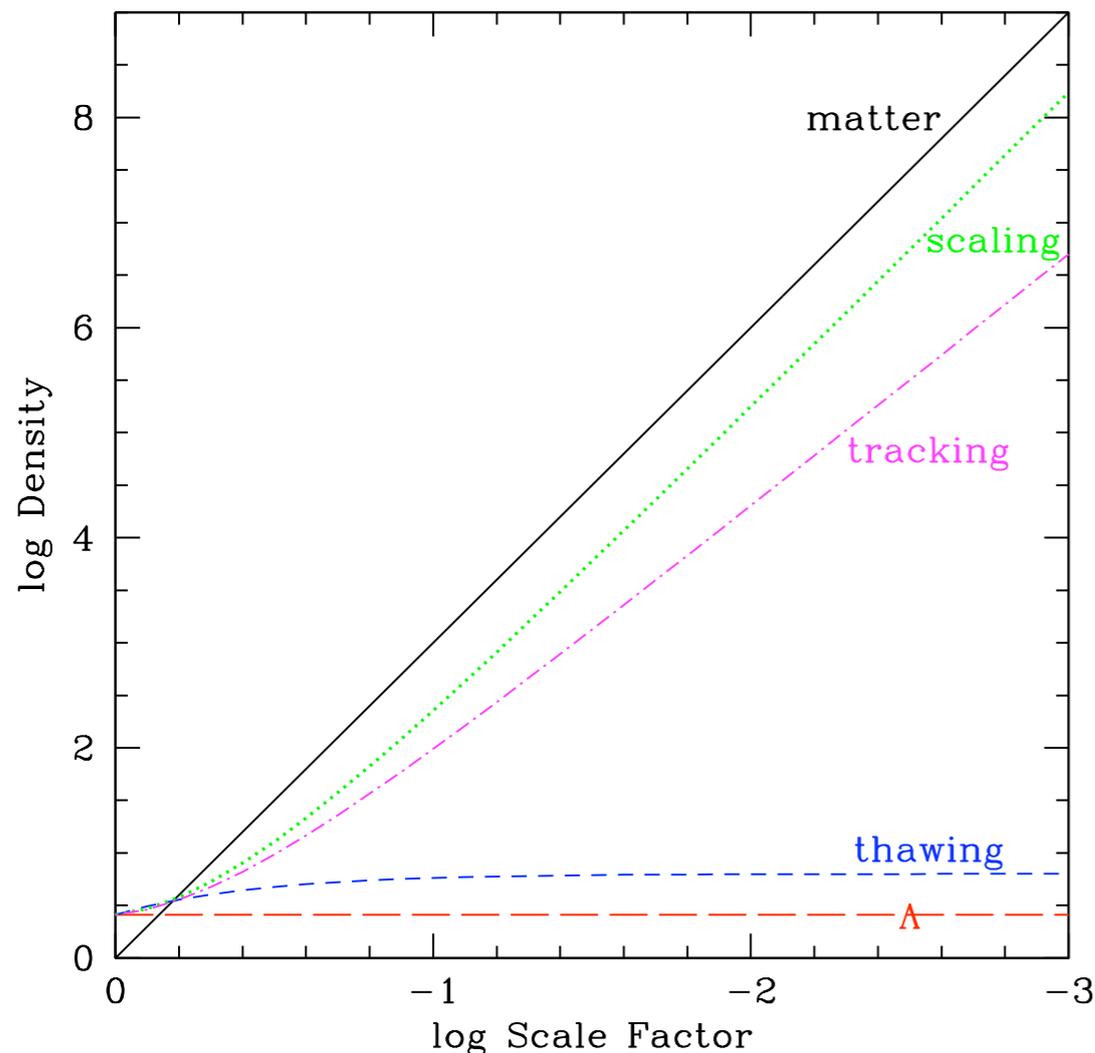
e.g. $U(1)$ symmetry of mexican hat



Freezing Models

Freezing: Field rolls to minimum during deceleration ($w > -1, w' < 0$), slows down as comes to dominate universe

Linder 0801.2968



Scaling fields: Attractor of Autonomous System, D.E. constant fraction of total

Tracking fields: Attractor in sense of independence of initial conditions, D.E. has different equation of state to total

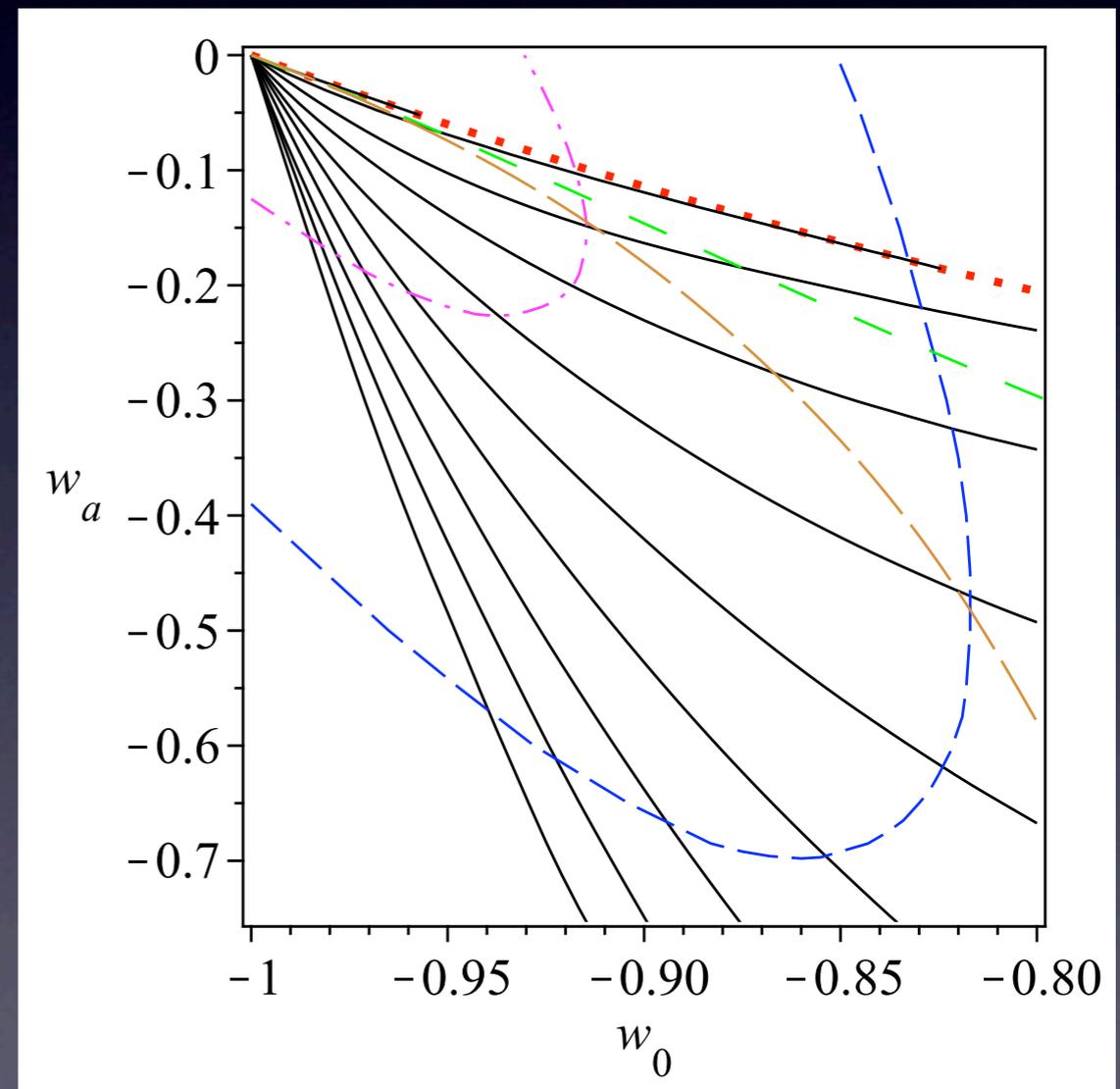
see e.g. Copeland et al. review
[hep-th/0603057](https://arxiv.org/abs/hep-th/0603057)

Thawing Models

Thawing: Field frozen away from minimum at early times by Hubble damping ($w \sim -1$) then thaw $w' > 0$

Thawing must be slow to be consistent with observations

$$w(a) = w_0 + w_a(1 - a)$$



Natural generalizations of quintessence:

k-essence

Armendariz-Picon et al. 2000

(modify kinetic term of quintessence field)

Assisted Quintessence

Kim et al. 2005

(many fields evolving in tandem)

Many attempts for example at
embeddings in Low Energy
Supergravity Models

e.g. Brax and Martin 1999

Many generalizations can be
incorporated into a unified EFT
approach for perturbations

Creminelli et al. 2008

Quintessence with a hint of modified gravity

Constraints on coupling of dark energy to visible matter are very strong (in absence of (kinetic) chameleon effects) but not so to dark matter!!

Vanilla model of dark sector interaction:

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{\text{pl}}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right) + S_{\text{visible}}[g_{\mu\nu}, \chi_i] + S_{\text{dark}}[\beta(\phi)g_{\mu\nu}, \xi_i]$$

N.B. Although we don't require the chameleon mechanism to be active here, that doesn't mean it won't take place!

or for example:

$$S_{\text{dark}} = \int d^4x \sqrt{-g} \left(-m(\phi) \bar{\psi} \psi + \mathcal{L}_{\text{kin}}(\psi) \right)$$

II. Making the coupling small environmentally

Theoretical Models: $\beta(\phi_b, \rho_b) \ll 1$

Symmetron - as yet no other explicit example

Symmetron

Consider a scalar with

Khoury and Hinterbichler 2010

1. Symmetry
2. Symmetry breaking potential
3. Non-minimal coupling to matter density

example

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} (\partial\phi)^2 - \frac{1}{4} \lambda \phi^4 + \frac{1}{2} \mu^2 \phi^2 + \mathcal{L}_M(g_{\mu\nu}(1 + \phi^2/M^2)) \right)$$

Z2 symmetry

$$\phi \rightarrow -\phi$$

Broken symmetry vev

$$\phi^2 = \mu^2/\lambda$$

Symmetron - effective potential

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\lambda\phi^4 + \frac{1}{2}\mu^2\phi^2 + \mathcal{L}_M(g_{\mu\nu}(1 + \phi^2/M^2)) \right)$$

As a result of non-minimal coupling, effective potential is

$$V_{\text{eff}}(\phi) = \frac{1}{2} \left(\frac{\rho}{M^2} - \mu^2 \right) \phi^2 + \frac{1}{4}\lambda\phi^4 \quad \beta \sim \frac{\phi M_{\text{PL}}}{M^2}$$

At low densities symmetry broken, coupling large

$$\rho < \mu^2 M^2 \quad \phi \sim \mu^2 / \lambda \quad \beta \sim \frac{\mu^2 M_{\text{Pl}}}{\lambda M^2}$$

At high densities symmetry recovered, coupling small

$$\rho > \mu^2 M^2 \quad \phi \sim 0 \quad \beta \sim 0$$

$$M \leq 10^{-3} M_{\text{Pl}} \quad \mu^{-1} \sim Mpc$$

III. Making the mass large environmentally

Theoretical Models:

Chameleon, Generalized Branes-Dicke models, $f(R)$

$$m(\phi_n, \rho_b) \gg \frac{1}{r_{exp}}$$

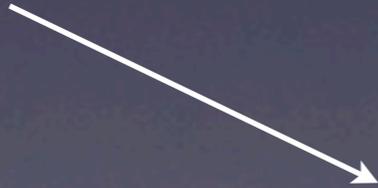
Chameleon effect

starts with
same idea:

Khoury and Weltman, 2003

$$S_{\text{cham}} = \int d^4x \sqrt{-g} \left(\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right) + S_{\text{matter}} \left[g e^{2\beta\phi/M_{\text{Pl}}} \right]$$

$$V_{\text{eff}}(\phi) = V(\phi) + \rho e^{\beta\phi/M_{\text{Pl}}}$$


$$m_{\text{eff}}^2 = V_{,\phi\phi}(\phi_{\text{min}}) + \frac{\beta^2}{M_{\text{Pl}}^2} \rho e^{\beta\phi/M_{\text{Pl}}}$$

Chameleon effect

$$m_{\text{eff}}^2 = V_{,\phi\phi}(\phi_{\text{min}}) + \frac{\beta^2}{M_{\text{Pl}}^2} \rho e^{\beta\phi/M_{\text{Pl}}}$$

Conditions necessary for chameleon mechanism to take place: $\beta > 0$

Balance

$$V_{,\phi} < 0$$

Stability

$$V_{,\phi\phi} > 0$$

m increase with density

$$V_{,\phi\phi\phi} < 0$$

easy to satisfy, e.g.

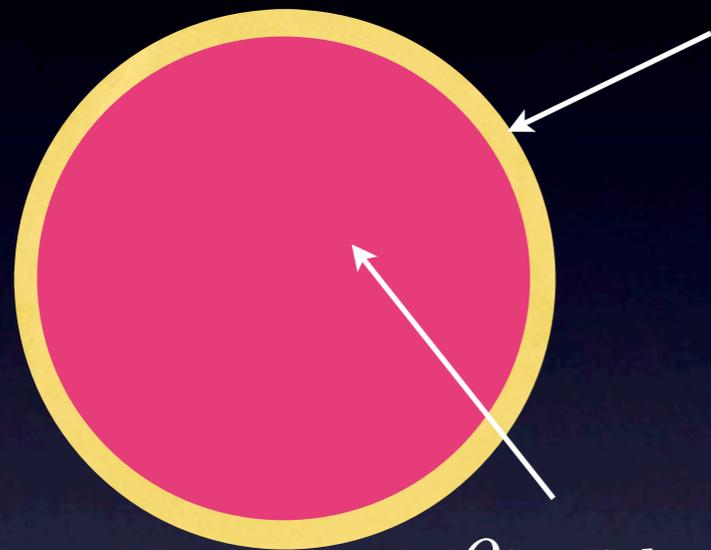
$$V(\phi) \sim \frac{M^{4+n}}{\phi^n}$$

To satisfy fifth force

$$M < 1\text{meV}$$

Thin shell effect

ρ_{outside}



Thin shell of gradient energy

$$\frac{\Delta R}{R} = \frac{1}{6\beta M_{\text{pl}}} \frac{\rho_{\text{inside}} \phi_{\text{outside}} - \phi_{\text{inside}}}{\Phi_N}$$

Khoury and Weltman 2004

ρ_{inside}

For most objects only a thin shell around the edge of the object contributes to the Newtonian potential

Cosmologically chameleon behaves like matter at early times and c.c. at late times (Like a freezing model)

If chameleons couple to photons (such couplings can be generated via loops) they affect astronomical observations of polarization and luminosity

Chameleons Issues

Naively not *technically natural*

$$m_\phi \sim \beta \frac{\Lambda_{UV}^2}{M_{\text{pl}}} \sim 1 \text{meV}$$

when

$$\beta \sim O(1)$$

$$\Lambda_{UV} \sim \text{TeV}$$

Adiabatic Instability (for
strongly coupled chameleons)

Bean et al. 2007

$$c_s^2 < 0$$

Type of Jeans instability, exponential
growth of small scale modes

IV. Making the kinetic term large environmentally

$$Z(\phi_b, \rho_b) \gg 1$$

Theoretical Models:

Vainshtein (or kinetic chameleon)
mechanism:

Massive Gravity, DGP, Cascading Gravity,
Galileon models and their generalizations!

Mechanism relies on a nontrivial reorganization of effective field theory to allow for large kinetic terms - arguably only natural in the context of massive gravity/DGP/Cascading

Vainshtein (Kinetic Chameleon) effect

$$\Lambda^3 \sim m^2 M_{\text{Pl}}$$

Allow in the action Irrelevant kinetic operators

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} (\partial\phi)^2 - \frac{1}{\Lambda^3} \square\phi (\partial\phi)^2 + \frac{\phi}{M_{\text{pl}}} \rho \right)$$

Expanding around background solution, generates large kinetic term

schematically: $\square\phi \sim \frac{\rho}{M_{\text{pl}}} \longrightarrow Z \approx 1 + \frac{\rho}{\Lambda^3 M_{\text{Pl}}}$

$$Z(\phi_b, \rho_b) \gg 1 \quad \text{when} \quad \rho_b \gg \Lambda^3 M_{\text{Pl}} \sim m^2 M_{\text{Pl}}^2$$

Galileon - a model that relies on Vainshtein

Logic: write down every term in
action consistent with symmetry

$$\pi \rightarrow \pi + c$$

$$\pi \rightarrow \pi + v_\mu x^\mu$$

$$\mathcal{L}_1 = \pi$$

$$\mathcal{L}_2 = -\frac{1}{2} \partial\pi \cdot \partial\pi$$

$$\mathcal{L}_3 = -\frac{1}{2} [\Pi] \partial\pi \cdot \partial\pi$$

$$\mathcal{L}_4 = -\frac{1}{4} ([\Pi]^2 \partial\pi \cdot \partial\pi - 2 [\Pi] \partial\pi \cdot \Pi \cdot \partial\pi - [\Pi^2] \partial\pi \cdot \partial\pi + 2 \partial\pi \cdot \Pi^2 \cdot \partial\pi)$$

$$\mathcal{L}_5 = -\frac{1}{5} ([\Pi]^3 \partial\pi \cdot \partial\pi - 3[\Pi]^2 \partial\pi \cdot \Pi \cdot \partial\pi - 3[\Pi][\Pi^2] \partial\pi \cdot \partial\pi + 6[\Pi] \partial\pi \cdot \Pi^2 \cdot \partial\pi + 2[\Pi^3] \partial\pi \cdot \partial\pi + 3[\Pi^2] \partial\pi \cdot \Pi \cdot \partial\pi - 6 \partial\pi \cdot \Pi^3 \cdot \partial\pi)$$

$$\Pi_\nu^\mu = \partial^\mu \partial_\nu \pi$$

Nicolis et al. 08 | 1.2 | 97

Self-acceleration
without ghosts!

Massive Gravity leads to light scalars

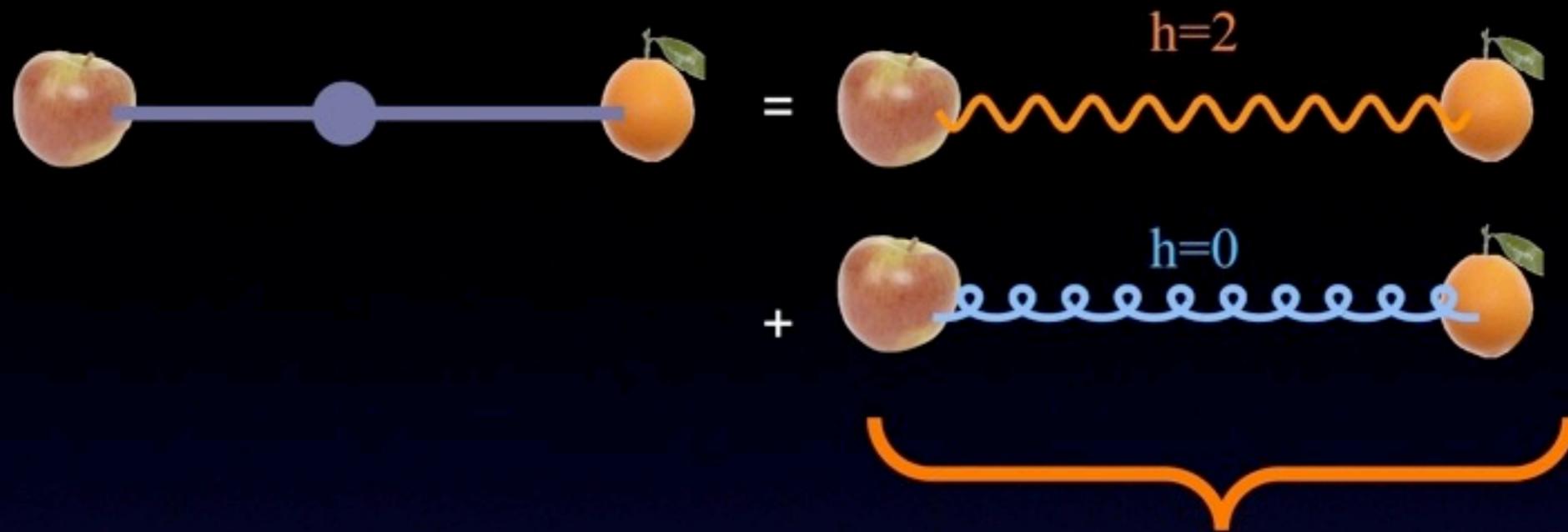
Massive spin-2 field, has 5 dof

$$h_{\mu\nu} \sim \frac{G_N}{\square_4 - m^2} T_{\mu\nu}$$

New scalar degree of freedom, exhibits Galileon symmetry and Galileon interactions

$$2 \oplus 1 \oplus 2$$

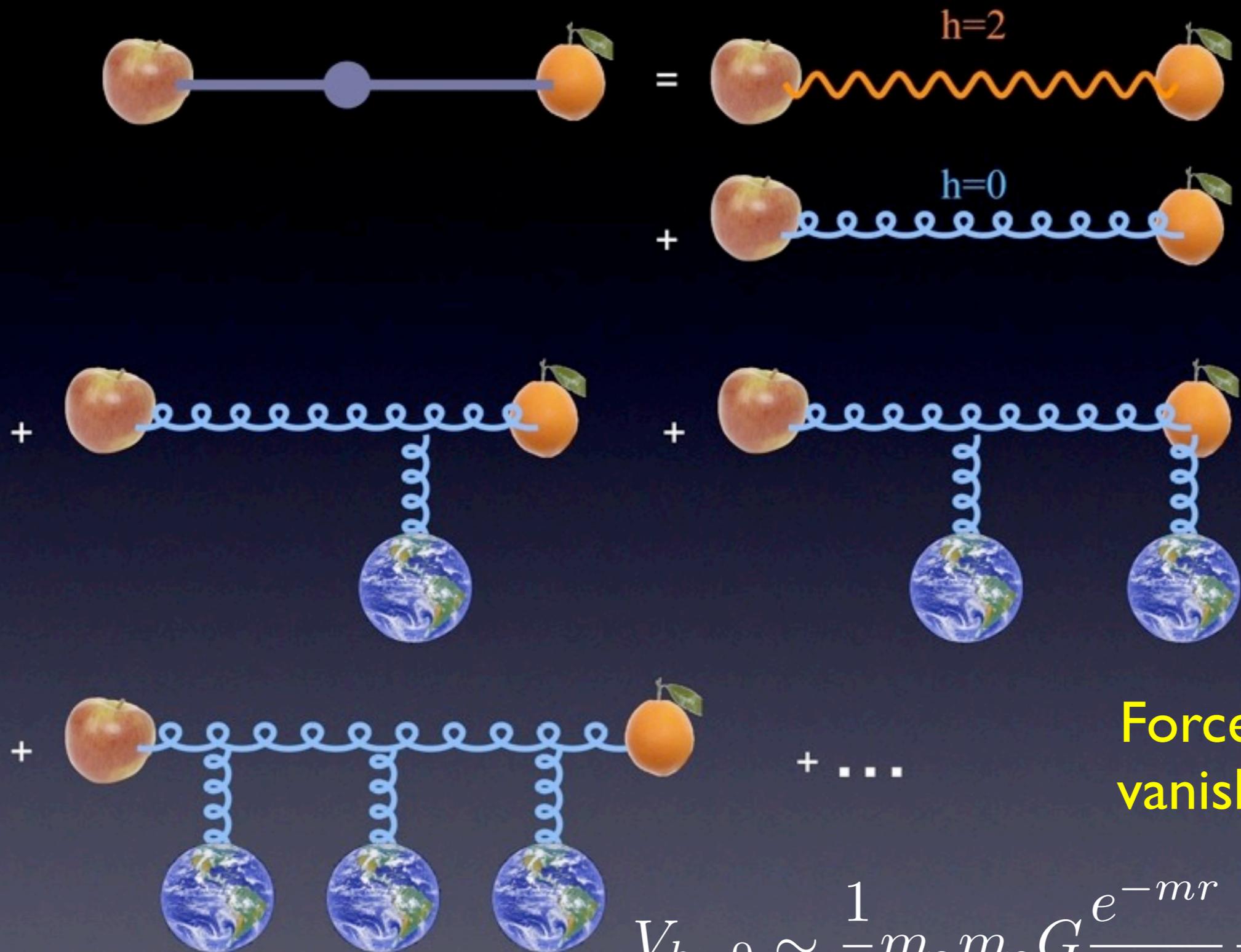

$$h_{\mu\nu} = h'_{\mu\nu} + \pi\eta_{\mu\nu}$$



Additional fifth force from scalar mode

$$V_{h=0} = \frac{1}{3} m_a m_o G \frac{e^{-mr}}{r}$$

at first sight such theories are ruled out!



Force now vanishes as

$$V_{h=0} \sim \frac{1}{3} m_a m_o G \frac{e^{-mr}}{r} \frac{1}{1 + \frac{\bar{\rho}_e}{m^2 M_{pl}^2}}$$

Vainshtein effect is strongly scale and density dependent

Characteristic radius from source

- Vainshtein radius

- helicity zero version of Schwarzschild radius

Strong coupling region

$$r \ll r_V$$

$$Z \gg 1$$

$$r_V = (r_s m^{-2})^{1/3}$$

$$\Lambda^3 \sim m^2 M_{\text{Pl}}$$

Weak coupling region

$$r \gg r_V$$

$$Z \sim 1$$

For Sun

$$r_V \sim 250 \text{ pc}$$

$$r_s \sim 3 \text{ km}$$

$$m^{-1} \sim 4000 \text{ Mpc}$$