

# Hide & Seek

In Modified Gravity



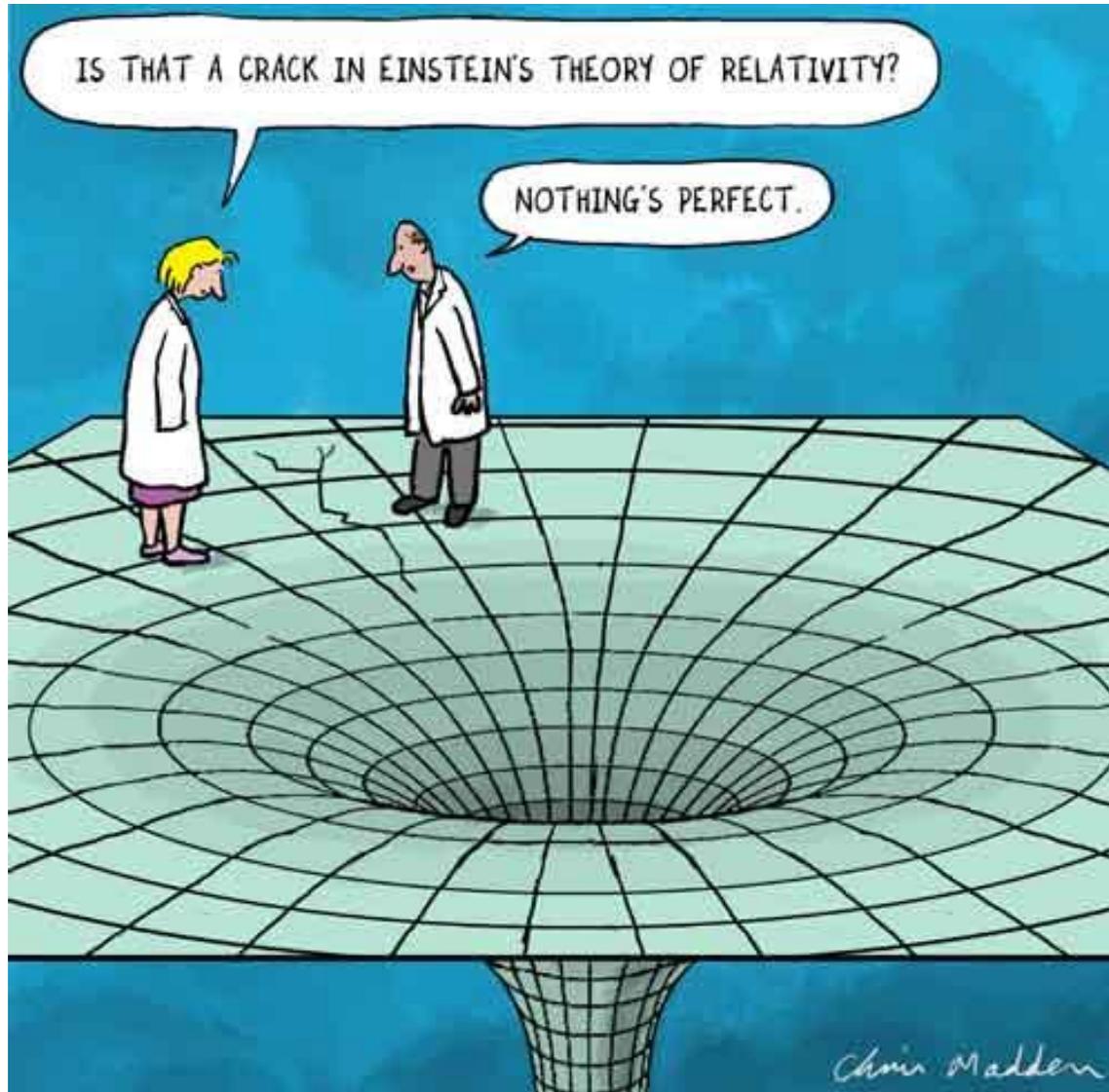
Laboratory Tests of Dark Energy

October, 28th 2011

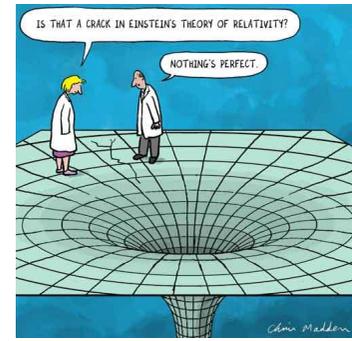
Claudia de Rham  
Université de Genève



# Modifying Gravity in IR



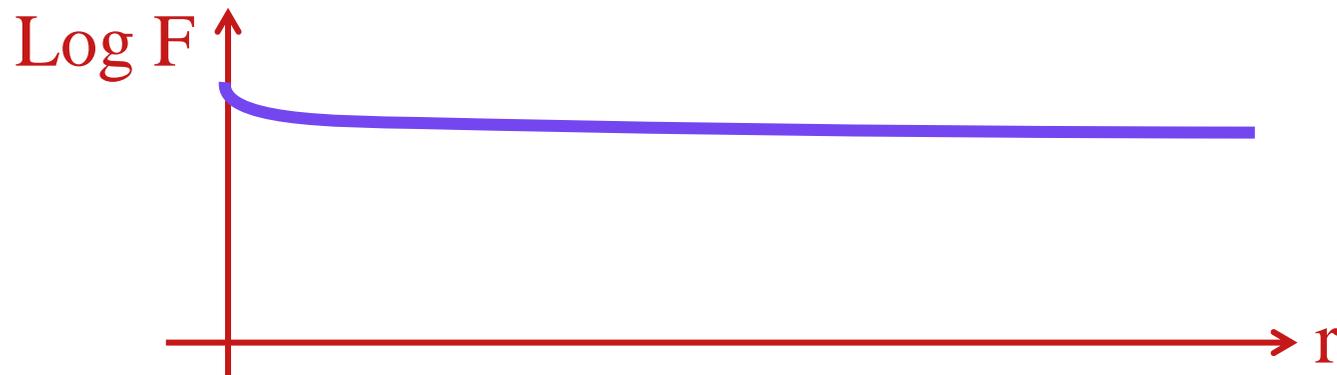
# Modifying Gravity in IR



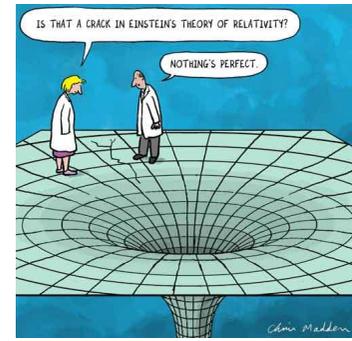
- One of the most natural way to modify Gravity in the IR is to give the graviton a mass (or resonance)

In GR, Gravity has an infinite range

$$F = \partial_r \left( \frac{M_1 M_2 G_N}{r} \right)$$



# Modifying Gravity in IR



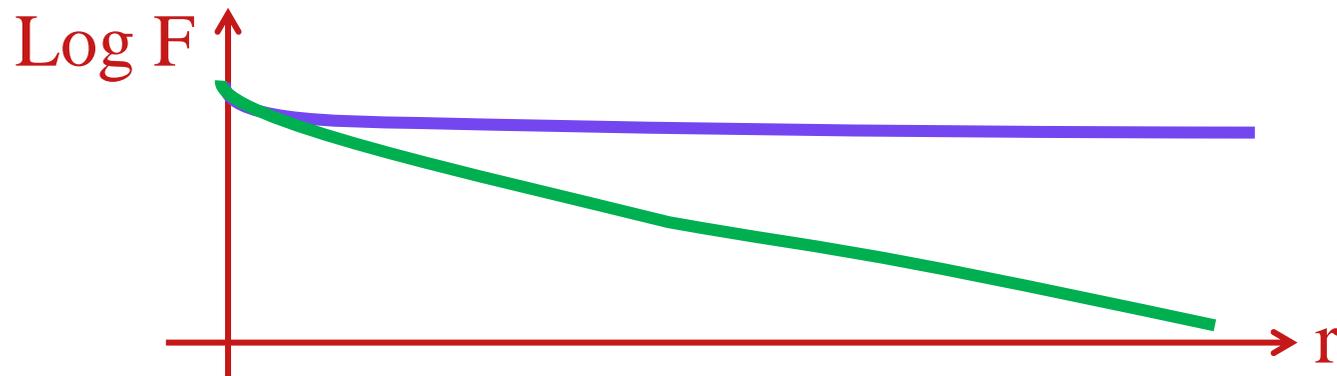
- One of the most natural way to modify Gravity in the IR is to give the graviton a mass (or resonance)

In GR, Gravity has an infinite range

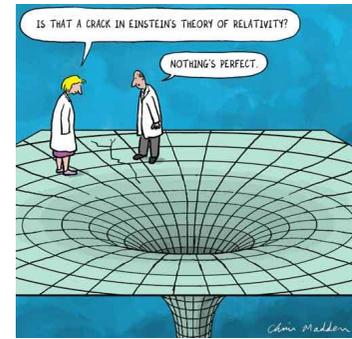
$$F = \partial_r \left( \frac{M_1 M_2 G_N}{r} \right)$$

In massive gravity, the force “shuts down” at some distance  $\lambda \sim m^{-1}$

$$F = \partial_r \left( \frac{M_1 M_2 G_N e^{-mr}}{r} \right)$$

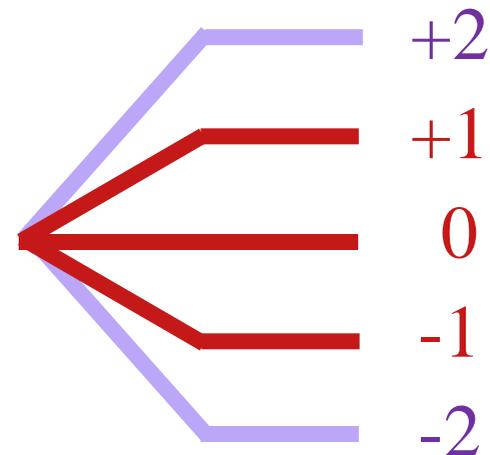
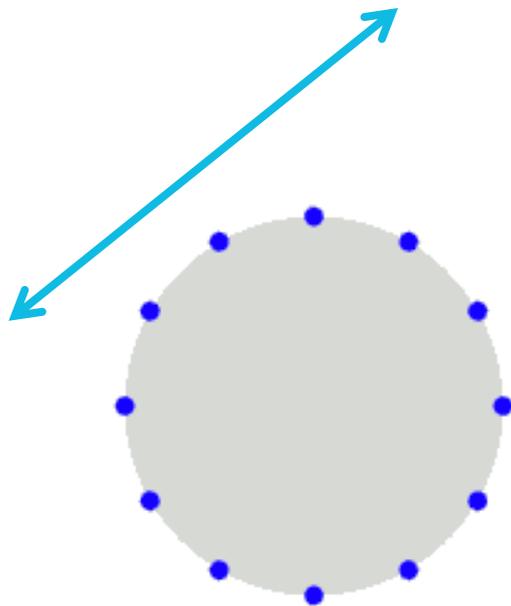


# Massive Gravity

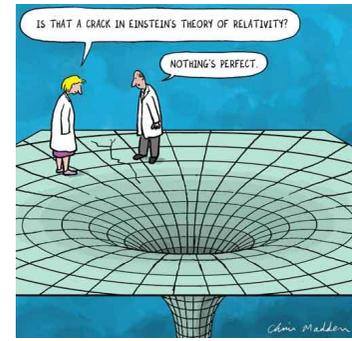


 In GR, the graviton has 2 polarizations

 In massive gravity, the graviton has 5 dof



# Massive Gravity

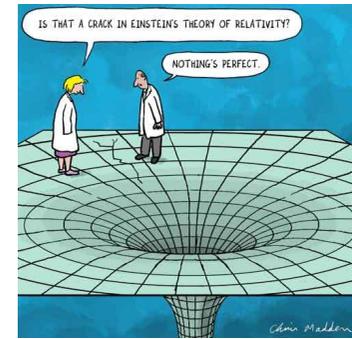


- 🔹 In GR, the graviton has 2 polarizations
- 🔹 In massive gravity, the graviton has 5 dof
- 🔹 These extra dof do not disappear in the massless limit  
 $m \rightarrow 0$

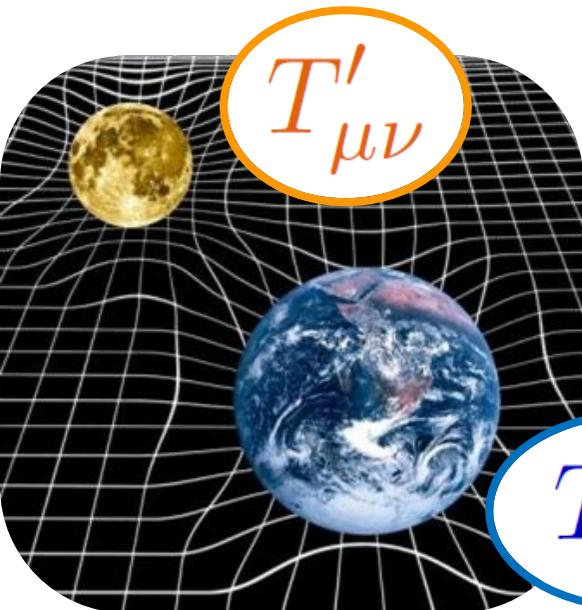
vDVZ discontinuity

van Dam-Veltman-Zakharov, 1970

# Massive Gravity



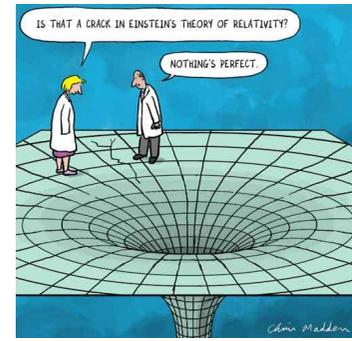
- ✿ In GR, the graviton has 2 polarizations
- ✿ In massive gravity, the graviton has 5 dof
- ✿ These extra dof do not disappear in the massless limit



$$\mathcal{A}_{\text{GR}} = T'_{\mu\nu} \frac{G_N}{\square} \left( T^{\mu\nu} - \frac{1}{2} T \eta^{\mu\nu} \right)$$
$$\mathcal{A}_m = T'_{\mu\nu} \frac{G_N}{\square - m^2} \left( T^{\mu\nu} - \frac{1}{3} T \eta^{\mu\nu} \right)$$

van Dam-Veltman-Zakharov, 1970

# Massive Gravity



- 🔹 In GR, the graviton has 2 polarizations
- 🔹 In massive gravity, the graviton has 5 dof
- 🔹 These extra dof do not disappear in the massless limit

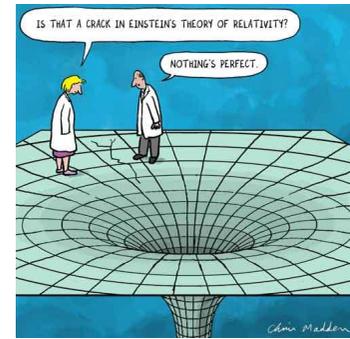
eg. for a static source,

for a radiation,

$$V_{\text{GR}}(r) = -\frac{G_N M}{r}$$
$$V_m(r) = -\frac{4}{3} \frac{G_N M}{r}$$

$$V_{\text{GR}}(r) = -\frac{G_N \hbar \nu}{r}$$
$$V_m(r) = -\frac{G_N \hbar \nu}{r}$$

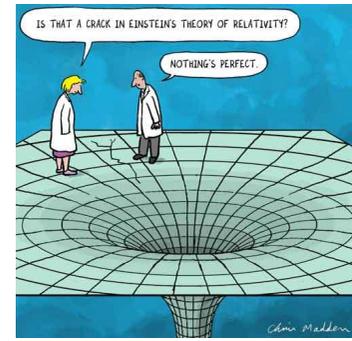
# Vainshtein Mechanism



- ✦ The resolution of the vDVZ discontinuity lies in the non-linear effects for the extra dof.
- ✦ This dof are screened at high energy via the Vainshtein mechanism
  - interactions of extra dof remain important down to much lower energy scale  $\Lambda \ll M_{\text{Pl}}$
  - effective coupling to matter depends on these interactions

$$\square \delta \pi \sim \frac{1}{M_{\text{Pl}}} \frac{1}{\left(1 + \frac{\partial^2 \pi}{\Lambda^3}\right)} J$$

# Vainshtein Mechanism



The Vainshtein mechanism can be seen explicitly at work in **Galileon** models

interactions

$$\square \delta\pi \sim \frac{1}{M_{\text{Pl}}} \frac{1}{\left(1 + \frac{\partial^2 \pi}{\Lambda^3}\right)} J$$

# How to Cook a Galileon:

Consider a scalar field  $\pi$ , living in flat spacetime

Consider all the interactions that:

1. Are local

$$\mathcal{L} = f(\pi, \partial\pi, \partial^2\pi, \partial^3\pi, \dots)$$

# How to Cook a Galileon:

Consider a scalar field  $\pi$ , living in flat spacetime

Consider all the interactions that:

1. Are local
2. Respect the shift and Galilean symmetry:  $\pi \rightarrow \pi + v_\mu x^\mu$

$$\mathcal{L} = f(\cancel{\pi}, \cancel{\partial\pi}, \partial^2\pi, \partial^3\pi, \dots) + (\partial\pi)^2 f_2(\partial^2\pi, \partial^3\pi, \dots)$$

# How to Cook a Galileon:

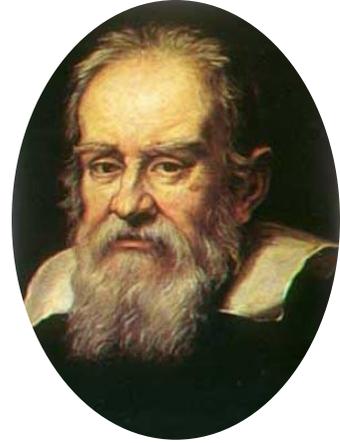
Consider a scalar field  $\pi$ , living in flat spacetime

Consider all the interactions that:

1. Are local
2. Respect the shift and Galilean symmetry:  $\pi \rightarrow \pi + v_\mu x^\mu$
3. Do not induce any ghost (are 2<sup>nd</sup> order in eom)

$$\mathcal{L} = f(\cancel{\pi}, \cancel{\partial\pi}, \cancel{\partial^2\pi}, \cancel{\partial^3\pi}, \cancel{\cdot}) + (\partial\pi)^2 f_2(\partial^2\pi, \cancel{\partial^3\pi}, \cancel{\cdot})$$

# Galileon Interactions



1. Are local
2. Respect the shift and Galilean symmetry:  $\pi \rightarrow \pi + v_\mu x^\mu$
3. Do not induce any ghost (are 2<sup>nd</sup> order in eom)

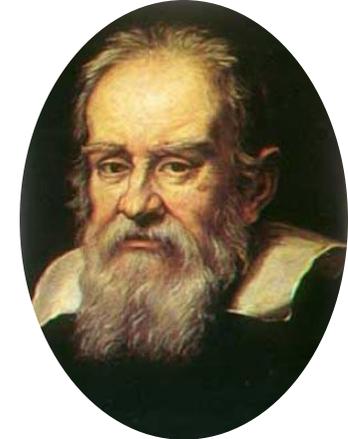
$$\mathcal{L}_2 = (\partial\pi)^2$$

$$\mathcal{L}_3 = (\partial\pi)^2 \square\pi$$

$$\mathcal{L}_4 = (\partial\pi)^2 \left( (\square\pi)^2 - (\partial_\mu\partial_\nu\pi)^2 \right)$$

$$\mathcal{L}_5 = (\partial\pi)^2 \left( (\square\pi)^3 - 3(\partial_\mu\partial_\nu\pi)^2 \square\pi + 2(\partial_\mu\partial_\nu\pi)^3 \right)$$

# Galileon Interactions



1. Are local
2. Respect the shift and Galilean symmetry:  $\pi \rightarrow \pi + v_\mu x^\mu$
3. Do not induce any ghost (are 2<sup>nd</sup> order in eom)

$$\mathcal{L}_2 = (\partial\pi)^2$$

$$\mathcal{L}_3 = (\partial\pi)^2 \square\pi$$

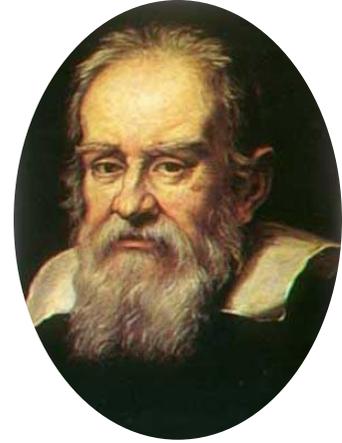
$$\mathcal{L}_4 = (\partial\pi)^2 ((\square\pi)^2 - (\partial_\mu\partial_\nu\pi)^2)$$

$$\mathcal{L}_5 = (\partial\pi)^2 ((\square\pi)^3 - 3(\partial_\mu\partial_\nu\pi)^2\square\pi + 2(\partial_\mu\partial_\nu\pi)^3)$$

The Galileon could mimic d.e.  
But remains shy on  
shorter distance scales

$$\mathcal{L}_{\text{Galileon}} = \sum_{n=2}^5 \frac{c_n}{\Lambda^{3(n-2)}} \mathcal{L}_n + \pi T$$

# Galileon Interactions

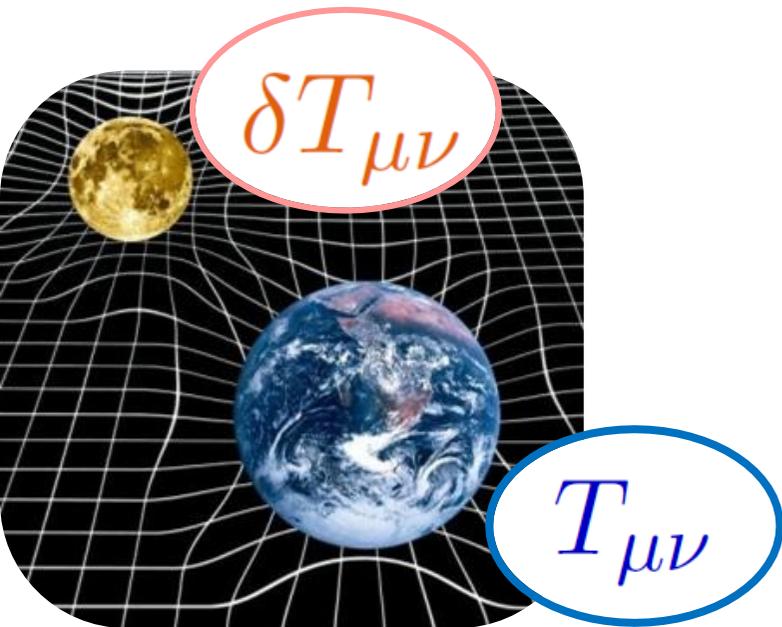


$$\mathcal{L}_{\text{Galileon}} = \sum_{n=2}^5 \frac{c_n}{\Lambda^{3(n-2)}} \mathcal{L}_n + \pi T$$

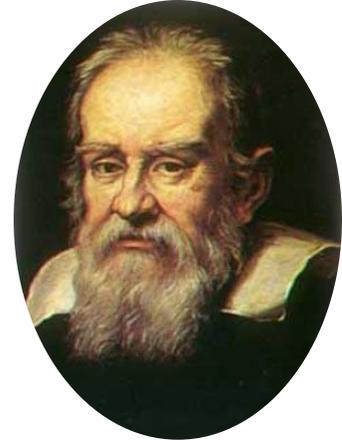
Consider a localized source

$$T = M\delta^{(3)}(r) + \delta T$$

$$\pi = \pi_0(r) + \phi(x^\mu)$$



# Galileon Interactions



$$\mathcal{L}_{\text{Galileon}} = \sum_{n=2}^5 \frac{c_n}{\Lambda^{3(n-2)}} \mathcal{L}_n + \pi T$$

Consider a localized source  $T = M\delta^{(3)}(r) + \delta T$

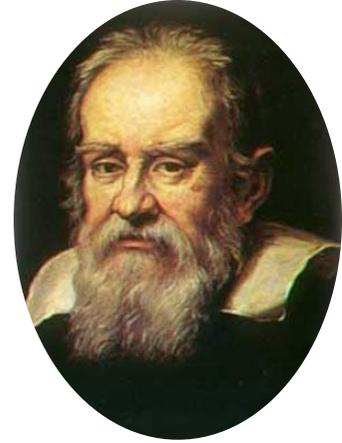
$$\pi = \pi_0(r) + \phi(x^\mu)$$

$\delta T_{\mu\nu}$

$$\mathcal{L} = \left( 1 + \frac{c_3}{\Lambda^3} \partial^2 \pi_0 + \frac{c_4}{\Lambda^6} (\partial^2 \pi_0)^2 + \dots \right) \partial\phi\partial\phi + \phi\delta T$$

$T_{\mu\nu}$

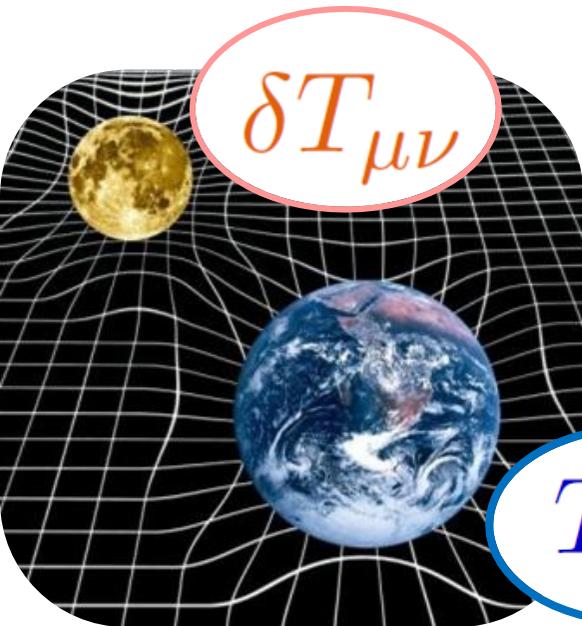
# Galileon Interactions



$$\mathcal{L}_{\text{Galileon}} = \sum_{n=2}^5 \frac{c_n}{\Lambda^{3(n-2)}} \mathcal{L}_n + \pi T$$

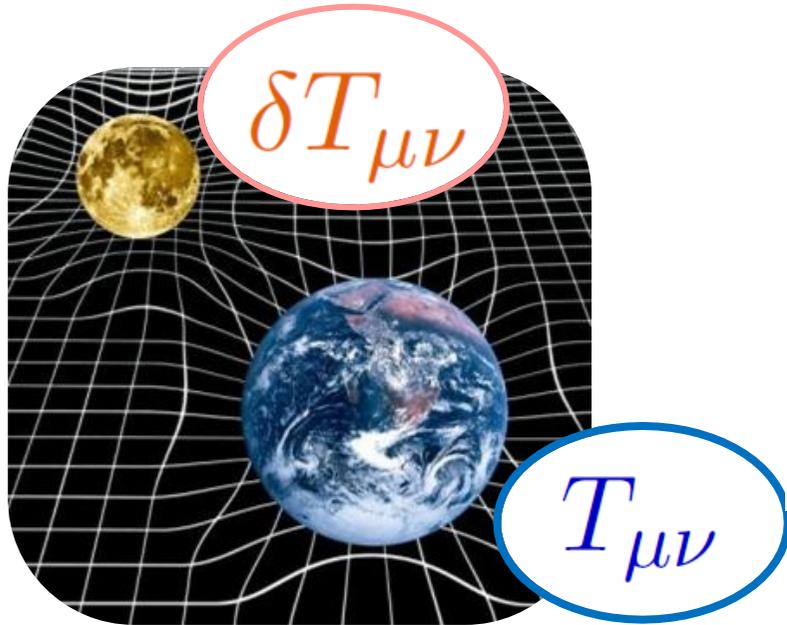
Consider a localized source  $T = M\delta^{(3)}(r) + \delta T$

$$\pi = \pi_0(r) + \phi(x^\mu)$$



$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{\phi\delta T}{\left(1 + \frac{c_3}{\Lambda^3}\partial^2\pi_0 + \frac{c_4}{\Lambda^6}(\partial^2\pi_0)^2 + \dots\right)}$$

# How big is the force

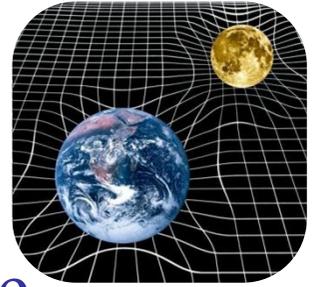


$$\frac{1}{\Lambda^3} \pi'' \sim \sqrt{\frac{M_{\text{Earth}}}{\Lambda^3 r^3}}$$

On the Moon, force is suppressed by a factor  $\left(\frac{M_{\text{Earth}}}{\Lambda^3 r_{\text{Moon}}^3}\right)^{-1/2} \sim 10^{-12}$

On Earth, force is suppressed by a factor  $\left(\frac{M_{\text{Earth}}}{\Lambda^3 r_{\text{Earth}}^3}\right)^{-1/2} \sim 10^{-15}$

# Kinetic Matrix

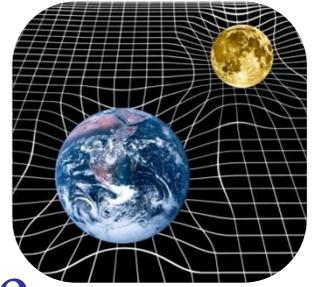


- 🔍 The Vainshtein mechanism works by modifying the Kinetic matrix

$$\mathcal{L} = \left( 1 + \frac{c_3}{\Lambda^3} \partial^2 \pi_0 + \frac{c_4}{\Lambda^6} (\partial^2 \pi_0)^2 + \dots \right) \partial \phi \partial \phi + \phi \delta T$$

1. The fifth force is still present, but suppressed  
LLR puts bounds on the scale  $\Lambda \lesssim 10^{-40} M_{\text{Pl}}$

# Kinetic Matrix



- 🌊 The Vainshtein mechanism works by modifying the Kinetic matrix

$$\mathcal{L} = \left( 1 + \frac{c_3}{\Lambda^3} \partial^2 \pi_0 + \frac{c_4}{\Lambda^6} (\partial^2 \pi_0)^2 + \dots \right) \partial \phi \partial \phi + \phi \delta T$$

1. The fifth force is still present, but suppressed  
LLR puts bounds on the scale  $\Lambda \lesssim 10^{-40} M_{\text{Pl}}$
2. The speed of the modes is medium-dependent !  
And is not the same along all directions (spontaneous breaking of Lorentz invariance)
  - some directions are typically superluminal
  - some others are subluminal  $\longrightarrow$  Cerenkov radiation

# Away from spherical sym.



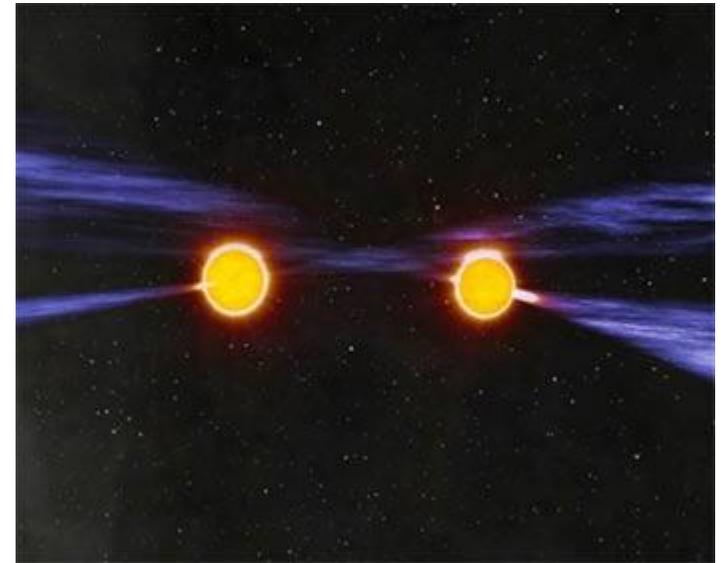
The Vainshtein mechanism is well understood analytically and numerically in **spherically symmetric configurations**

# Away from spherical sym.



🔹 The Vainshtein mechanism is well understood analytically and numerically in **spherically symmetric configurations**

🔹 When that symmetry is broken (eg. pulsars), the low speed along the angular directions implies that dipole and quadrupole radiation could be enhanced



🔹 Effects in labs and within the solar system ???

# The Galileon from Massive Gravity



So far the Galileon was considered as fundamental dof in its own right

The Galileon symmetry is broken when coupled to gravity

How far can we trust the Galileon description  
at higher energy scales ??

# The Galileon from Massive Gravity



- So far the Galileon was considered as fundamental dof in its own right
- In its natural habitat, the Galileon really arises as the helicity-0 mode of the graviton
- In DGP and massive gravity, the Galileon arises in the decoupling limit,

$$M_{\text{Pl}} \rightarrow \infty, m \rightarrow 0 \text{ with } m^2 M_{\text{Pl}} \rightarrow \text{const}$$

$$\Lambda^3 = m^2 M_{\text{Pl}}$$

# Massive Gravity



🔹 In the decoupling limit,  $M_{\text{Pl}} \rightarrow \infty$ ,  $m \rightarrow 0$  with  $m^2 M_{\text{Pl}} \rightarrow \text{const}$   
massive gravity takes the form

$$\mathcal{L} = h^{\mu\nu} \partial^2 h_{\mu\nu} + h^{\mu\nu} \left( X_{\mu\nu}^{(1)} + \frac{\alpha_2}{\Lambda^3} X_{\mu\nu}^{(2)} + \frac{\alpha_3}{\Lambda^6} X_{\mu\nu}^{(3)} \right) + h_{\mu\nu} T^{\mu\nu}$$

$\alpha_2$  and  $\alpha_3$  are the only free parameters

With  $X_{\mu\nu}^{(1)} = \partial_\mu \partial_\nu \pi - \square \pi \eta_{\mu\nu}$

$$X_{\mu\nu}^{(2)} = (\partial_\mu \partial_\nu \pi)^2 + \dots$$

$$X_{\mu\nu}^{(3)} = (\partial_\mu \partial_\nu \pi)^3 + \dots$$

# Massive Gravity



1. If  $\alpha_3 = 0$  the theory is diagonalizable, and resemble the Galileon,

$$\mathcal{L} = \frac{3}{2}\mathcal{L}_2 + \frac{3\alpha_2}{2\Lambda^3}\mathcal{L}_3 + \frac{\alpha_2^2}{2\Lambda^6}\mathcal{L}_4 + \underbrace{\left(\pi\eta_{\mu\nu} + \frac{\alpha_2}{\Lambda^3}\partial_\mu\pi\partial_\nu\pi\right)}_{\text{Characteristic}} T^{\mu\nu}$$

# Massive Gravity



1. If  $\alpha_3 = 0$  the theory is diagonalizable, and resemble the Galileon,
2. If  $\alpha_3 \neq 0$  the theory is not generically diagonalizable, but when it is, it comes with **specific new interactions...**

# Massive Gravity



1. If  $\alpha_3 = 0$  the theory is diagonalizable, and resemble the Galileon,
2. If  $\alpha_3 \neq 0$  the theory is not generically diagonalizable, but when it is, it comes with **specific new interactions...**

Eg. around spherically symmetric configurations,

$$y + \frac{\alpha_2}{\Lambda^3} y^2 + \frac{\alpha_2^2 + \alpha_3}{\Lambda^6} y^3 + \frac{\alpha_3 \alpha_2}{\Lambda^9} y^4 + \frac{\alpha_3^2}{\Lambda^{12}} y^5 = M$$

$\downarrow$                      $\downarrow$                      $\downarrow$                      $\downarrow$                      $\downarrow$

$\mathcal{L}_2$                      $\mathcal{L}_3$                      $\mathcal{L}_4$                      $\mathcal{L}_5$                     ???                     $y = \pi'(r)/r$

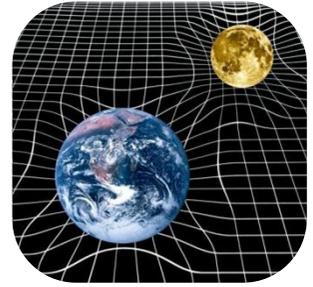
# Massive Gravity



1. If  $\alpha_3 = 0$  the theory is diagonalizable, and resemble the Galileon,
2. If  $\alpha_3 \neq 0$  the theory is not generically diagonalizable, but when it is, it comes with **specific new interactions...**
3. If  $\alpha_2 = \alpha_3 = 0$ , the interactions for the helicity-0 mode arise at a higher energy scale

$$\Lambda > (m^2 M_{\text{Pl}})^{1/3}$$

# Massive Gravity

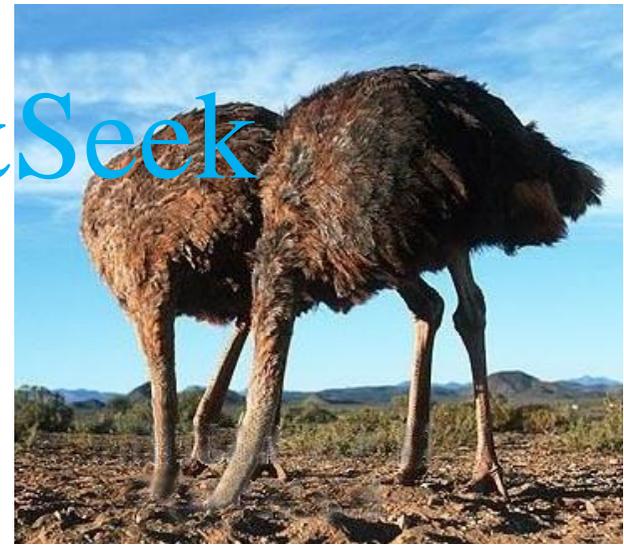


1. If  $\alpha_3 = 0$  the theory is diagonalizable, and resemble the Galileon,
2. If  $\alpha_3 \neq 0$  the theory is not generically diagonalizable, but when it is, it comes with **specific new interactions...**
3. If  $\alpha_2 = \alpha_3 = 0$ , the interactions for the helicity-0 mode arise at a higher energy scale

$$\Lambda > (m^2 M_{\text{Pl}})^{1/3}$$

**Implications for lab tests and solar system tests are not fully understood...**

# Massive Gravity Hide&Seek



- ▼ A natural framework for the Galileon can be found within Massive Gravity.
- ▼ Galileon present a natural realization of the Vainshtein mechanism. *This mechanism seems to come in hands with:*
  - Specific astronomical signatures such as LLR, weak lensing...
  - Already quite well constrained by solar system tests
  - Modifications of the speed of propagation around dense objects
- ▼ Massive Gravity comes with:
  - specific couplings between the different Galileon terms
  - specific couplings with matter
  - potential new interactions