Shedding Light on Dark Matter: How Faraday Rotation Can Limit a Dark Magnetic Moment

Susan Gardner

Department of Physics and Astronomy
University of Kentucky
Lexington, KY 40506

gardner@pa.uky.edu

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UNIVERSITY OF KENTUCKY

Context

Dark matter is observed through its gravitational interactions, but to know its nature we probe it with <u>other</u> (i.e., electromagnetic, weak) interactions.

E.g., we constrain DM through its putative annihilation to γ, e^+, ν, ... channels.

But we cannot really answer, "How <u>dark</u> is 'dark'?"

[with thanks to Sigurdson et al., PRD 70, 083501 (2004).]

Suppose a dark matter particle, though electrically neutral, has a non-zero magnetic moment μ.
How could such be observed?

$\Longrightarrow$ Enter the gyromagnetic Faraday effect. [SG, astro-ph/0611684, PRL 100, 041303 (2008).]

Outline

- Why would one want to look?

  The Universe we know is *complex*.

- Whence a dark magnetic moment?
- What limits on electromagnetic interactions already exist?
- What is the gyromagnetic Faraday effect?

  First review "usual" Faraday effect in the ISM.

- How can one use the gyromagnetic Faraday effect to constrain DM properties?

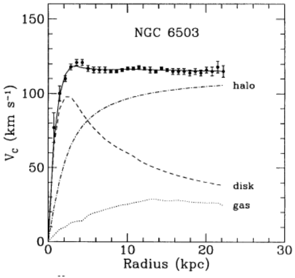
  Through the measured polarization of the CMB radiation.

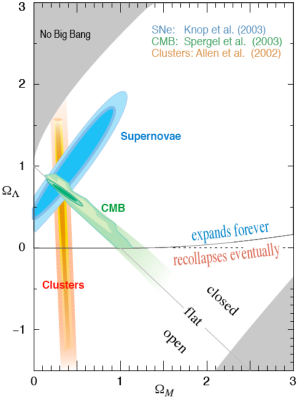
  Through terrestrial optical rotation studies.
  Enter a PVLAS-*like* experiment.

- How stringent are the attainable constraints?

Why Look for a DM Magnetic Moment?

Overwhelming evidence exists in support of “dark matter”.
Note galactic rotation curves [V. Rubin et al., astro-ph/9904050]
and a cosmic “concordance” [PDG, RPP, 2006.]

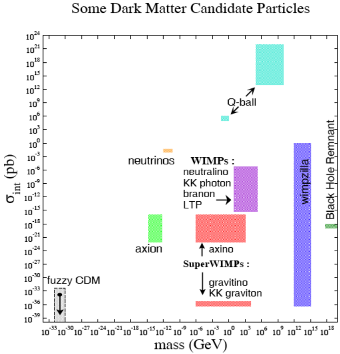




The WIMP “miracle” tells us a single species can reproduce this relic density if $M_{\rm WIMP} \sim \mathcal{O}(100)$ GeV.
However, the Universe we know is complex. A variety of particles exist, many with Dirac and/or Pauli moments.

Whence a Dark Magnetic Moment?

All DM candidates are exotic....



[E.-K. Park, contribution to DMSAG report, July, 2007.]

Majorana fermions have no magnetic moments if CPT is conserved $\Longrightarrow$
SUSY models do not yield DM magnetic moments!?

However, one can realize Dirac neutralinos in SUSY models.

[Haber and Kane (1985); Fox, Nelson, and Weiner (2002)]

Extra dimension models can also yield Dirac DM.

[Arkani-Hamed et al. (2002); Servant and Tait (2003), Cheng, Feng, Matchev (2002); Hsieh, Mohapatra, Nasri (2006)]

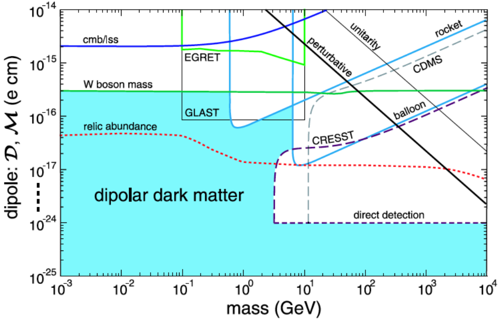
Existing Limits on a DM Magnetic Moment

The limits on the recoil signal from DAMA data (NaI detector) yields $\mu/\mu_N < 1.4 \cdot 10^{-4}$ for a WIMP mass of 100 GeV.

[Pospelov and ter Veldhuis, PLB 480 (2000) 181.]

The constraints on the electric dipole moment and magnetic dipole moment of a DM particle as a function of mass have also been analyzed.

[Sigurdson et al., PRD 70, 083501 (2004); PRD 73, 089903 (E) (2006).]



Only "W mass" constraint is deemed reliable by Sigurdson et al.

N.B. $3 \cdot 10^{-16}$e cm $\sim 1.6 \cdot 10^{-5}\mu_B$.

Existing Limits on a DM Magnetic Moment

For a DM particle of mass $\mathcal{O}(1\text{GeV})$ or less, the best terrestrial limit comes from precision electroweak constraints: [Sigurdson et al., PRD 70, 083501 (2004); PRD 73, 089903 (E) (2006).]

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F}\frac{1}{(1 - M_W^2/M_Z^2)(1-\Delta r)}$$

$$\Delta r^{\rm SM} = 0.0355 \pm 0.0019 \pm 0.0002 \quad ; \quad \Delta r^{\rm exp} = 0.0326 \pm 0.0023$$

$$\Delta r^{\rm new} < 0.003 \quad \text{at} \quad 95\%\text{CL} \quad \text{(sic)}$$

Now [Profumo and Sigurdson, PRD 75, 023521 (2007)]

$$\mathcal{L}_{\rm eff} = -\frac{i}{2}\bar{\chi}\sigma_{\mu\nu}\frac{a + b\gamma_5}{\tilde{M}}\chi F^{\mu\nu}$$

yields via $\Pi^{\mu\nu}(q^2) = \Pi(q^2)(q^2 g^{\mu\nu} - q^\mu q^\nu)$ for $\tilde{M} \ll M_Z$

$$\Delta r = \Pi(M_z^2) - \Pi(0) - M_Z^2\left(\frac{\partial\Pi(q^2)}{\partial q^2}(0)\right) \simeq \frac{M_Z^2}{3\pi^2 M^2} \quad \text{with} \quad M^2 \equiv \frac{\tilde{M}^2}{|a|^2 + |b|^2}$$

Existing Limits on a DM Magnetic Moment

Thus

$$\Delta r^{\rm new} < 0.003$$

yields

$$M \gtrsim 3.4 M_Z \Longrightarrow \frac{a}{2\tilde{M}} < 6 \cdot 10^{-6} \mu_B$$

With $b = 0$, $a = \kappa e$, $\bar{a} \equiv (M_Z/\tilde{M})^2$, and with $\tilde{M} \ll M_Z$ I find

$$\Delta r = -\frac{\kappa^2 \alpha}{4\pi} \left(\frac{\bar{a}}{6} \log \bar{a} - \frac{\bar{a}}{9} \right)$$

Using [PDG, 2007 partial update]

$$|\Delta r^{\rm new}| < 0.0010 \quad \text{at} \quad 95\%\text{CL}$$

yields weakly varying bounds with $\tilde{M}$,

$$\tilde{M} \sim m_p \,, \quad \mu \lesssim 0.013 \mu_N \approx 6 \cdot 10^{-6} \mu_B \ \text{ and } \ \tilde{M} \sim m_e \,, \quad \mu \lesssim 4 \cdot 10^{-6} \mu_B$$

This constraint can be evaded! Enter the **neutron**. (PDG 2006)

$$\mu = -1.9130427 \pm 0.0000005 \mu_N \approx 1 \cdot 10^{-3} \mu_B$$

Existing Limits on a DM Magnetic Moment

Keeping the neutron in mind, we note the apparent constraint can be evaded by compositeness.

The precision ew constraint is thus modified to

$$\Delta r = -\frac{\kappa^2\alpha}{4\pi}\left(\frac{\bar{a}}{6}\log\bar{a} - \frac{\bar{a}}{9}\right)(1 - M_Z/M_c)^{-4}$$

where M_c is the compositeness scale. For $M_c \sim 10$ GeV, e.g.,

$$\tilde{M} \sim m_p\,, \quad \mu \lesssim 0.013\mu_N \approx 6\cdot 10^{-6}\mu_B \to 0.7\mu_B$$
$$\tilde{M} \sim m_e\,, \quad \mu \lesssim 4\cdot 10^{-6}\mu_B \to 0.03\mu_B$$
$$\tilde{M} \sim m_e/10\,, \quad \mu \lesssim 3\cdot 10^{-6}\mu_B \to 0.02\mu_B$$

For $M_c \sim 2$ GeV, the last bound relaxes to 15 μ_B.

Other mechanisms for relaxing the precision ew bounds exist.

We now turn to how we might constrain the DM μ directly.

The (Gyroelectric) Faraday Effect

A medium with free charges becomes *circularly birefringent* if $|\mathbf{B_0}| \neq 0$.
An electron with charge $-e$ and mass m suffers a displacement $\mathbf{s}$

$$m\ddot{\mathbf{s}} = -e(\mathbf{E} + \dot{\mathbf{s}} \times \mathbf{B}_{\rm tot}) \,,$$

and $\mathbf{B}_{\rm tot} = \mathbf{B}_0 + \mathbf{B}$. Let $\mathbf{B}_0 \parallel \hat{\mathbf{x}}$ and $\mathbf{E}(\mathbf{x}, t) = E_\pm \mathbf{e}_\pm \exp(ik_\pm x - i\omega t)$, with $\mathbf{e}_\pm \equiv \hat{\mathbf{y}} \pm i\hat{\mathbf{z}}$.
Assuming $|\mathbf{B}_0| \gg |\mathbf{B}|$, the steady-state solution for $\mathbf{s}$ yields, for a medium of electrons with number density n_e, the polarization $\mathbf{P} = -n_e e\mathbf{s}$ or $\mathbf{P}_\pm = \epsilon_0 \chi_{e\,\pm} \mathbf{E}_\pm$. Thus

$$\frac{\epsilon_\pm}{\epsilon_0} \equiv 1 + \chi_{e\,\pm} = 1 - \frac{\omega_P^2}{\omega(\omega \mp \omega_B)} \,,$$

where ω_P is the plasma frequency, $\omega_P^2 \equiv n_e e^2/\epsilon_0 m$ and $\omega_B = eB_0/m$.
With $k_\pm = (\omega/c)\sqrt{\epsilon_\pm/\epsilon_0}$ and with $\omega \gg \omega_B, \omega_P$, we have $\phi \equiv (k_+ - k_-)l/2 = -\omega_P^2 \omega_B l/2c\omega^2$ to leading order in ω.
Note ω^{-2} dependence!

The (Gyroelectric) Faraday Effect

Linearly polarized light propagating in the direction of $\mathbf{B_0}$ suffers a rotation

$$\phi = -\frac{e^3}{2c\omega^2\epsilon_0 m^2}\int_0^l dz\, n_e(z) B_0(z)$$

and a time delay $(\tau(\omega) = l/v_g,\; v_g^{-1} = dk_{\rm avg}/d\omega)$

$$\tau_{\rm delay} = \tau(\omega) - \lim_{\omega\to\infty}\tau(\omega) = \frac{e^2}{2c\omega^2\epsilon_0 m}\int_0^l dz\, n_e(z)$$

Studies of the ϕ and $\tau_{\rm delay}$ using radio pulsar sources

[A. G. Lyne and F. G. Smith, Nature **218**, 124 (1968).]

yield n_e and B_0 averaged along the line of sight in the warm ISM.

N.B. ω dependence makes knowledge of the source polarization unnecessary.

Modern surveys map the galactic magnetic field. [e.g., Han et al, ApJ 642 (2006) 868.]

Magnetic field strengths are of few μG scale.

Note $\langle n_e \rangle$ is of few $10^{-2}\,\mathrm{cm}^{-3}$ scale. For $\lambda \sim 20$ cm, ϕ is 10's of radians.

The Gyromagnetic Faraday Effect

A medium with free magnetic moments can become *circularly birefringent* if $|\mathbf{B_0}| \neq 0$. [D. Polder, Phil. Mag. **40**, 99 (1949).]

If $|\mathbf{B_0}|$ induces a magnetization $\mathbf{M}_{\rm tot}$, then

$$\dot{\mathbf{M}}_{\rm tot} = \gamma \mathbf{M}_{\rm tot} \times \mathbf{B}_{\rm tot} \,,$$

γ is the gyromagnetic ratio, $\mathbf{M}_{\rm tot} = \hat{\mathbf{x}} M_0 + \mathbf{M}$, and M_0 results from B_0 alone.

If the constituents carry an EDM as well, an additional term appears.

[Bargmann, Michel, Telegdi, PRL 2 (1959) 435.]

With $|\mathbf{B}_0| \gg |\mathbf{B}|$, $|\mathbf{M}_0| \gg |\mathbf{M}|$, the steady-state solution is

$$M_\pm = \pm \frac{\chi_0 \omega_B}{\omega \pm \omega_H} B_\pm \equiv \chi_\pm B_\pm \,,$$

where $\chi_0 \equiv M_0/B_0$ and $\omega_B \equiv \gamma B_0$. Recall χ_m obeys $\mathbf{M} = \chi_m \mathbf{B}$, so that

$$\frac{\mu_\pm}{\mu_0} \equiv 1 + \chi_{m\pm} = 1 \pm \frac{\chi_0 \omega_B}{\omega \pm \omega_B} \,,$$

where $k_\pm = (\omega/c)\sqrt{\mu_\pm/\mu_0}$.

The Gyromagnetic Faraday Effect

To leading order in ω_B/ω

$$k_{\rm diff} \equiv k_+ - k_- = \frac{\chi_0 \omega_B}{c} + \frac{\chi_0 \omega_B^3}{c\,\omega^2} + \frac{\chi_0^2 \omega_B^3}{2c\,\omega^2} + \dots$$

engenders Faraday rotation

$$k_{\rm avg} \equiv \frac{1}{2}(k_+ + k_-) = \frac{\omega}{c}\left(1 - \frac{1}{2}\chi_0\left(\frac{\omega_B}{\omega}\right)^2 - \frac{1}{8}\chi_0^2\left(\frac{\omega_B}{\omega}\right)^2 + \dots\right)$$

engenders time delay

The frequency-dependence of the rotation and the time delay are trivial.

In thermal equilibrium for a spin-1/2 system

$$M_0 = n_e \mu \tanh\left(\frac{\mu B_0}{k_B T}\right) \approx n_e \left(\frac{\mu^2 B_0}{k_B T}\right) \text{ with } \mu B_0 \ll k_B T$$

If $T = T(z)$, gyroelectric Faraday rotation and ϕ_0 share a common integral.

$$\phi_0 = \frac{\mu^2 \gamma}{2ck_B T}\int_0^l dz\, n_e(z) B_0(z)$$

The Gyromagnetic Faraday Effect

For **electrons**

$$|\tilde{\chi}| \equiv \frac{|\gamma|\mu^2}{k_B T} = \frac{|g|\mu^2 \mu_B}{\hbar k_B T} \sim 4.6 \cdot 10^{-19} \left[\frac{300\,\mathrm{K}}{T}\right] \frac{\mathrm{cm}^3}{\mathrm{G\,s}}$$

cf.

$$\chi \equiv \frac{e^3}{\omega^2 \epsilon_0 m^2} \sim 1.6 \cdot 10^{-6} \left[\frac{\lambda}{1\,\mathrm{cm}}\right]^2 \frac{\mathrm{cm}^3}{\mathrm{G\,s}},$$

The relative size of the two effects depends on wavelength and temperature.
In the warm ISM, $T \sim 5000^\circ$ K, with $\lambda = 6 - 20$ cm, the gyromagnetic Faraday effect is **negligible**.
The gyromagnetic Faraday effect can be much larger for dark matter.

- It is denser, and “clumpiness” helps.
- It can accrue over longer distances.

  Lighter candidate masses give larger rotations for fixed μ.

We shall consider $\mathcal{O}$(MeV) DM candidates henceforth.

[C. Boehm et al., PRL 92, 101301 (2004); Beacom, Bell, Bertone, PRL 94, 171301 (2005).]

We evade the Lee-Weinberg constraint thr. magnetic moment annihilation.
We note $M_{WDM} \geq 4$ keV (2σ) for thermal relics. [M. Viel et al., PRL 100, 041304 (2008).]

Faraday Effects on the CMB Polarization

To realize a constraint on the EDM or μ of a DM particle, we must use a photon source of known polarization.

Thus we turn to the polarization of the CMB.

Scalar gravitational perturbations give rise to E-mode ("gradient type") polarization exclusively.

There are countable sources of B-mode ("curl type") polarization.

The magnetic Faraday effect from DM acts as a "foreground" source of B-mode polarization, so that we wish to study frequency-independent, $E \cdot B$ correlations in the CMB polarization.

It is distinguishable from gravitational lensing, e.g., as it cannot impact the temperature correlations.

Although the polarization of the CMB was observed by DASI in 2002, B-mode polarization has not yet been observed.

[C. Pryke et al., QUaD, arXiv:0805.1944]

Magnetizing Dark Matter

A dilute gas in an external magnetic field polarizes through spontaneous emission.

The rate for this process can be extremely slow.

$$W = \frac{4}{3\hbar}\left(\frac{\omega}{c}\right)^3 (g\mu_B)^2 = g^5\,[B(\mu G)]^3 \cdot 1.087 \cdot 10^{-29}\,\mathrm{s}^{-1}$$

For $g \sim 10^{-3}$, $B \sim 1\mu$G, $W \sim 1.1 \cdot 10^{-44}\,\mathrm{s}^{-1}$ but $\tau_{\rm univ} \sim 13\,\mathrm{Gyr} \sim 4 \cdot 10^{17} s$.

We must magnetize DM while it is still in thermal equilibrium in the early Universe.

Once DM decouples there is no mechanism to polarize it, and concommitantly its primordial polarization cannot be lost.

The magnetization tracks the magnetic field as the Universe evolves, as $\omega_{\rm cycl} \gg \dot{B}/B$, where $B \sim B_{\rm emit} a^{-2}$ so that $\dot{B}/B$ is set by H_0!

A non-zero, frequency-independent B-mode polarization at small angular scales, which survives the removal of possible gravitational lensing contamination, can be directly attributed to the presence of DM.

Terrestrial Studies

Terrestrial studies are also possible and possibly yield better control on DM couplings.

- We can apply a strong magnetic field of known strength.
- Measurements of very small rotation angles are possible.
- Faraday rotation accrues coherently under momentum reversal.
- Vacuum pumps do not "pump" dark matter!

Enter the PVLAS experiment. Measure the polarization parameters of laser light after travel through vacuum in a magnetic field. [E. Zavattini et al., PRL 96, 110406 (2006)]
The proposed experiment differs crucially from the PVLAS experiment in that the applied magnetic field must be parallel and not perpendicular to the light.
Note PVLAS parameters. $l_{\rm eff} \sim 4.4 \cdot 10^6$ cm, $B_0 = 5\,\mathrm{T} = 5 \cdot 10^{10}\,\mu$G, $\delta\phi_0 \sim 1 \cdot 10^{-8}$ rad.
The uncertainty principle limits polarization measurements; one can do better. [D. Budker, priv. comm.]

N.B. the DM magnetization reflective of primordial magnetic fields is very small.

Terrestrial Studies

Natural time scale of photon polarization measurement implies that one can be sensitive to both daily and annual variations in the WIMP wind.

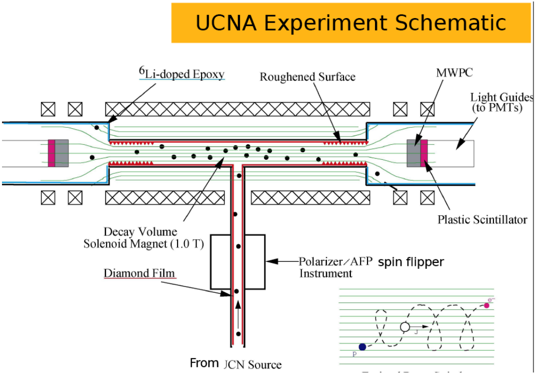
To realize a useful terrestrial experiment, one must "magnetize" the DM.

This can be done using a longitudinal Stern Gerlach device. This is used to polarize UCNs. The "wrong" spin is stopped by the magnetic fields.

[G. L. Greene, private communication.]

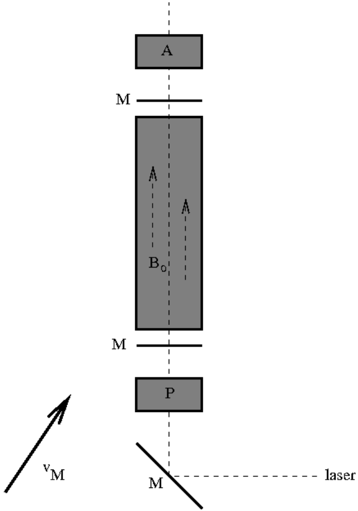
Polarizing Dark Matter

This can be done by applying a magnetic field along the WIMP's momentum in the Earth-based frame. This technique is used to polarize ultra-cold neutrons (UCNs) in the UCNA experiment at Los Alamos. The "wrong" (higher energy) spin state cannot enter the magnetic field region if it has a sufficiently low kinetic energy. [G. L. Greene, private communication.]



[slide from J. Martin.]

Schematic of a Faraday Rotation Experiment



Polarizing Dark Matter

Here we employ usual assumptions to assess the efficacy of the UCNA mechanism in this context. [S. Golwala, Ph.D. thesis (CDMS); J.D. Lewin and P.F. Smith, Astropart. Physics, 6 (1996) 87.]

Namely, the dark matter velocity distribution in the galaxy rest frame is an "isothermal sphere":

$f(\mathbf{v}_M, \mathbf{v}_E) = \frac{1}{\pi^{3/2} v_0^3} \exp(-(\mathbf{v}_M + \mathbf{v}_E)^2 / v_0^2)$.

$\mathbf{v}_M$ is the DM velocity with respect to the Earth.

$\mathbf{v}_E$ is the Earth velocity with respect to the nonrotating halo of the galaxy.

We must determine $\mathbf{v_E}$:

$\mathbf{v}_E = \mathbf{u}_r + \mathbf{u}_s + \mathbf{u}_e$

$\mathbf{u}_r$ is the velocity of the galactic disk.

$\mathbf{u}_s$ is the velocity of the Sun w.r.t. the galactic disk.

$\mathbf{u}_E$ is the velocity of the Earth about the Sun.

We ignore the Earth's rotation about its axis as it is small cf. to errors.

In galactic coordinates, in km/s, to good approximation,

$\mathbf{u}_r = (0, 220, 0) \qquad \mathbf{u}_s = (9, 12, 7)$

$\mathbf{u}_e \cdot \hat{\mathbf{l}} = 30 \cos 60^\circ \cos(2\pi(\frac{t - 152.5}{365.25}))$

$\mathbf{v}_E = 232 + 15 \cos(2\pi(\frac{t - 152.5}{365.25}))$.

Polarizing Dark Matter

To estimate the polarization we assume DM of mass M is incident on an interface perpendicular to $\mathbf{B}_0$.

Our polarization condition is $\mathbf{v}_M \cdot \hat{\mathbf{B}}_0 \leq v_{\rm stop}$ with $Mv_{\rm stop}^2/2 = \mu B_0$.

The DM fraction with 100% polarization is
$f_{\rm pol} = \frac{1}{2} \int d^3 v_M\, f(\mathbf{v}_M, \mathbf{v}_E) \Theta(v_{\rm stop} - |\mathbf{v}_M \cdot \hat{\mathbf{B}}_0|)$

The net polarization of the DM in the magnetic field region is
$\mathcal{P} = f_{pol}/(1 - f_{pol})$ with $n_M = \rho(1 - f_{\rm pol})/M$. Note $M_0 = n\mu\mathcal{P}$.

Choosing $\mathbf{v}_E \parallel \hat{\mathbf{B}}_0$ minimizes $f_{\rm pol}$ to yield

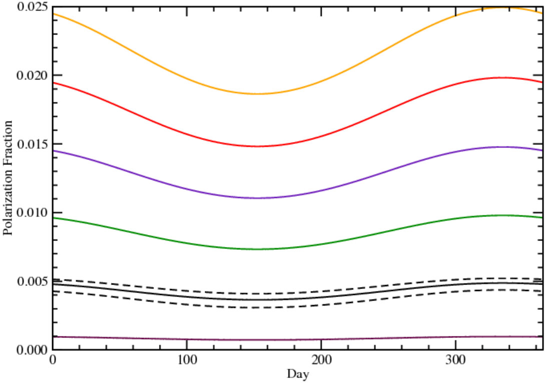
$$f_{\rm pol}^{\parallel} = \frac{1}{4}\left(\mathrm{erf}\left(\frac{v_{stop} - v_E}{v_0}\right) + \mathrm{erf}\left(\frac{v_{stop} + v_E}{v_0}\right)\right)$$

With $\mu \equiv \kappa e\hbar/2M$ we have $v_{stop} = 4.51\text{km/s}\sqrt{\kappa B_0[\mathrm{T}]}(m_e/M)$.

f_{pol} grows larger as $|\mathbf{v_E}|$ decreases.

Annual Variation in DM Polarization

Here $\kappa = 1$, $M = m_e$, and $v_0 = 220$ km/s.



We choose t =335 to set limits on μ, where $\mu \equiv \kappa e\hbar/2M$.
N.B. The temporal variation is sensitive to astronomical input, as well as to the orientation of $\mathbf{B}$ and $\mathbf{v}_E$.

Limits on κ

If $\omega_B \ll \omega$, $\phi_0 = \chi_0 \omega_B l/2c = \mu M_0 l/\hbar c$ and

$$\phi_0 = \frac{\mu^2 n_M \mathcal{P} l}{\hbar c}$$

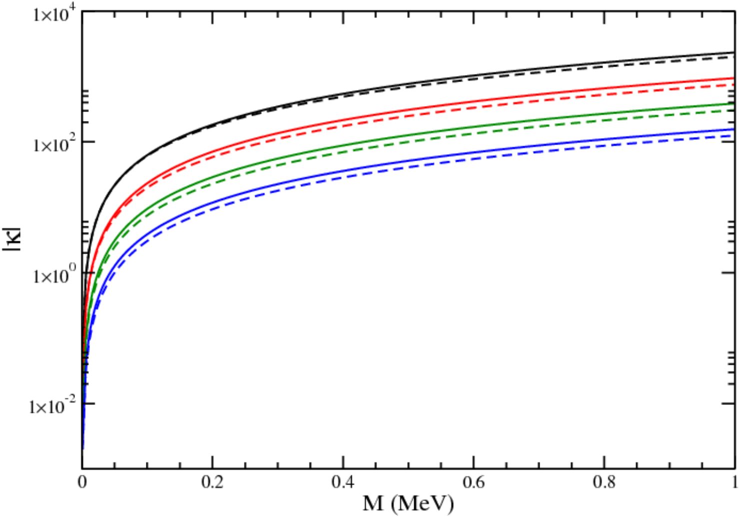
No dependence on DM temperature!

We use $\rho \sim 0.3\mathrm{GeV/cm^3}$ to find

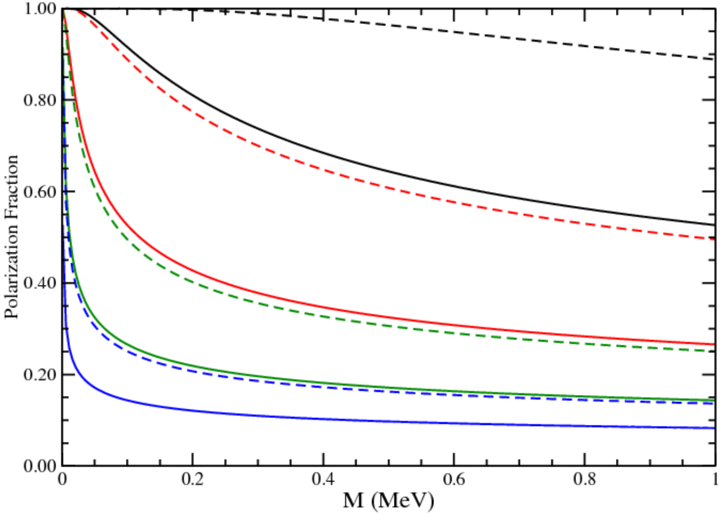
$$\phi_0 \simeq (6.84 \cdot 10^{-23}\,\mathrm{cm}^2) n_e[\mathrm{cm}^{-3}] l[\mathrm{cm}] \mathcal{P} \kappa^2 \left(\frac{m_e}{M}\right)^3$$

In PVLAS the parameter $l = 4.4 \cdot 10^6$ cm and $\delta(\phi_0/l) \approx 10^{-12}$ rad at 95% CL.
We choose $v_0 = 220$ km/s and $B_0 = 7$ T to yield....

Limits on κ at 95% CL



Limits on κ at 95% CL



Summary and Outlook

We have considered the possibility of observing a DM candidate particle with a non-zero magnetic moment through the gyromagnetic Faraday effect.

The effect can serve to generate an appreciable source of CMB B-mode polarization and can be studied terrestrially as well.

Backup Slides

Constraining DM

To estimate the Faraday rotation due to DM, we must integrate over the past light cone of the photon.
For CDM, the cosmological scale dependence of B/T cancels. We assume B/T to be constant.

$$\int_0^l dx n(x) \to n_o c \int_0^z dz' H(z')^{-1}(1+z')^3 \equiv n_o \tilde{l}$$

to yield the effective path length $\tilde{l}$ and $\phi_0 = \mu^2 \gamma H^o_{\rm prim} n_o \tilde{l}/2ck_B T^o$.
$B^o_{\rm prim}$, n_o, and T_o are the primordial magnetic field, DM number density, and temperature, scaled to the present epoch.
We compute $H(z)$ as per the Friedmann equation in a flat ΛCDM cosmology,

$$H(z) = H_0(1+z)\left[1 + \Omega_M z + \Omega_\Lambda(1/(1+z)^2 - 1)\right]^{1/2}$$

with $\Omega_M = 0.27$ and $H_o = 71\,\mathrm{km\,s^{-1}Mpc^{-1}}$. [D.N. Spergel et al., astro-ph/0603449.]
For a spin-$1/2$ DM particle of mass M we define $\mu \equiv \kappa\mu_M$ and $\gamma = 2\kappa\mu_M/\hbar$ with $\mu_M = e/2M$.

Constraining DM

We compute the Faraday rotation accrued by photons travelling from $z \sim 1100$ to the present epoch.
We use the gravitational infall velocity of galactic DM to estimate the DM temperature today. Assuming a Maxwell-Boltzmann distribution, $v_{\rm rms}/c = \sqrt{3k_B T/Mc^2}$.
Thus

$$\begin{aligned} \phi_0 \quad &\sim \quad 3.6 \cdot 10^{-18} \frac{\mathrm{cm}^3}{\mu\mathrm{G\,Mpc}} \left(\frac{\mu}{\mu_B}\right)^3 \left(\frac{m_e}{M}\right)^2 \left(\frac{v_{\rm rms}}{c}\right)^{-2} \\ &\times \quad \bar{n}_o[\mathrm{GeV\,cm}^{-3}] B^o_{\rm prim}[\mu\mathrm{G}]\tilde{l}[\mathrm{Mpc}]\,, \end{aligned}$$

where $\bar{n}_o \equiv \rho_{\rm cdm}/m_e \sim 2.17 \cdot 10^{-3}$, $v_{\rm rms} \sim 200$ km/s, and $\tilde{l} \sim 1.3 \times 10^{10}$ Mpc. We choose $M \sim m_e/10$ and use the bound $B^o_{\rm prim} \lesssim 10^{-3}\,\mu$G, for primordial magnetic fields coherent across the present horizon [Blasi, Burles, Olinto, ApJ 514 (1999) L79.] to yield $|\kappa| \lesssim 0.8$ if ϕ_0 can be determined to $\phi_0 \sim 10^{-2}$ rad.
A μ of this size is not yet ruled out by other data.

Existing Limits on a DM Electric Charge

Suppose the DM constituents have a charge of $|e|$.

[De Rújula, Glashow, Sarid, NPB 333 (1990), 173.]

"Champs of charge + 1 should now appear as super-heavy isotopes of hydrogen...."

What constraints exist?

1) Neutron star survival $\Longrightarrow$ charged particle fraction f of dark halo $f \lesssim 10^{-5}$

[Gould, Draine, Romani, Nussinov, PLB 238 (1990), 337.]

2) Fraction of superheavy hydrogen in sea water $f \lesssim 6 \cdot 10^{-15}$ excludes charged DM particles between 10^4 and 10^7 GeV.

[Verkerk et al., PRL 68 (1992), 1116.]

Milli-charged dark matter, however, is not ruled out.

For particles of charge ϵe and mass m_ϵ, laboratory and observational constraints yield $m_\epsilon \lesssim m_e$ and $10^{-15} \lesssim \epsilon < 1$. Weaker constraints exist at larger masses.

[Davidson, Hannestad, Raffelt, JHEP 005 (2000) 003]

Presumably the study of substructure in galactic halos can also constraint this effect.

[Spergel, Steinhardt, PRL 84 (2000), 3760.]

Direct Detection Limits on WIMPS