

Holographic Noise

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Fermilab and U. Chicago

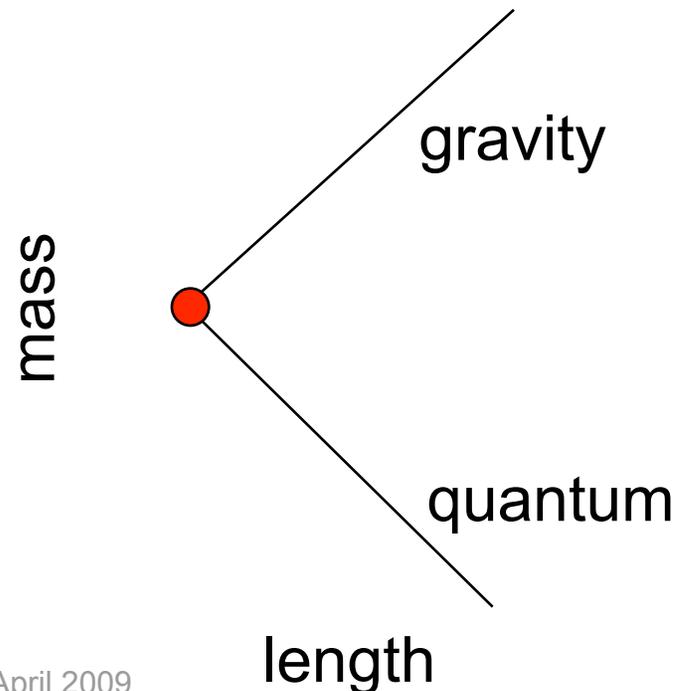
The smallest interval of time

- Quantum gravity suggests a minimum (Planck) time,

$$t_P \equiv l_P/c \equiv \sqrt{\hbar G_N/c^5} = 5 \times 10^{-44} \text{ seconds}$$

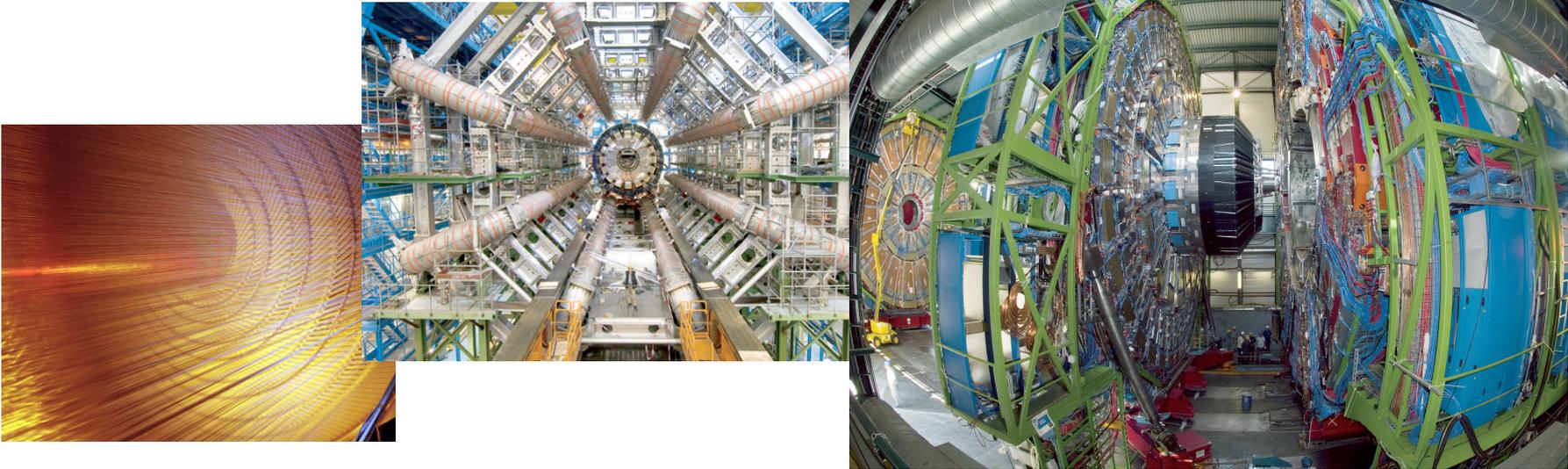
$$l_P = \sqrt{\hbar G_N/c^3} = 1.616 \times 10^{-33} \text{ cm}$$

- ~ particle energy 10^{16} TeV



Best microscopes vs best microphones

CERN/Fermilab: $\text{TeV}^{-1} \sim 10^{-18}$ m: particle interactions



LIGO/GE0600: $\sim 10^{-18}$ m, **coherent over $\sim 10^3$ m baseline:**
Positions of massive bodies



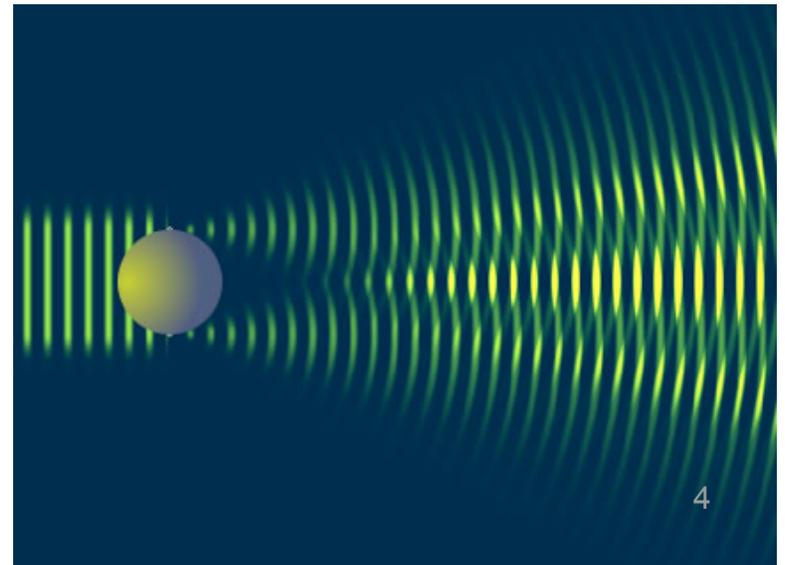
A new phenomenon: holographic noise

- The minimum interval of time may be studied directly using interferometers
- Not gravitational waves
- Wavefunction of spacetime: macroscopic limit of holographic theories
- Transverse uncertainty in position from Planck limit
- “Holographic Noise”: precise, zero-parameter prediction

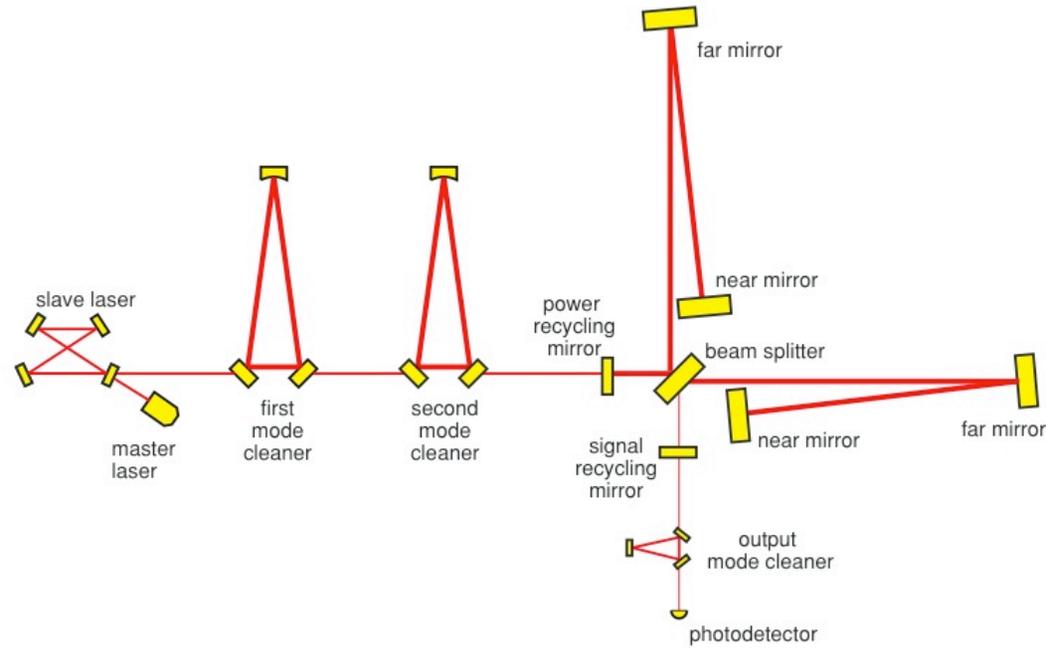
“Planck diffraction limit” at L

$$\Delta x \sim \sqrt{\lambda L}$$

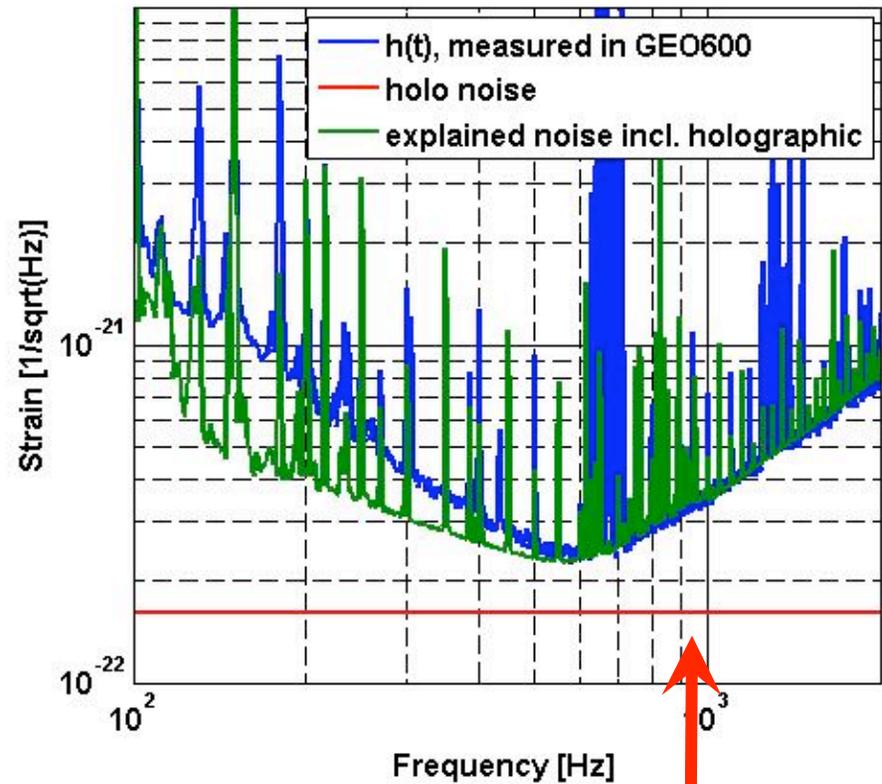
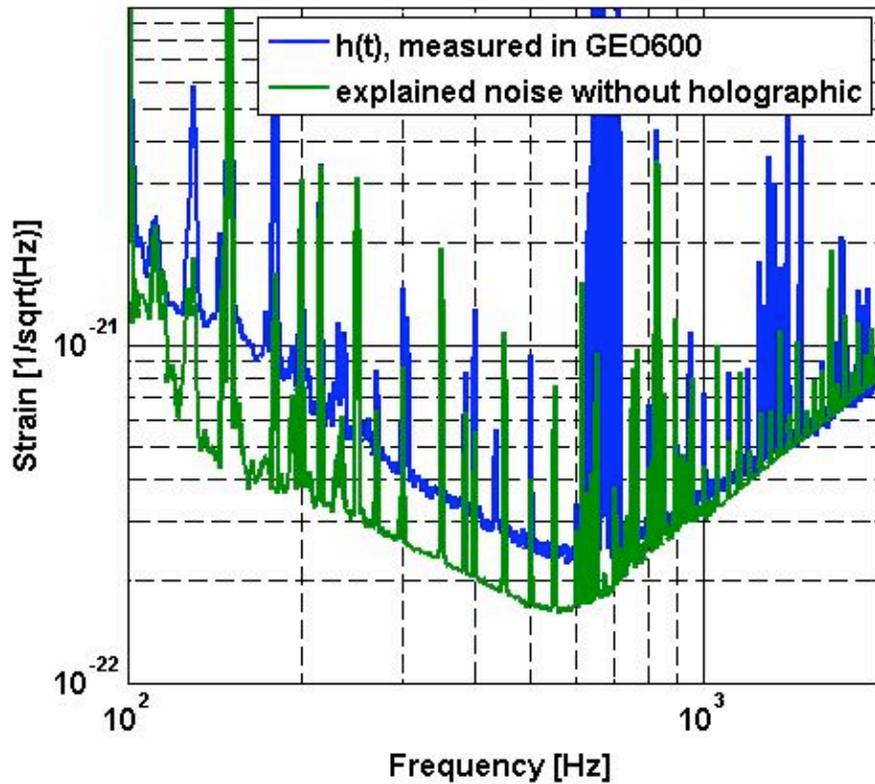
is \gg Planck length



GEO-600 (Hannover): best displacement sensitivity



“Mystery Noise” in GEO600



Data: S. Hild (GEO600)

Prediction: CJH, arXiv:0806.0665
(Phys Rev D.78.087501)

$$\sqrt{t_{Planck}} / 2$$

zero-parameter prediction for holographic noise in GEO600 (equivalent GW strain)

Total noise: not fitted JPL seminar, April 2009

Measurement of holographic noise

- Holographic wave geometry predicts a new detectable effect: "holographic noise"
- Not the same as zero-point field mode fluctuations
- Spectrum and distinctive spatial character of the noise is predicted with no parameters
- It may already be detected
- An experimental program is motivated

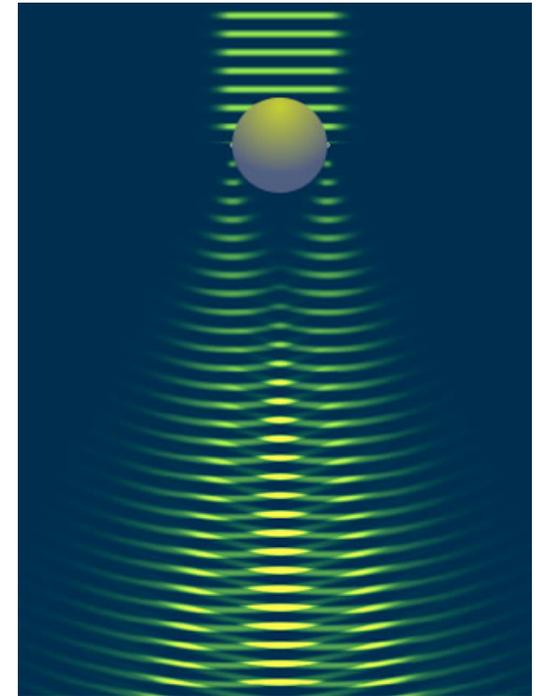
CJH: [arXiv:0806.0665](#) *Phys Rev D* 78, 087501 (2008)

CJH: [arXiv:0712.3419](#) *Phys Rev D* 77, 104031 (2008)

In Matrix theory: CJH and M. Jackson, arXiv:0812.1285

Holographic Wave Geometry

- Spacetime is a quantum system, not a continuous classical manifold
- theory in 2+1 dimensions: $z=t$
- “Planck photon’s view” of the universe
- Positions are transverse wavefunctions on light sheets or wavefronts
- Planck maximum frequency
- Transverse wavefunction spreads over macroscopic distances
- transverse indeterminacy in geometry much larger than Planck length

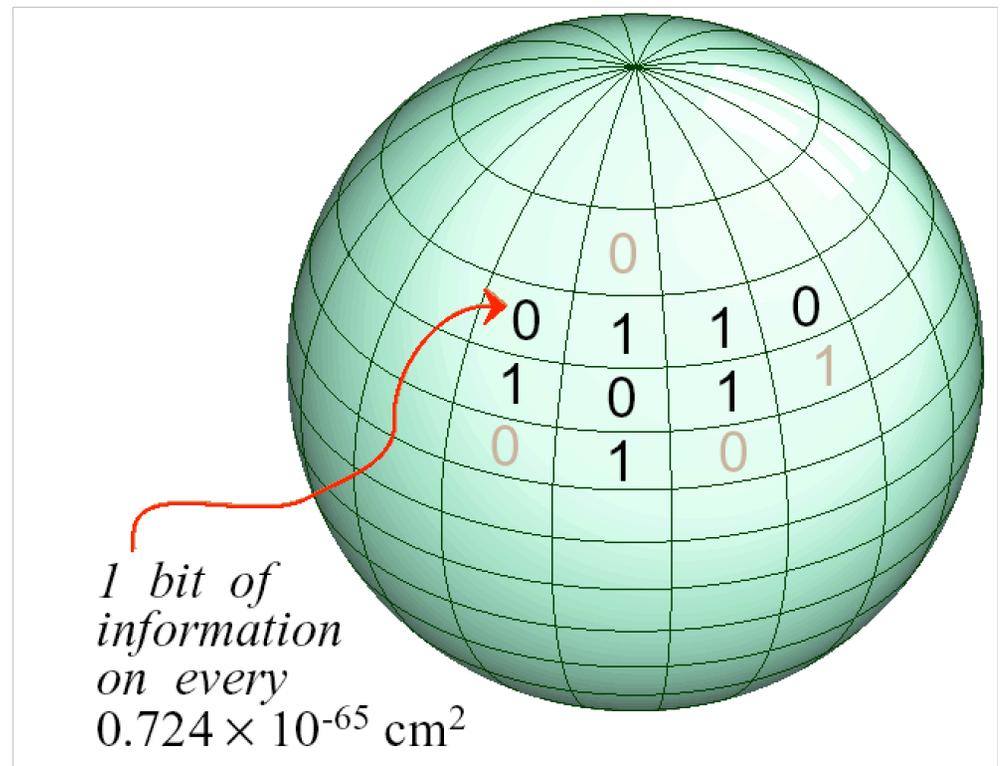


Holographic Theories of Everything

“This is what we found out about Nature’s book keeping system: the data can be written onto a surface, and the pen with which the data are written has a finite size.”

-Gerard 't Hooft

Everything about the 3D world can be encoded on a 2D null surface at Planck resolution



Holographic Quantum Geometry: theory

- Black holes: entropy=area/4 $S = A/l_P^2 4 \ln 2$
- Black hole evaporation
- Einstein's equations from heat flow
- Classical GR from surface theory
- Universal covariant entropy bound
- Exact state counts of extremal holes in large D
- AdS/CFT type dualities: N-1 dimensional duals
- Matrix theory
- All suggest theory on 2+1 dimensional null surfaces with Planck frequency bound

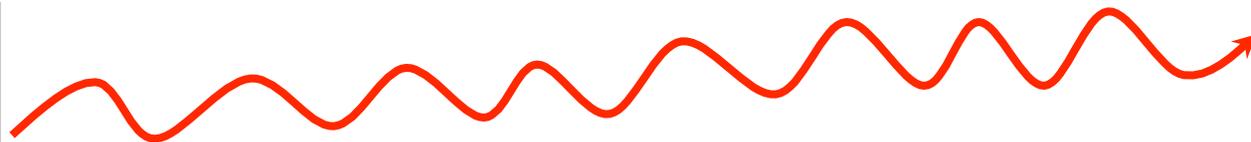
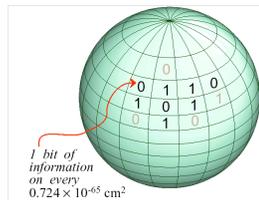
Beckenstein, Hawking, Bardeen et al., 'tHooft, Susskind, Bousso, Srednicki, Jacobson, Padmanabhan, Banks, Fischler, Shenker, Unruh

Holography 1: Black Hole Thermodynamics

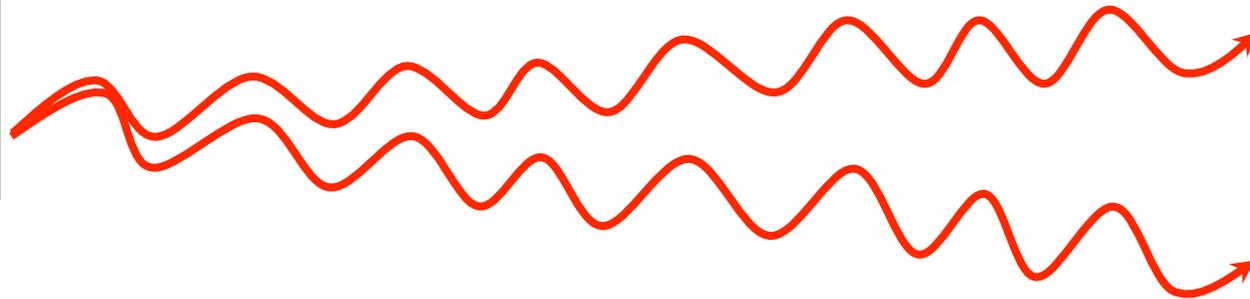
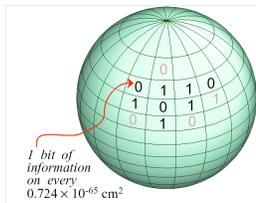
- Beckenstein, Bardeen et al. (~1972): laws of black hole thermodynamics
- Area of (null) event horizon, like entropy, always increases
- Entropy is identified with 1/4 of event horizon **area** in Planck units (not volume)
- Is there is a deep reason connected with microscopic degrees of freedom of spacetime encoded on the surface?

Holography 2: Black Hole Evaporation

- Hawking (1975): black holes radiate ~thermal radiation, lose energy and disappear
- Is information lost? Or is quantum unitarity preserved?
- Degrees of freedom: evaporated quanta carry degrees of freedom (~ 1 per particle) as area decreases
- Black hole entropy may completely account for information of evaporated states, also assembly histories
- Is black hole completely described by information on 2+1D event horizon?
- Information of evaporated particles=entropy of hole

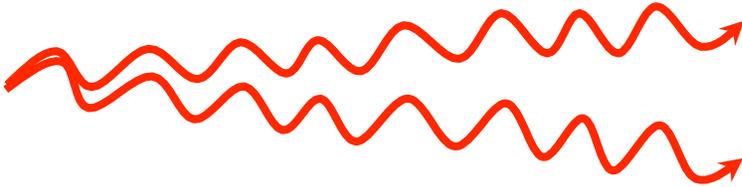
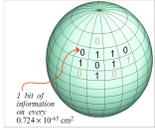


Holographic indeterminacy of distant spacetime allows black hole evaporation to be a reversible unitary quantum process



If the quantum states of the evaporated particles allowed relative transverse position observables with arbitrary angular precision, at large distance they would contain more information than the hole

Direct connection to black hole evaporation



$$(L / \Delta x)^2 < (R / \lambda)^2$$

$$\Delta x > R$$

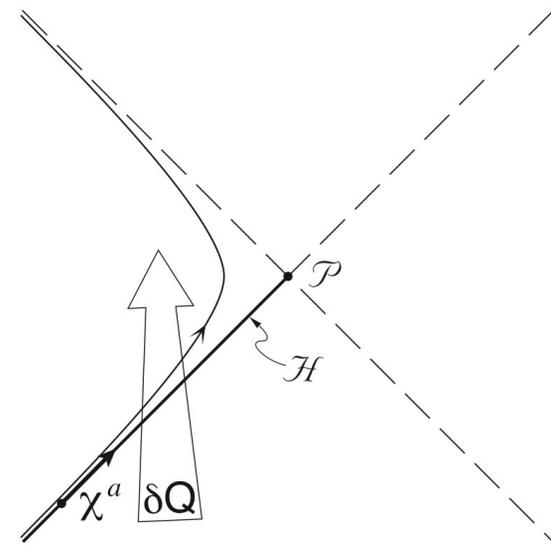
- ~ One particle evaporated per Planck area
- position recorded on film at distance L
- wavelength \sim hole size R , also standard position uncertainty
- Particle images on distant film: must have fewer “pixels” than hole
- Requires transverse uncertainty at distance L independent of R

$$\Delta x > \sqrt{\lambda L}$$

- Property of flat spacetime independent of hole
- Similarly for number of position states of an interferometer

Holography 3: nearly-flat spacetime

- Unruh (1976): Hawking radiation seen by accelerating observer
- Appears with any event horizon, not just black holes: identify entropy of thermal radiation with missing information
- Jacobson (1995): Einstein equation derived from thermodynamics (\sim equation of state)
- Classical GR from 2+1D null surface (Padmanabhan 2007)



Holography 4: Covariant (Holographic) Entropy Bounds

- 't Hooft (1985): black holes are quantum systems
- 't Hooft, Susskind et al. (~1993): world is "holographic", encoded in 2+1D at the Planck scale
- Black hole is highest entropy state (per volume) and sets bound on entropy of any system (includes quantum degrees of freedom of spacetime)
- All physics within a 3D volume can be encoded on a 2D bounding surface ("holographic principle")
- Bousso (2002): holographic principle generalized to "covariant entropy bound" based on causal diamonds: entropy of 3D light sheets bounded by area of 2D bounding surface in Planck units
- Suggests that 3+1D geometry emerges from a quantum theory in 2+1D: light sheets

Holography 5: Exact dual theories in $N-1$ dimensions

- Maldacena, Witten et al. (1997...): AdS/CFT correspondence
- N dimensional conformal field "boundary" theory exactly maps onto (is dual to) $N+1$ dimensional "bulk" theory with gravity and supersymmetric field theory
- Is nearly flat $3+1$ spacetime described as a dual in $2+1$?

Holography 6: string/M theory

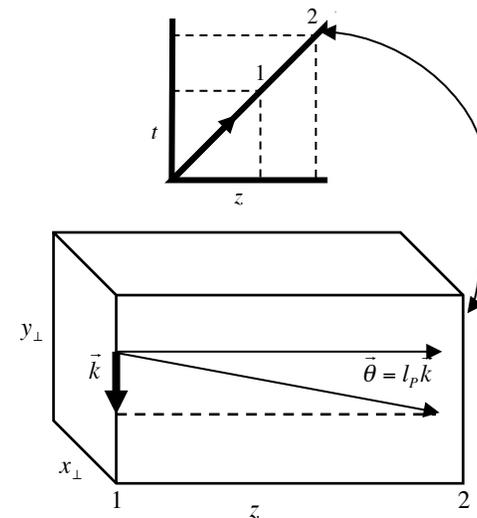
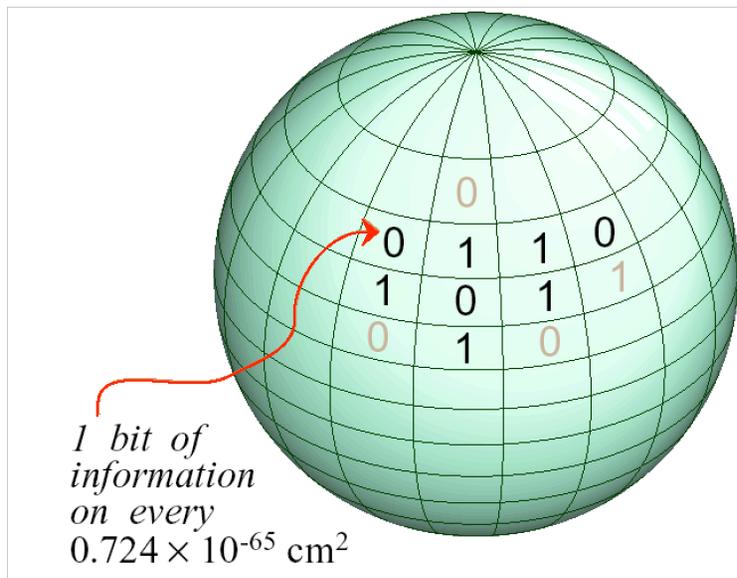
- Strominger, Vafa (1996): count degrees of freedom of extremal higher-dimension black holes using duality
- All degrees of freedom appear accounted for
- Agrees with Hawking/Beckenstein thermodynamic count
- Unitary quantum system
- **Strong indication of a minimum length \sim Planck length**
- What do the degrees of freedom look like in a realistic system?
- **Matrix theory: wavefunctions of transverse position Matrix Hamiltonian (CJH& M. Jackson)**

Holographic Geometry

- Spacetime+mass-energy is a holographic quantum system
- the world in any frame can be described by Planck-scale null waves
- "from inside": transverse indeterminacy in position much larger than Planck length

Holographic geometry implements holographic entropy bound in emergent 3+1D spacetime

- 3+1D space = Hilbert space of 2+1D theory
- By construction, follows light sheets: covariant formulation
- fewer independent modes than field theory quantized in 3+1D
- independent pixels in 3D volume \sim area of 2D null surface element
- “bandwidth limit” of spacetime states



Theories with holographic noise

Two conditions are sufficient:

1. Maximum Planck frequency in any frame
2. Planck wavelength resolution on light sheets

Count degrees of freedom with Shannon/Nyquist sampling: 2 degrees of freedom per wavelength

1D segment of length L on
null wavefront

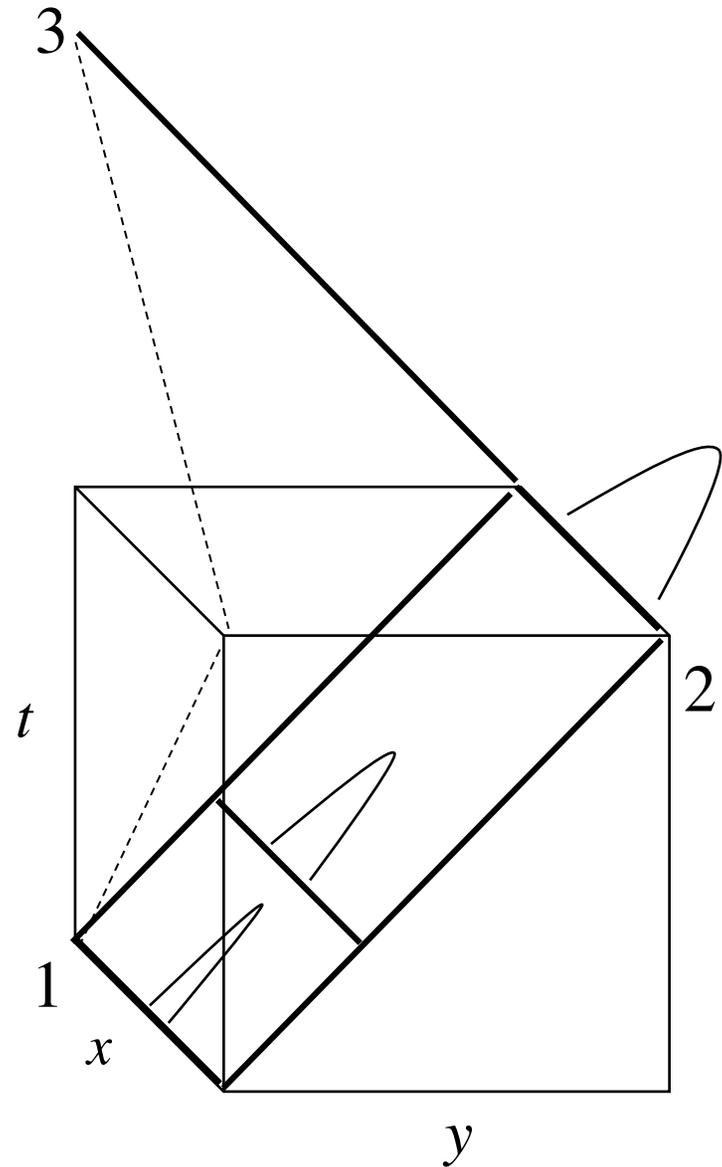
Sweeps out 2D surface:

$$(L / \Delta x)^2 \approx L / l_P$$

independent position
degrees of freedom

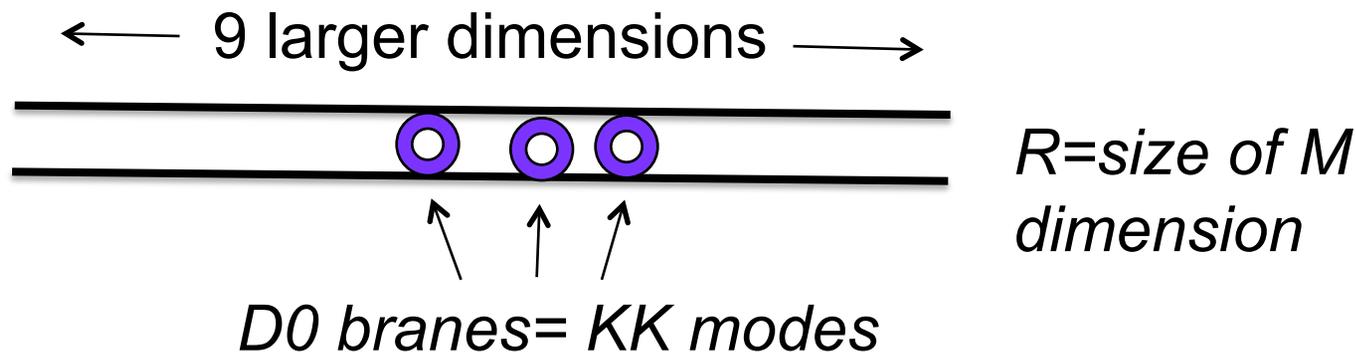
Position variance in 2D

$$\Delta x^2 \approx L l_P$$

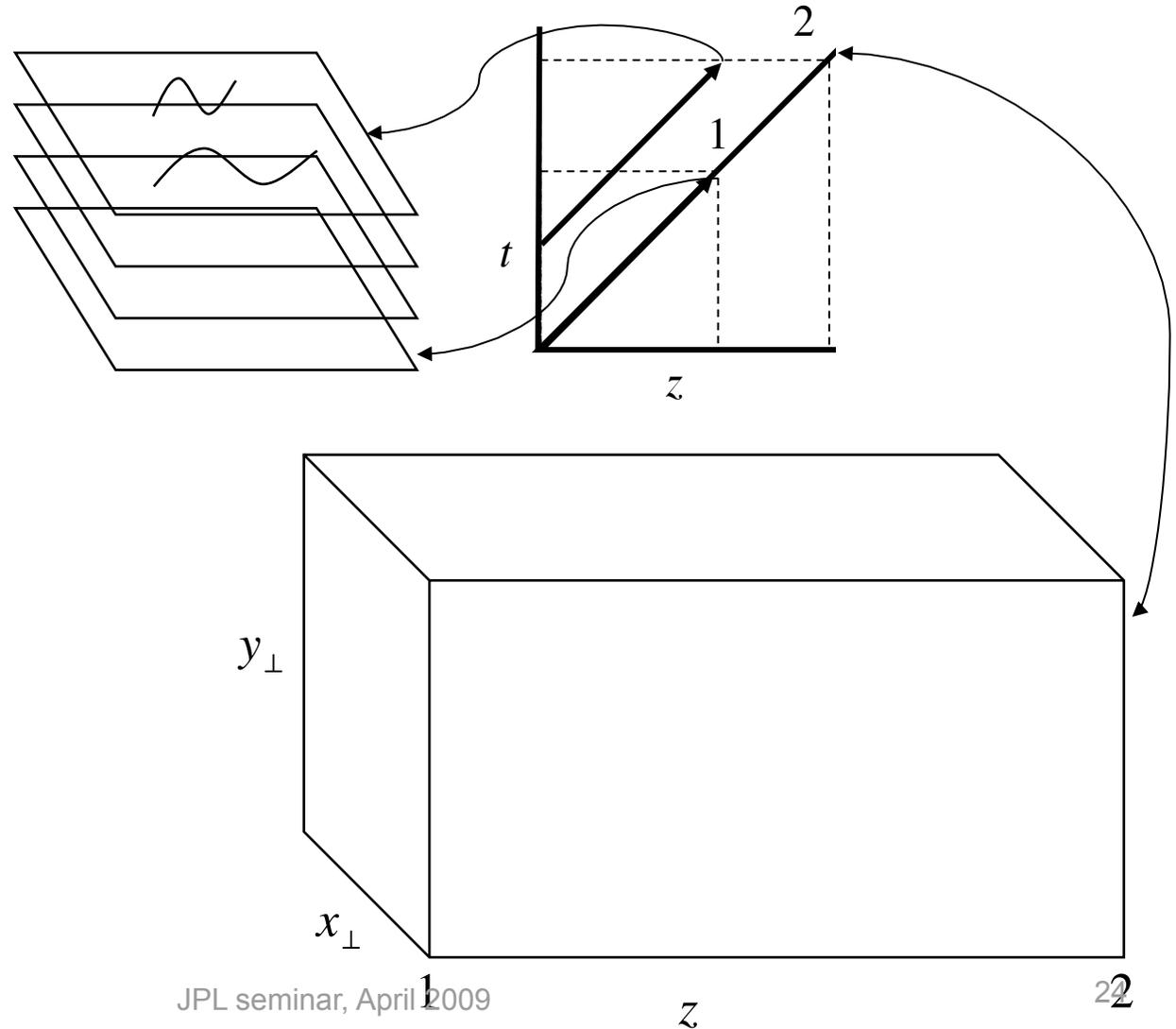


Example: Matrix theory

- Banks, Fischler, Shenker, & Susskind 1997: a candidate theory of everything
- Fundamental objects are $9 N \times N$ matrices, describing N “D0 branes” (particles)
- Dual relationship with string theory
- Gives rise to 10 space dimensions, 1 compact, plus time



- Only 2 of the 9 space dimensions survive to be macroscopic
- The third space dimension is virtual, swept out by 2D null sheet
- Einstein's "ride on a photon": what does the world look like?



*3+1D spacetime
emerges from
2+1D: light
sheet with $z=t$*

Holographic spacetime: wave theory from M theory

- N D0 branes, N x N matrices X_i , $i= 1$ to 9, compact M dimension with radius $R \sim$ Planck length
- Hamiltonian from Banks, Fischler, Shenker, & Susskind:

$$H = R \operatorname{tr} \left\{ \frac{\Pi_i \Pi_i}{2} + \frac{1}{4} [X_i, X_j]^2 + \theta^T \gamma_i [\theta, X_i] \right\}$$

- Notions of position, distance emerge on scales $\gg R$
- local in 2+1 D, “incompressible” on Planck scale: holographic
- Center of mass position of macroscopic body, $x = \operatorname{tr} X$
- Macroscopic longitudinal position encoded by first (kinetic) term, conjugate momenta to position matrices

CJH & M. Jackson, arXiv:0812.1285

Macroscopic wave equation from M theory

- M Hamiltonian

$$H = \frac{R}{\hbar} \text{tr} \sum_i \left\{ \frac{\Pi_i \Pi_i}{2} + \sum_j \frac{1}{4} [X^i, X^j]^2 \right\}$$

- Leads to wave equation in each transverse dimension x

$$\frac{\partial^2 u}{\partial x^2} + \frac{2i}{R} \frac{\partial u}{\partial z^+} = 0$$

- Quantum mechanics without Planck's constant
- Schrodinger equation
- Solutions display diffusion, diffraction

Paraxial wave equation

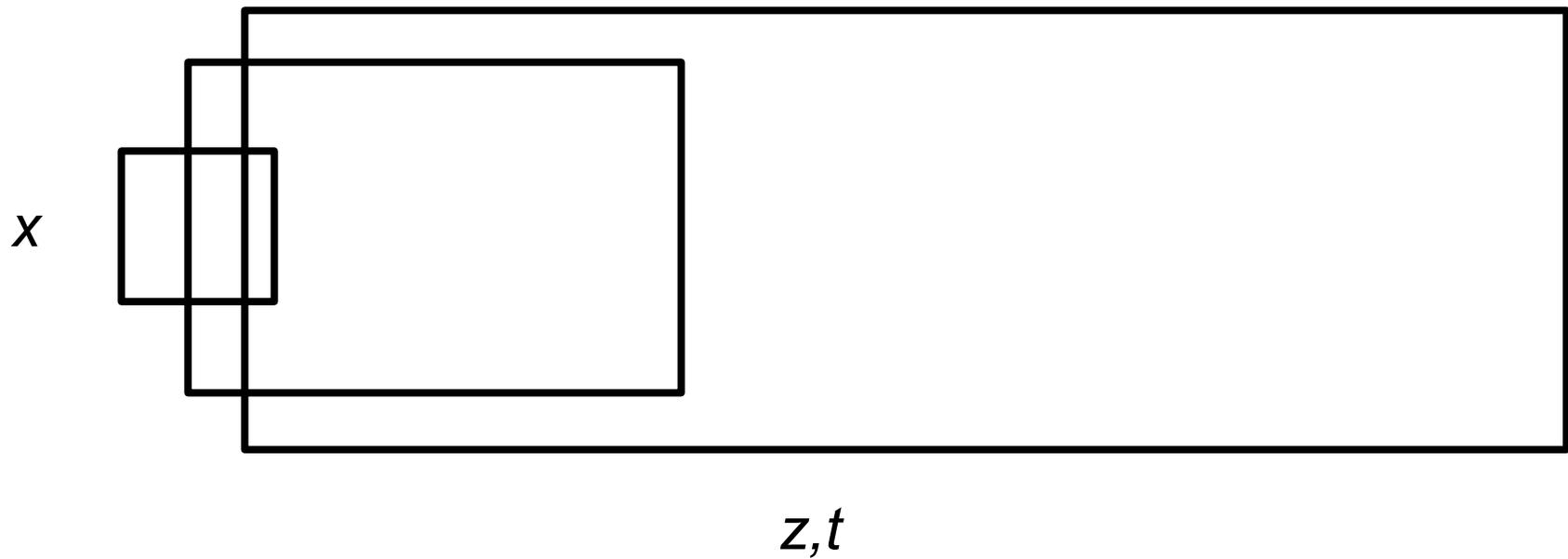
- phasors in wavefronts
- wave equation in each transverse dimension x

$$\frac{\partial^2 u}{\partial x^2} - \frac{2i}{R} \frac{\partial u}{\partial z^+} = 0$$

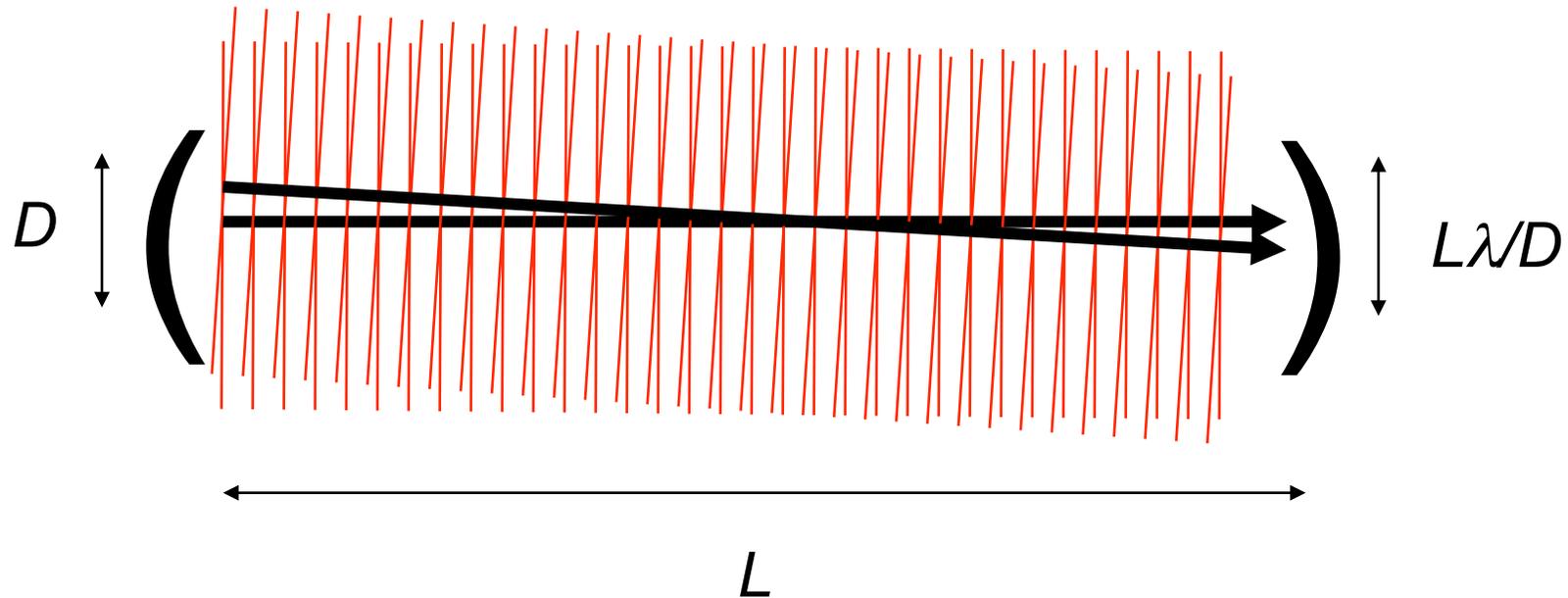
- “Paraxial Wave Equation:” generic, quasi-optical behavior
- Solutions display diffraction: e.g. laser cavities

Nonlocal modes connect longitudinal and transverse positions

- Wave solutions: “Holographic geometry”
- Transverse gaussian beam solutions from wave optics
- New macroscopic behavior, not the same as field theory limit



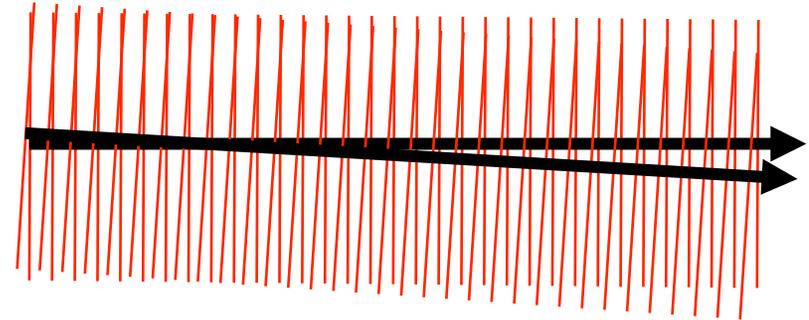
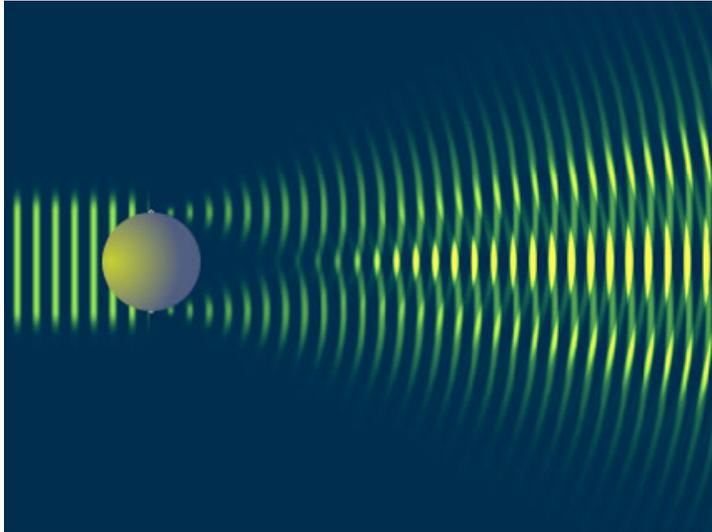
Rayleigh range and transverse uncertainty



- Aperture D , wavelength λ : angular resolution λ/D
- Size of diffraction spot at distance L : $L\lambda D$
- Endpoints of a ray can be anywhere in aperture, spot
- path is determined imprecisely by waves
- Minimum uncertainty at given L when aperture size = spot size, or

$$D = \sqrt{\lambda L}$$

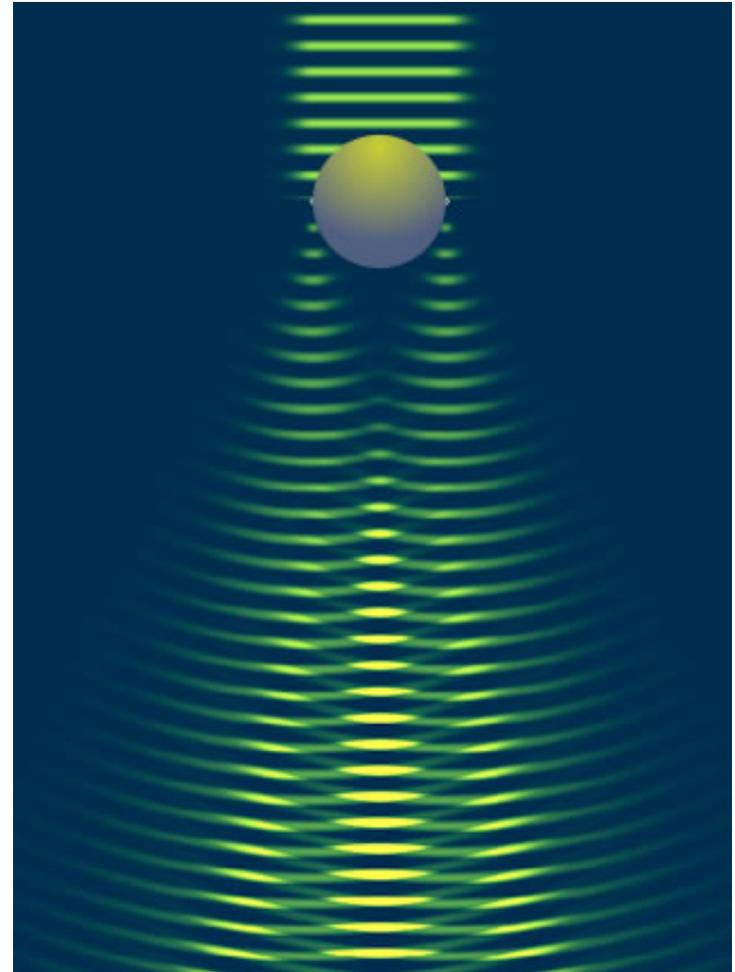
Indeterminacy of a Planckian path



- Classical spacetime manifold defined by paths and events
- Complementarity: path \sim ray approximation of wave physics
- Transverse wavefunction of position displays indeterminacy formally identical to optical wave correlations
- Position \sim endpoint of ray
- Indeterminacy of geometry reflects limited information content of band-limited waves

Wave Theory of Spacetime Indeterminacy

- Adapt theory of transverse correlation in wave optics
- theory of “position wavefunctions”
- Complex amplitude \sim wavefunction
- Complex correlation \sim quantum correlation
- Intensity \sim probability
- Position at first reflection=“aperture”
at second= intensity at “screen”
- Set wavelength to match
holographic degrees of freedom
- **Allows calculation of noise with
numerical factors, no parameters
for a given apparatus**



Uncertainty of transverse position

Spacetime positions are wavefunctions. Transverse positions at normal separation L are Fourier transforms of each other and have standard deviations related by:

$$\sigma' \sigma = \lambda L$$

For macroscopic L the “uncertainty” is much larger than the wavelength

Controlled theory based on wave optics:
CJH, arXiv: **0806.0665**

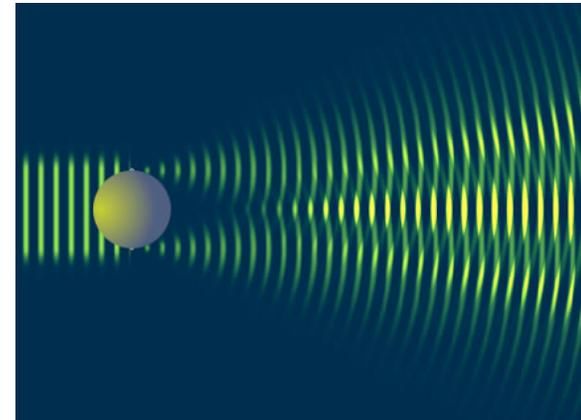
holographic approach to the classical limit

- **Angles** are indeterminate at the Planck scale, and become better defined at larger separations:

$$\Delta\theta(L) = (l_P/L)^{1/2}$$

- But uncertainty in **relative transverse position increases** at larger separations:

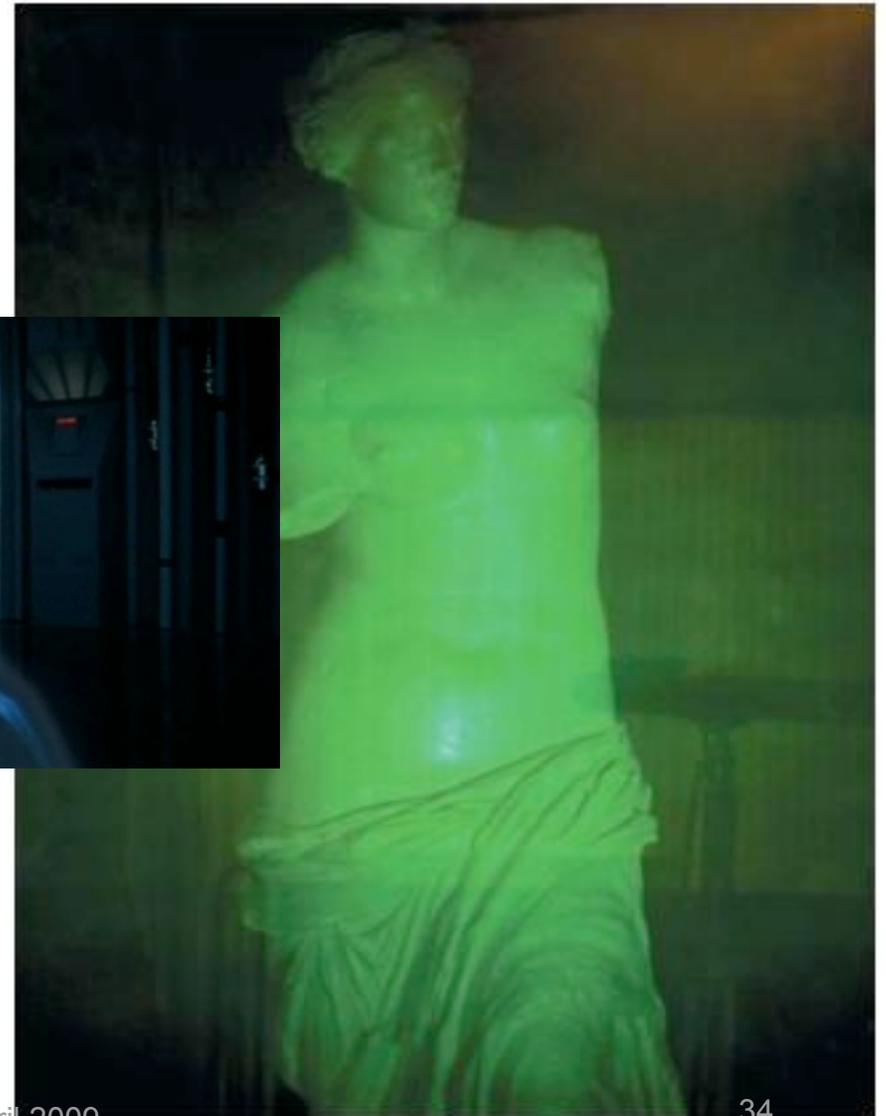
$$\Delta x_{\perp}^2 > l_P L$$



- Not the classical limit of field theory
- Indeterminacy and nonlocality persist to macroscopic scales

A holographic world is blurry

limited information content



What does it look like
"from inside"?
("Flatland" realized with
waves)

The case of a real hologram

- For optical light and a distance of about a meter,

$$D = \sqrt{\lambda L}$$

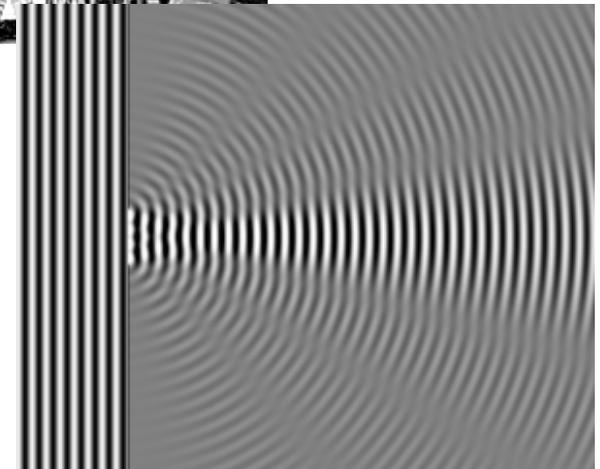
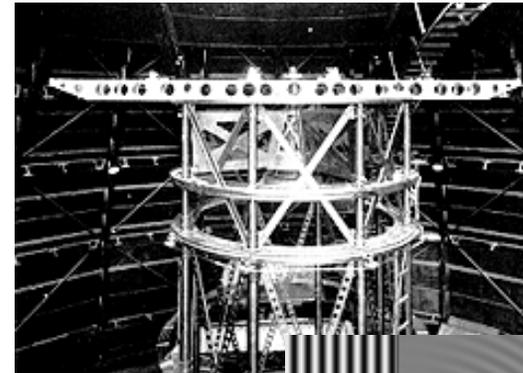
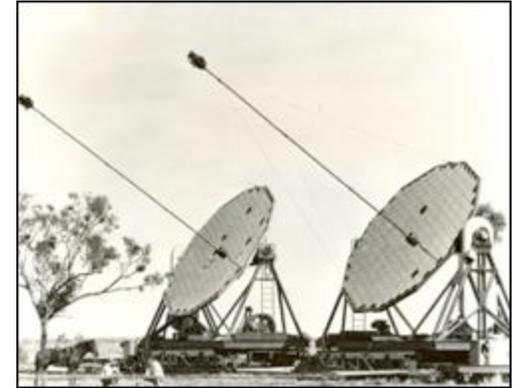
is about a millimeter

- Larger aperture gives sharper image but then photon paths and arrival positions cannot be measured so well
- If you "lived inside" a hologram, you could tell by measuring the blurring/indeterminacy



Familiar examples from the world of optics

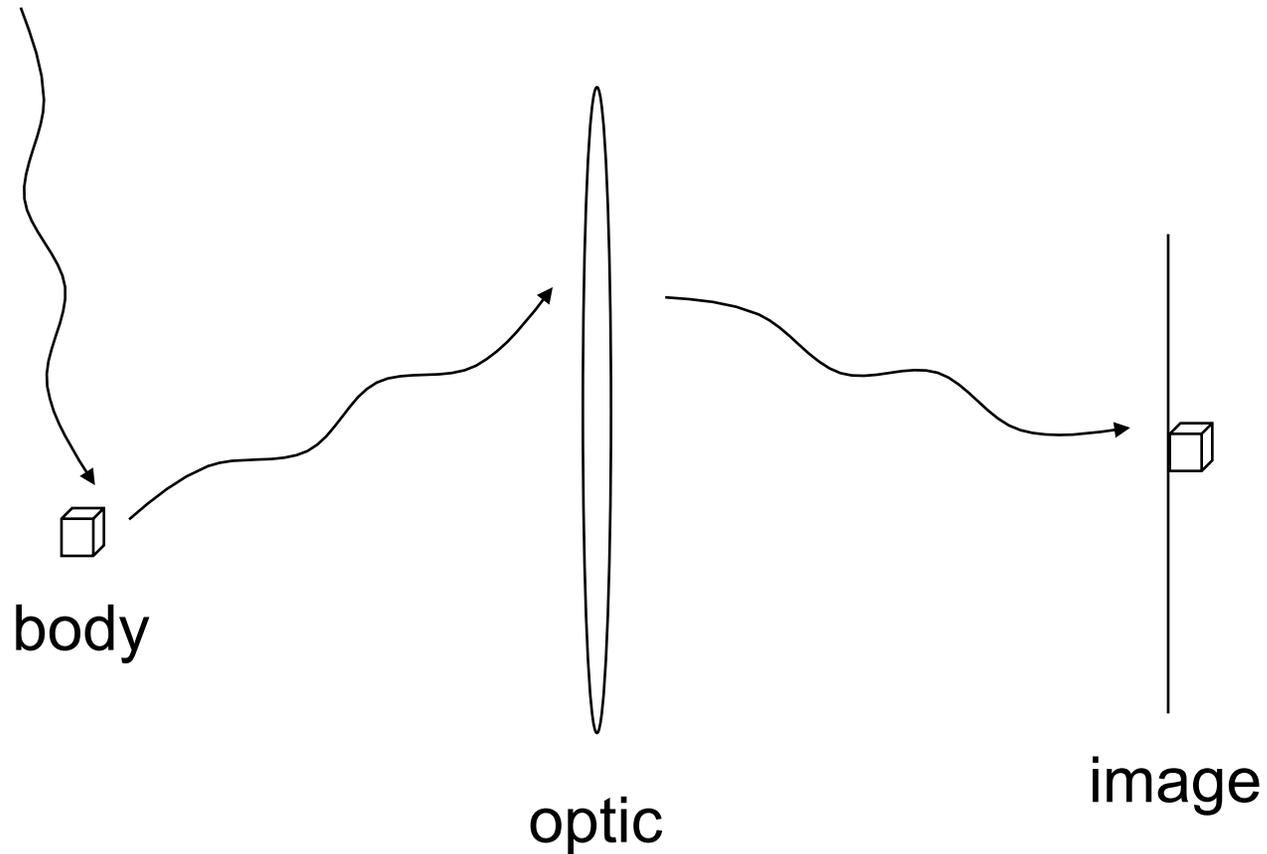
- Hanbury Brown-Twiss interferometry: correlation of intensity from distant star in widely separated apertures
- Michelson stellar interferometer: fringes from star
- Diffraction in the lab: shadow of plane wave cast by edge or aperture



All display similar optical examples of wave phenomena much larger than the waves,

$$\sigma' \sigma = \lambda L$$

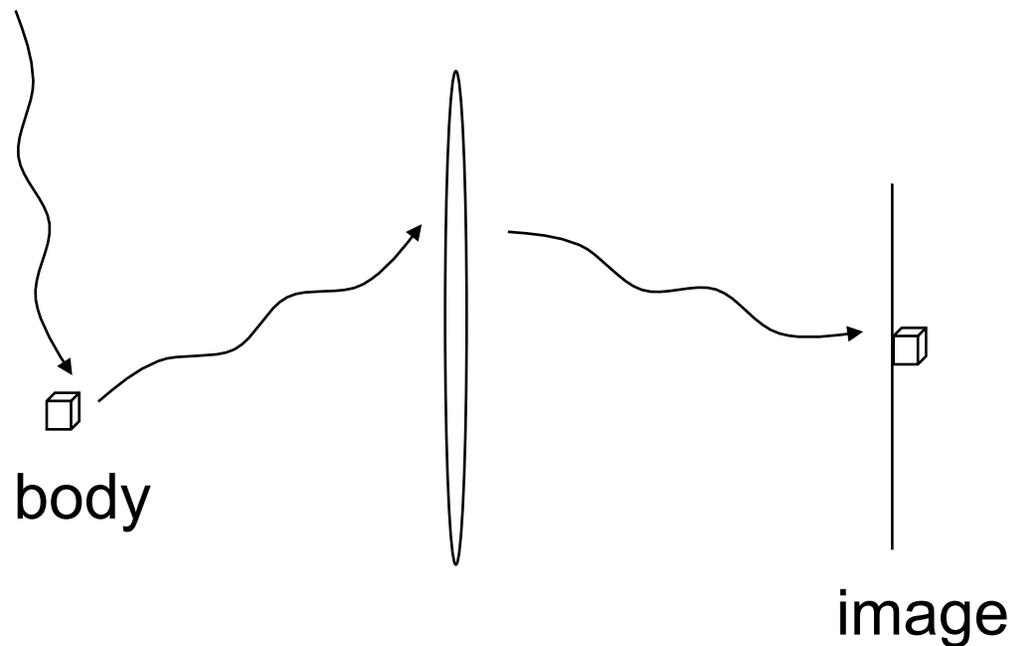
"Heisenberg microscope"



$$\Delta(\text{measured position}) \times \Delta(\text{momentum of perturbation}) > \hbar/2$$

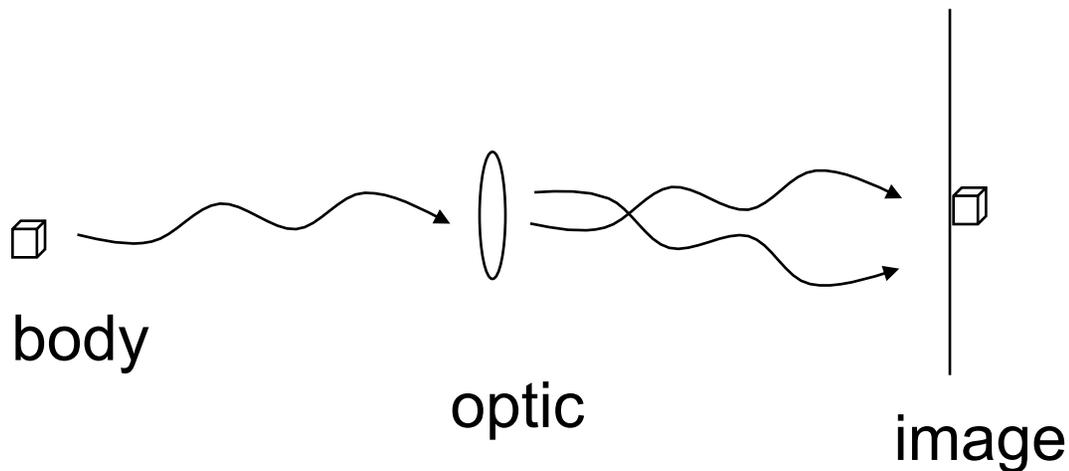
Heisenberg Microscope

- Measures transverse position by imaging using scattered light
- Uncertainty relation between measured position, transverse photon momentum
- **observables do not have independent classical meaning**



"Planck telescope"

- Create "image" on a screen
- Wavelength = Planck
- Minimum uncertainty in angle or transverse position difference when size of optic \sim size of its own diffraction spot
- Wavefronts can't transport or encode more transverse information
- **Transverse positions of body, optic, image, do not have independent classical meaning**



Uncertainty: Heisenberg and Holographic

- **"Heisenberg microscope"**: transverse position of a remote body measured by angular position ~ detected position of radiation particle in image
- Fixed 3D classical space
- $\Delta(\text{measured transverse position of a body}) \times \Delta(\text{momentum of measuring radiation}) > \hbar/2$
- Δ independent of microscope aperture, focal length; depends on mass of body
- State of body, radiation depends on measurement
- **"Planck telescope"**: remote transverse positions measured by Planck radiation
- Fixed wavelength in a given frame
- $\Delta(\text{position 1}) \times \Delta(\text{position 2}) > (\text{Planck length}) \times (\text{separation})$
- Δ position depends on separation, independent of mass
- Property of holographic spacetime geometry: **limiting precision of Planck waves**
- **State of position of everything depends on measurement**

Holographic geometry: dark energy physics in the lab?

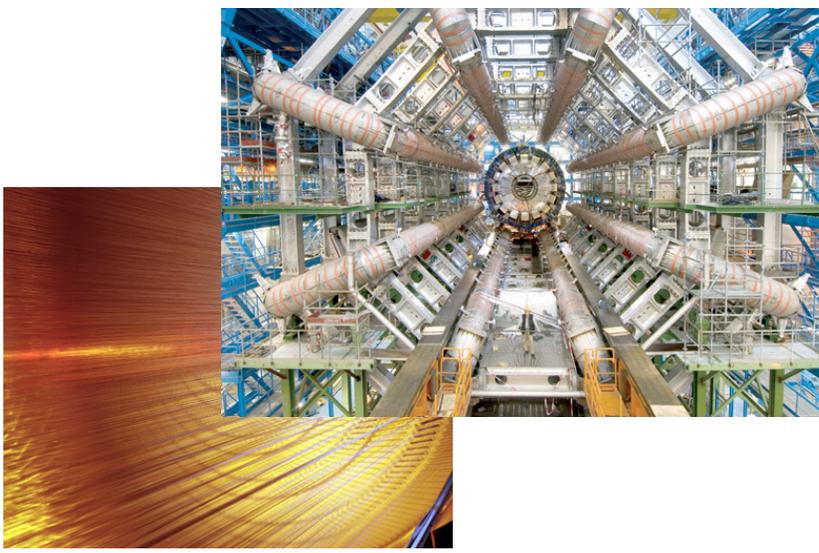
- Holographic blurring is $\sim 0.1\text{mm}$ at the Hubble length
- $\sim (0.1\text{mm})^{-4}$ is the dark energy density
- “Nonlocality length” for dark energy is holographic displacement uncertainty, scaled to Hubble length
- (literature on “holographic dark energy” centers on same numerology)
- Does not “explain” dark energy, but a piece of the puzzle: quantum physics of empty space = 2+1D quantum theory

Interferometers as Planck telescopes

- Nonlocality, coherence: relative positions at km scale
- Fractional precision: angle $< 10^{-21}$, $>$ "halfway to Planck"
- Transverse position measured in Michelson layout
- Heavy proof masses, small Heisenberg uncertainty (SQL): positions measure spacetime wavefunction
- holographic noise appears in signal

measuring holographic geometry requires coherent transverse position measurement over macroscopic distance

CERN/FNAL: $\text{TeV}^{-1} \sim 10^{-18} \text{ m}$

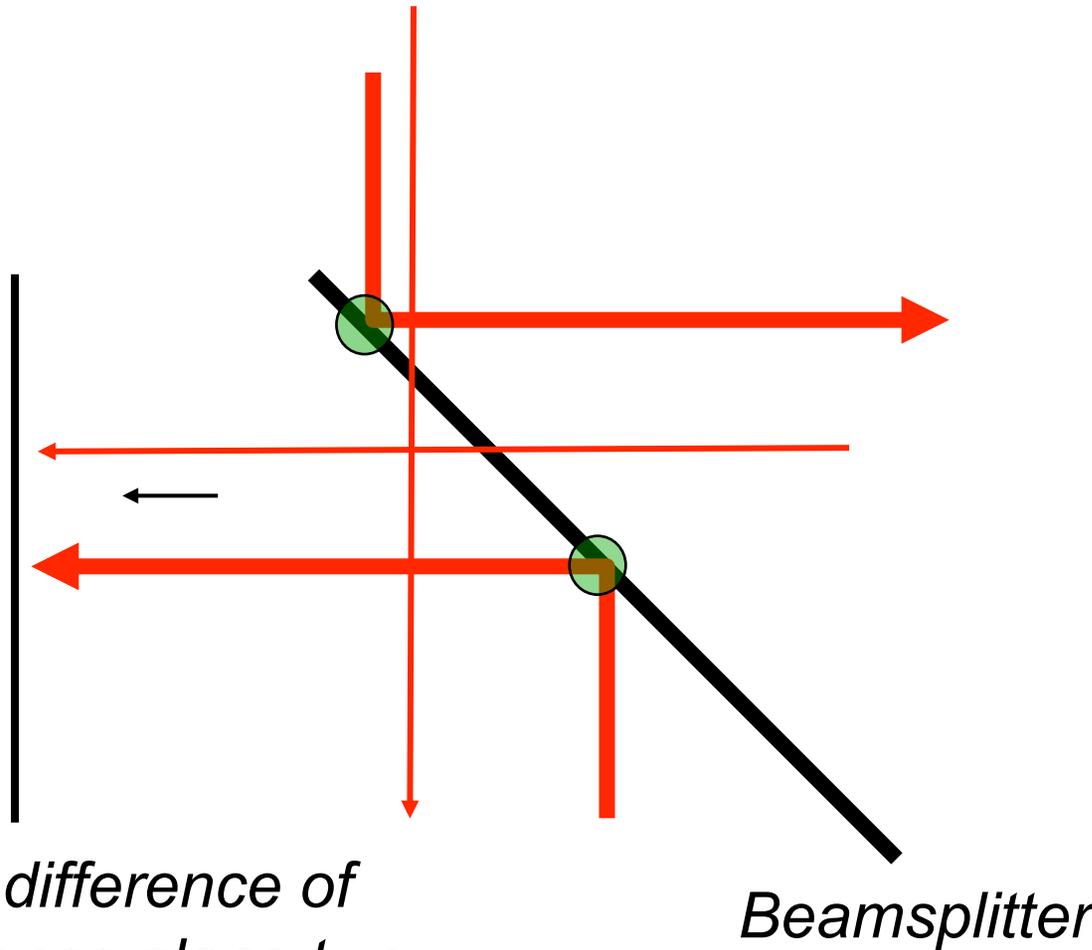


LIGO/GEO: $\sim 10^{-18} \text{ m}$
over $\sim 10^3 \text{ m}$ baseline



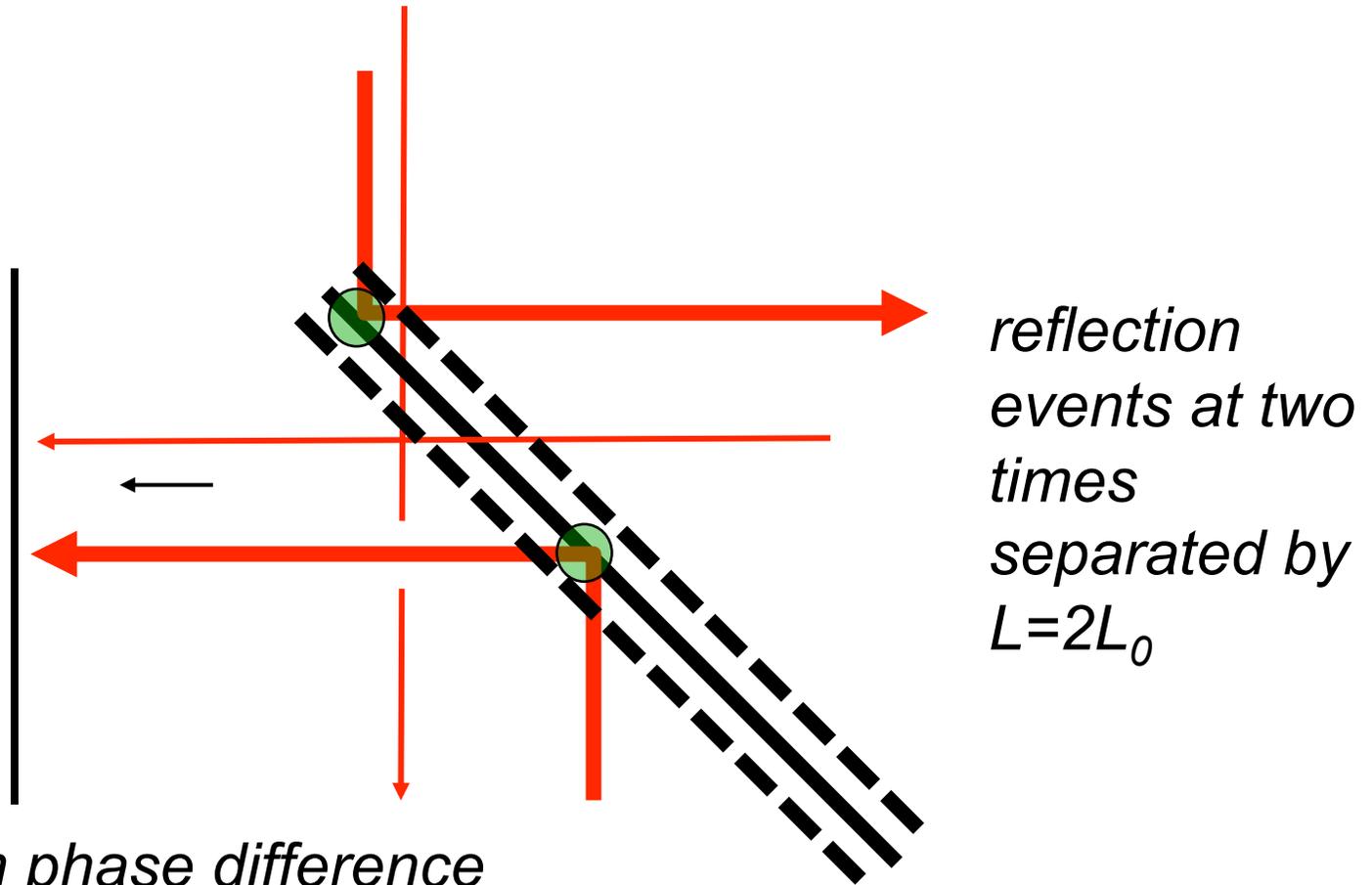
seminar April 2019

Beamsplitter and signal in Michelson interferometer



Signal phase ~ difference of integrated distance along two orthogonal arms

Holographic noise in the signal of a Michelson interferometer



Signal: random phase difference of reflection events from indeterminate position difference of beamsplitter at the two events

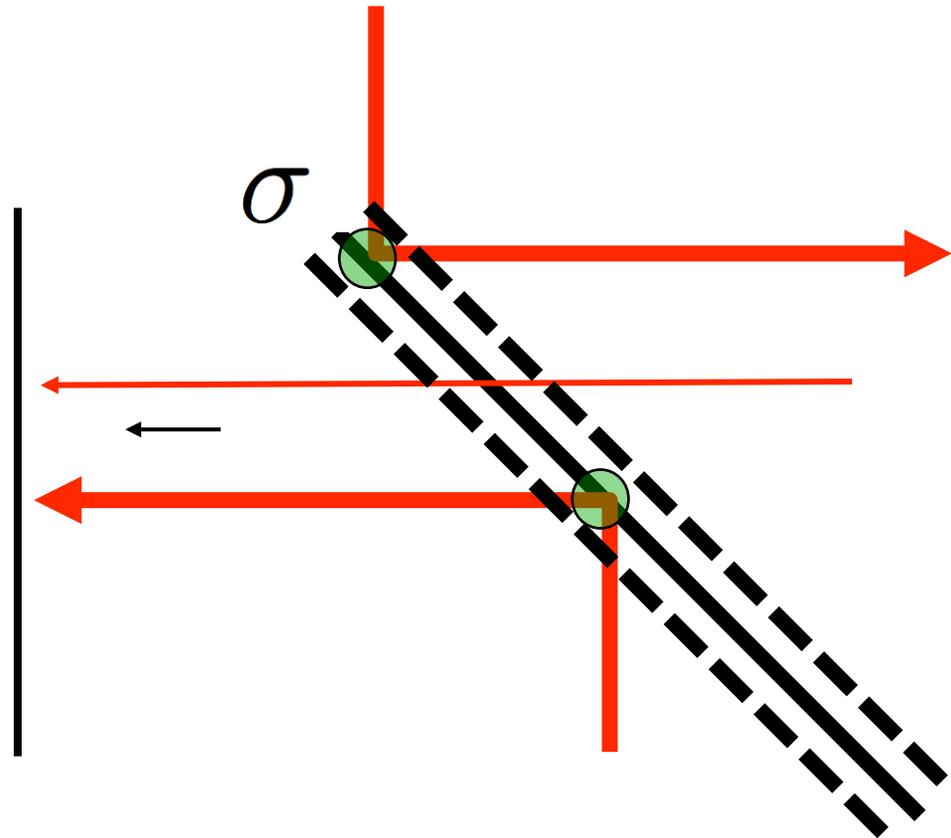
Quantum uncertainty of transverse positions of beamsplitter

- Position wavefunction widths of beamsplitter at reflection events related by

$$\sigma' \sigma = \lambda L$$

- apparent arm length difference is a random variable, with variance:

$$\Delta L_0^2 = \sigma^2 + \sigma'^2 = 2\sigma^2 = 4l_P L_0$$



this is a new effect predicted with no parameters

Power Spectral Density of Shear Noise

Uncertainty in angle \sim dimensionless shear

$$\Delta\theta(L) = (l_P/L)^{1/2}$$

At $f=c/2L$, shear fluctuations with *power spectral density*

$$h_H^2 \simeq L\Delta\theta^2 \approx t_P$$

h_H^2 = mean square perturbation per frequency interval

(no parameters, Planck length is the only scale)

Universal Holographic Noise

*flat power spectral density of **shear** perturbations:*

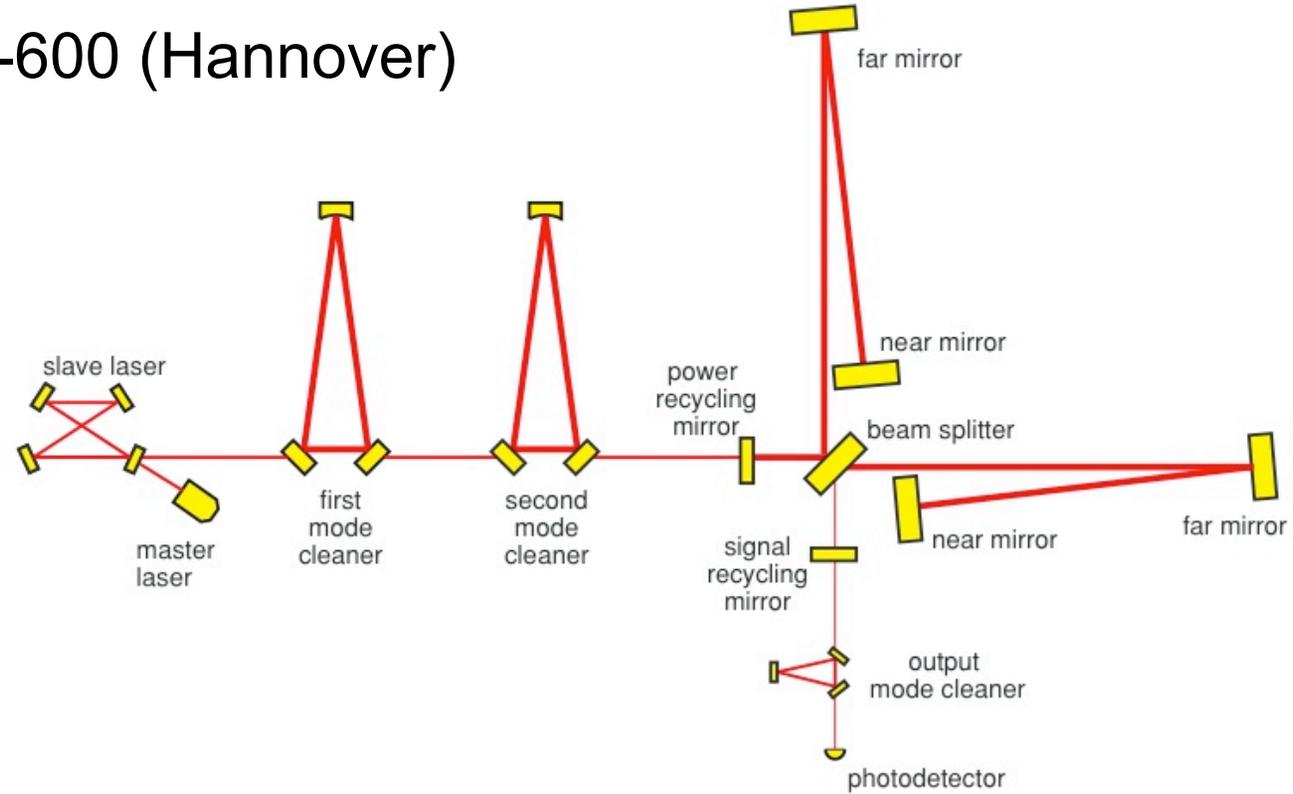
$$h \approx \sqrt{t_P} = 2.3 \times 10^{-22} \text{Hz}^{-1/2}$$

- general property of holographic quantum geometry
- Prediction of spectrum with no parameters
- Prediction of spatial shear character: only detectable in transverse position observables
- Definitely falsifiable

Holographic noise does not carry energy or information

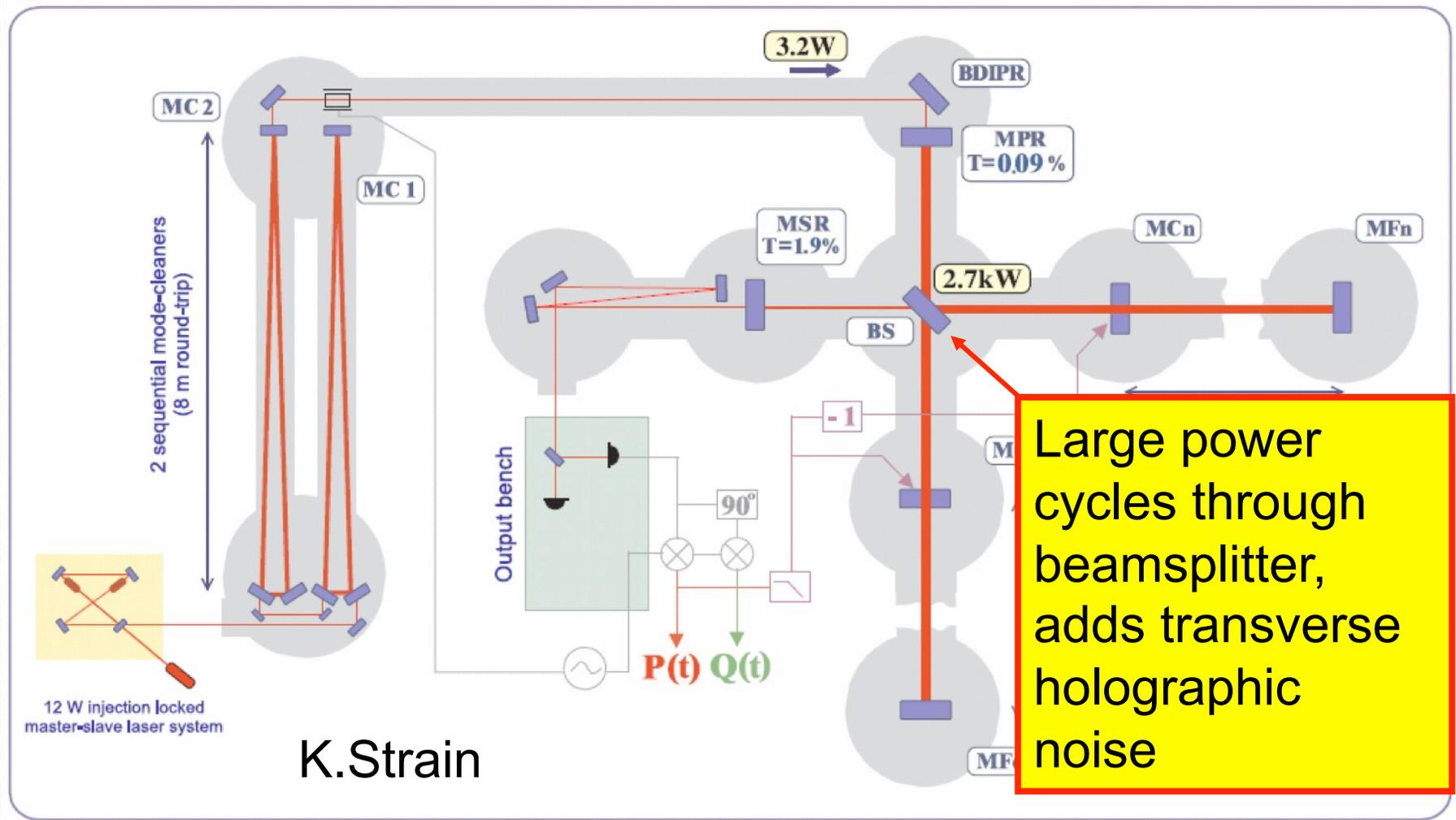
- ~ classical gauge mode (flat space, no classical spacetime degrees of freedom excited)
- ~ sampling or pixelation noise, not thermal noise
- Bandwidth-limited precision
- Necessary so the number of distinguishable positions does not exceed holographic bound on Hilbert space dimension
- No curvature
- no strain, just shear
- no detectable effect in a purely radial measurement

GEO-600 (Hannover)

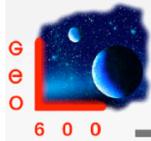




The GEO600 Interferometer

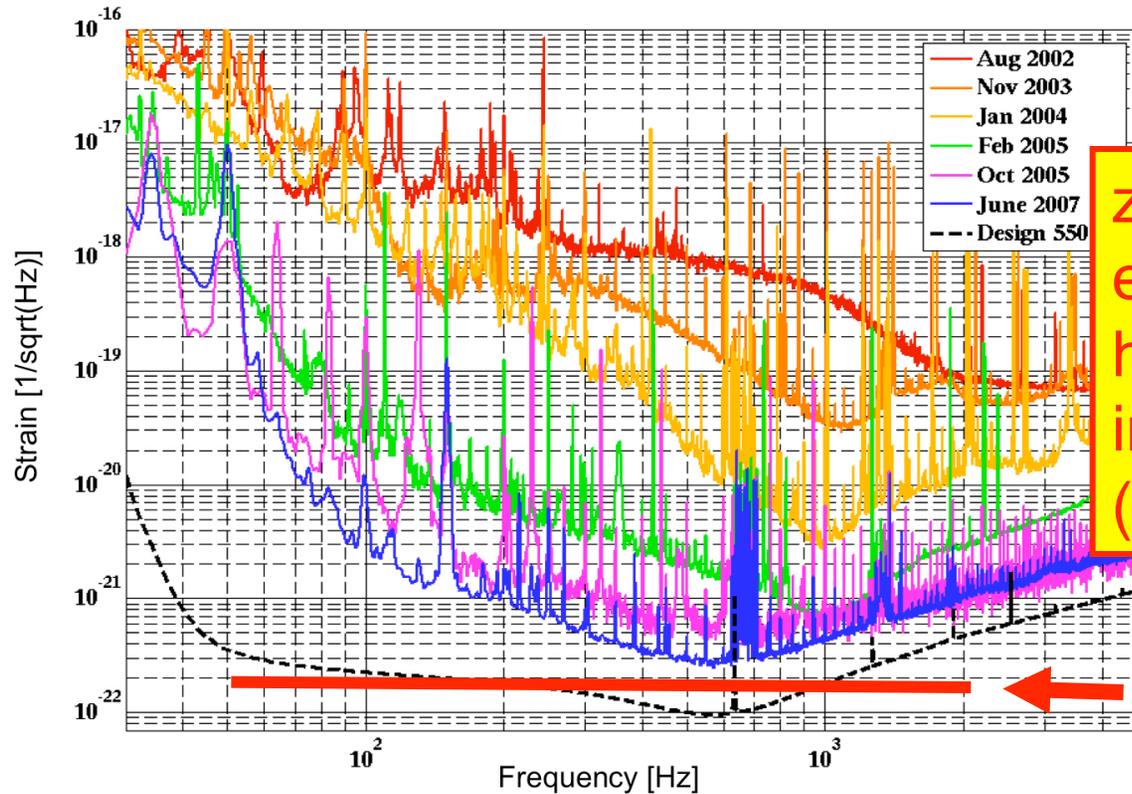


Noise in GEO600



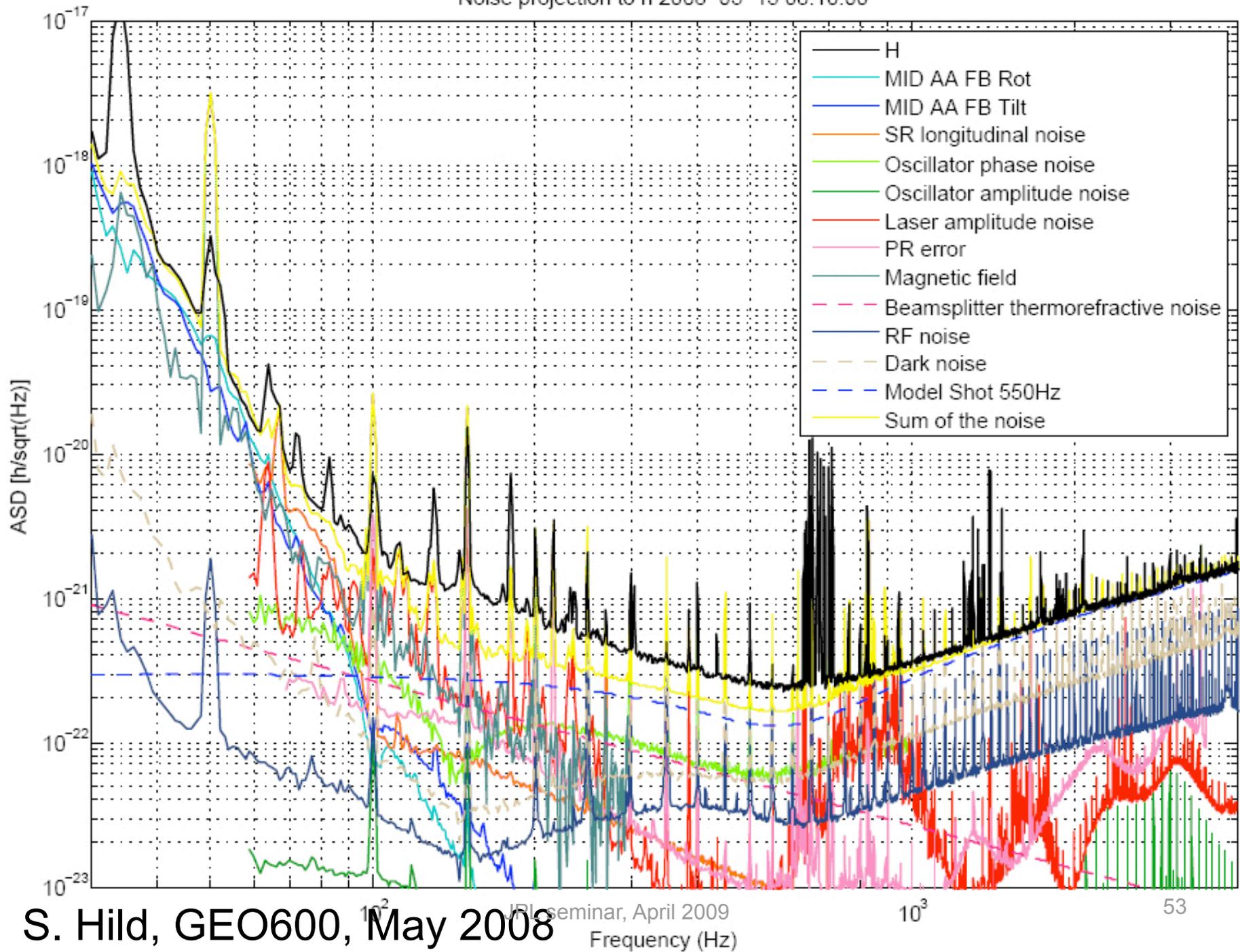
GEO Sensitivities

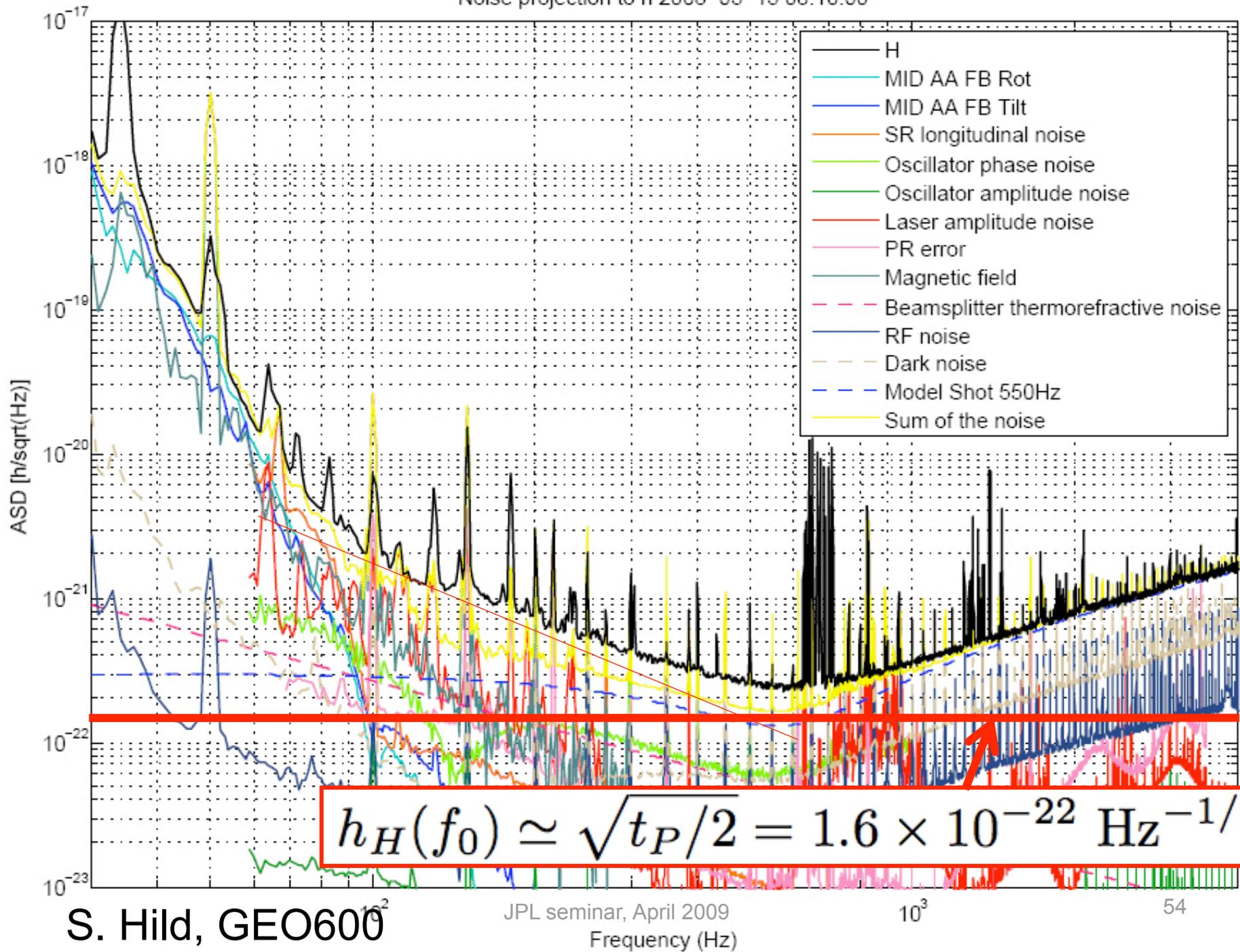
K. Strain



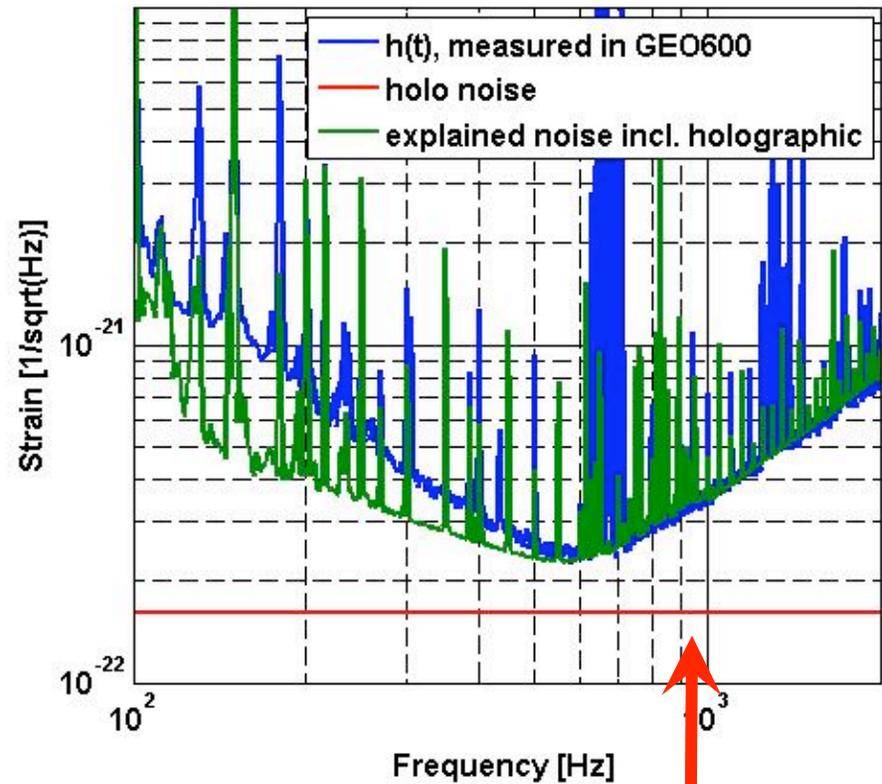
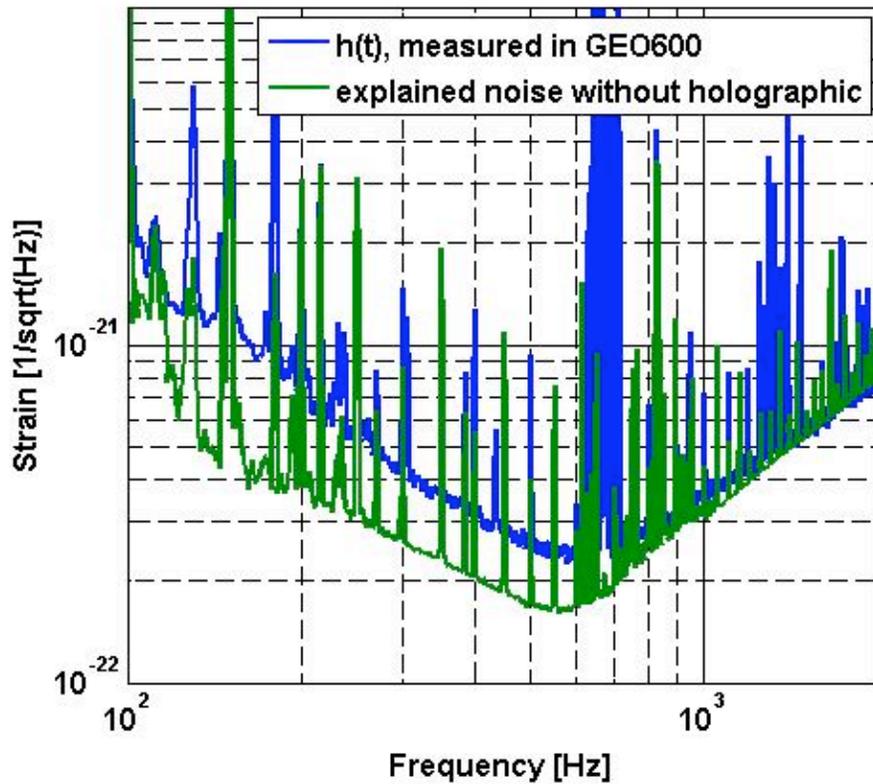
zero-parameter estimate of holographic noise in GEO600 (equivalent strain)

$$\sqrt{t_{Planck}} / 2$$





“Mystery Noise” in GEO600



Data: S. Hild (GEO600)

Prediction: CJH, arXiv:0806.0665
(Phys Rev D.78.087501)

$$\sqrt{t_{Planck}} / 2$$

zero-parameter prediction for holographic noise in GEO600 (equivalent GW strain)

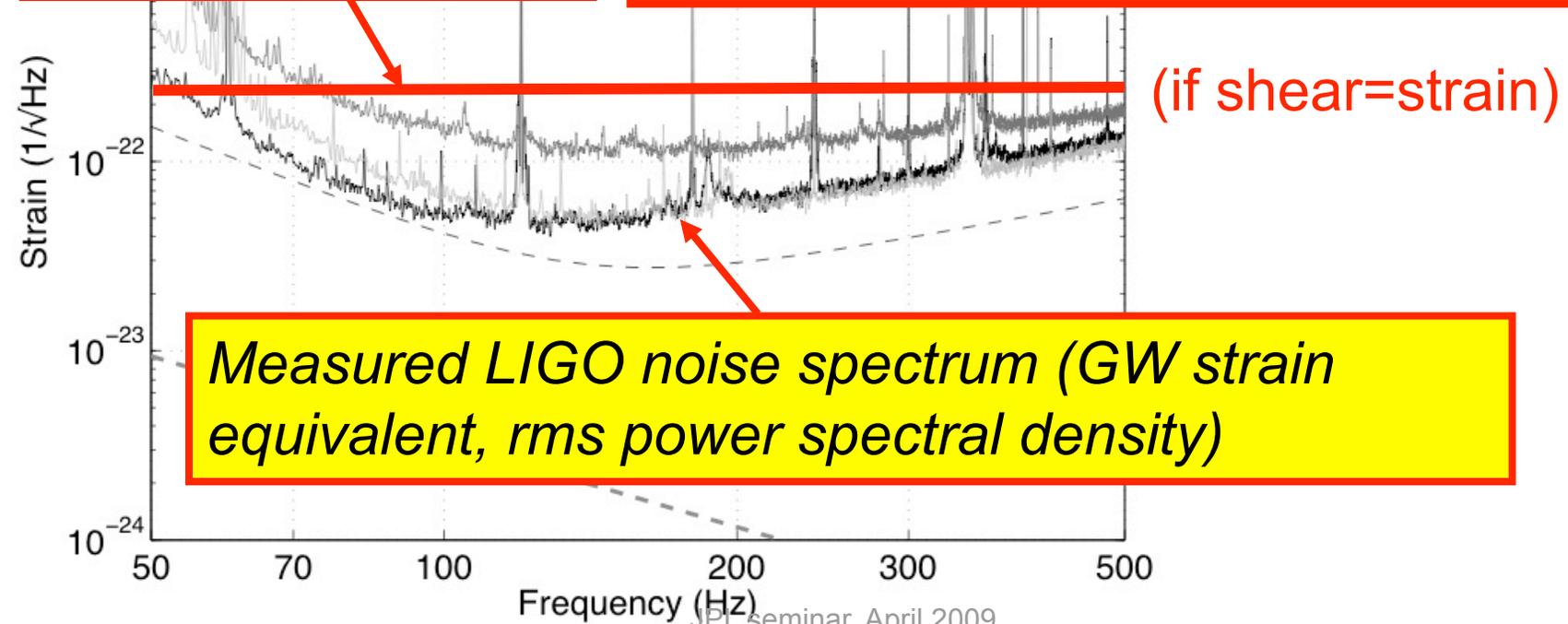
Total noise: not fitted JPL seminar, April 2009

LIGO noise (astro-ph/0608606)



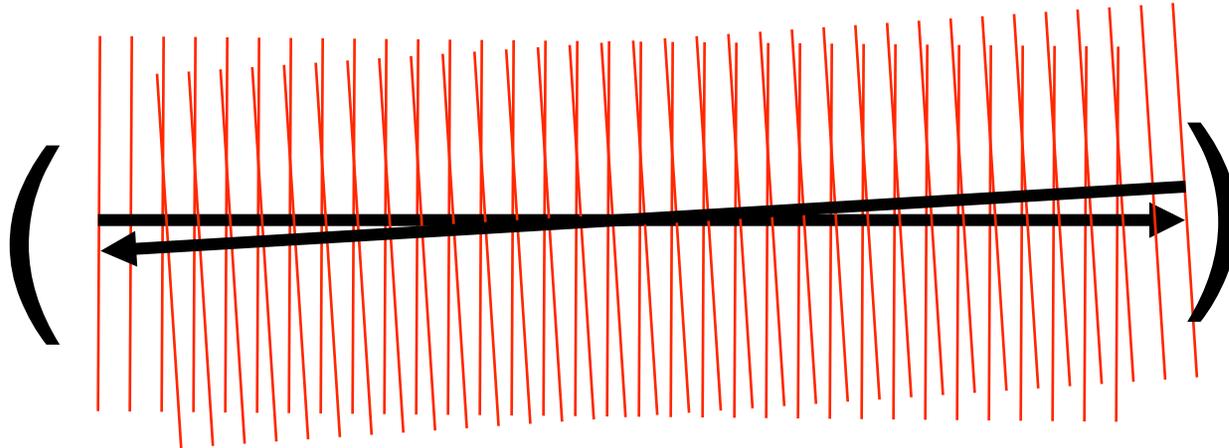
holographic noise spectrum (shear)

$$\sqrt{t_P} = 2.3 \times 10^{-22} / \sqrt{\text{Hz}}$$

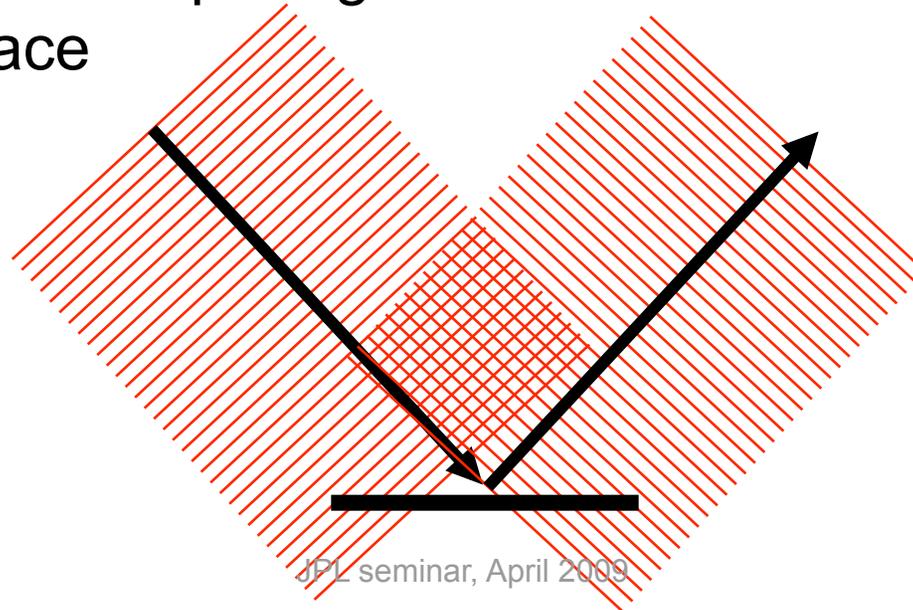


Measured LIGO noise spectrum (GW strain equivalent, rms power spectral density)

Normal incidence optics: phase signal does not record the transverse position of a surface



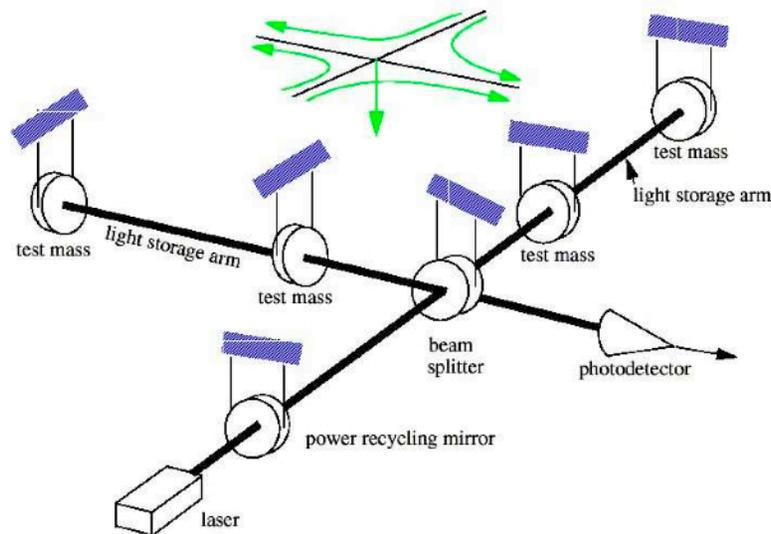
- But phase of beam-split signal is sensitive to transverse position of surface



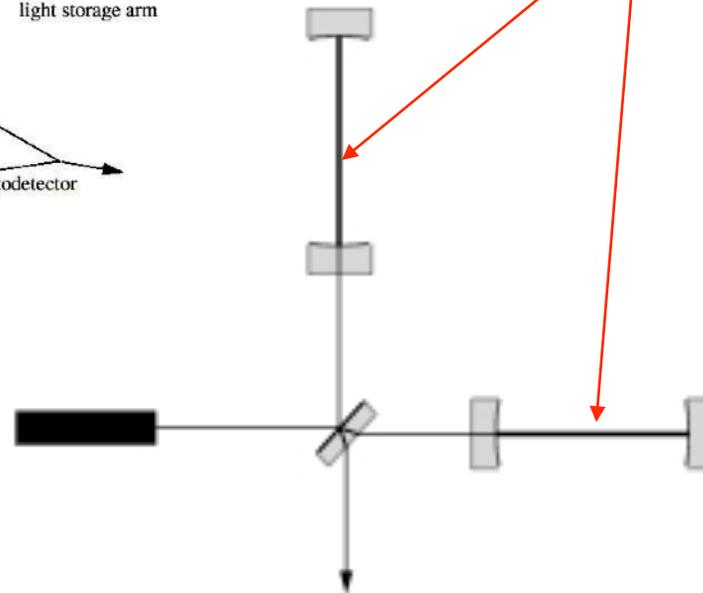
Why doesn't LIGO detect holographic noise?

- LIGO design is not as sensitive to transverse displacement noise as GEO600
- relationship of holographic to gravitational wave depends on details of the system layout

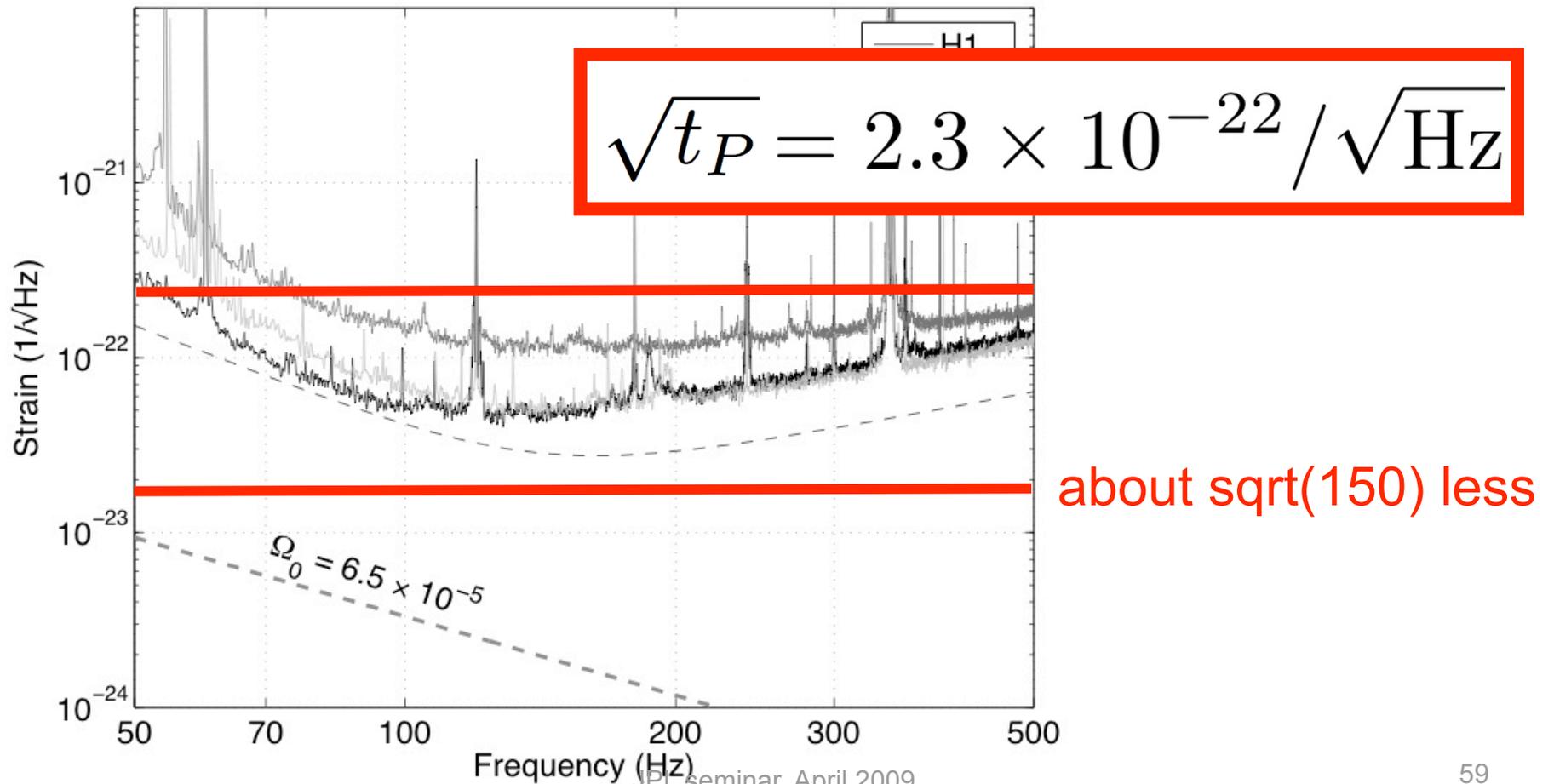
Fig. 1. Schematic layout of a LIGO interferometer.



Transverse position measurement is not made in FP cavities



LIGO noise, and holographic noise prediction based on square root of arm cavity finesse



Interferometers can detect quantum indeterminacy of holographic geometry

- Beamsplitter position indeterminacy inserts holographic noise into signal
- **system with GEO600 technology can detect holographic noise if it exists**
- Signatures: spectrum, spatial shear

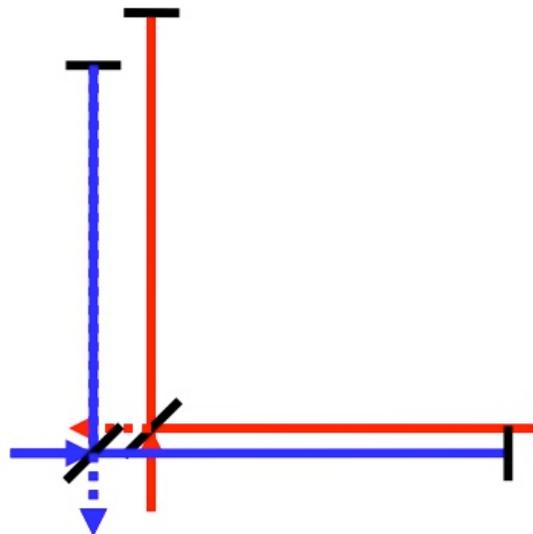
CJH: Phys. Rev. D 77, 104031 (2008); [arXiv:0806.0665](https://arxiv.org/abs/0806.0665)

Current experiments: summary

- Most sensitive device, GEO600, sees noise compatible with holographic spacetime indeterminacy
- requires testing and confirmation!
- H. Lück: "...it is way too early to claim we might have seen something."
- But GEO600 is operating at holographic noise limit
- LIGO: current system not sensitive enough, awaits upgrade
- Followup possible at higher frequency
- Proof: new apparatus, coherence of adjacent systems

Holographic quantum geometry experiments: beyond GW detectors

- $f \sim 100$ to 1000 Hz with GW machines
- $f \sim$ MHz possible with new apparatus on ~ 40 m scale
- Easier suspension, isolation, optics, vacuum, smaller scale
- Correlated holographic noise in adjacent paths:
signature of holographic effect

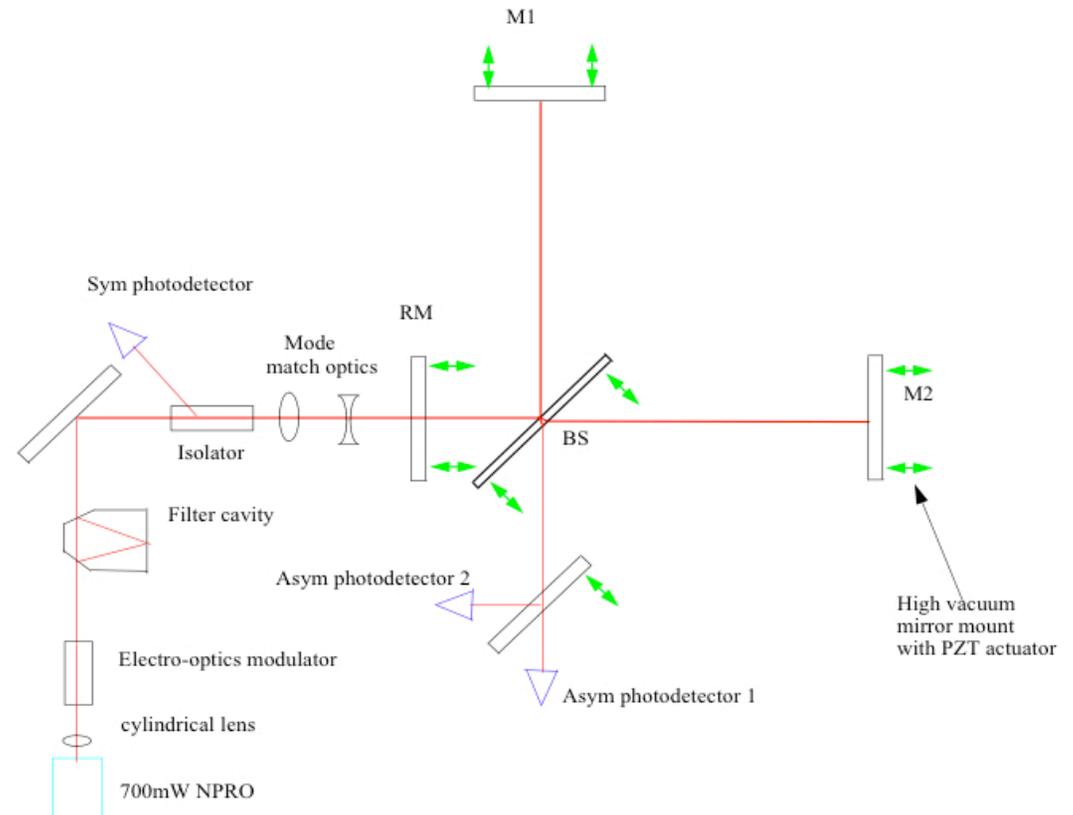


A dedicated experiment?

Two ~40m Michelson interferometers in coincidence

~1000 W cavity

holographic noise = laser photon shot noise in ~5 minutes (1 sigma)



Currently discussing: Fermilab (CJH, Chou, Wester, Steffen), MIT (Weiss, Waldman), Caltech (Whitcomb, Ballmer), AEI (Danzmann, Lück, Hild, Grote), UC (Meyer)

Experimental science of holographic noise

- Direct measurement of the fundamental minimum time interval, total number of physical degrees of freedom
- Precisely measure Planck time: compare with value derived from Newton's G , test fundamental theory
- Test predictions for spectrum and spatial correlations: properties of holographic geometry
- Connects with quantum physics of Dark Energy, inflationary fluctuations

Next Steps

- LIGO 2km/4km correlation at high frequency
- GEO600 upgrades/retuning/ sample at free spectral range (125 kHz)
- Conceptual experimental design at higher frequencies
- Future: other technologies for measuring high precision, low noise, nonlocal relative transverse positions (e.g., atom interferometers)
- Improve/axiomatize connections with M theory, field theory