The Quest for Primordial Non-Gaussianity

Overview and some recent developments;
skewed toward data/observations

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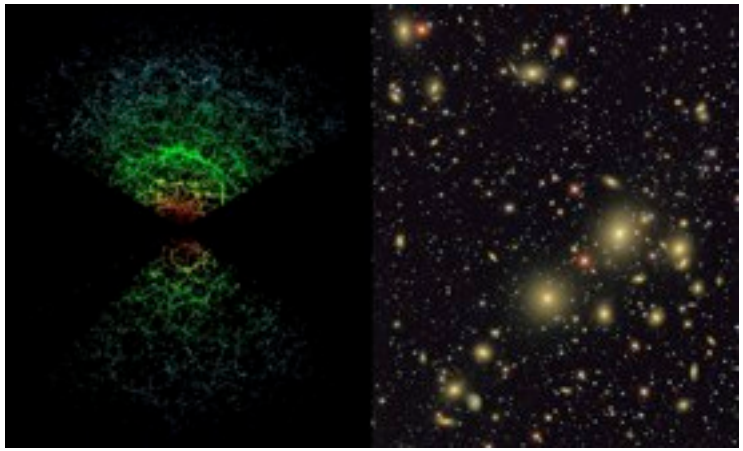
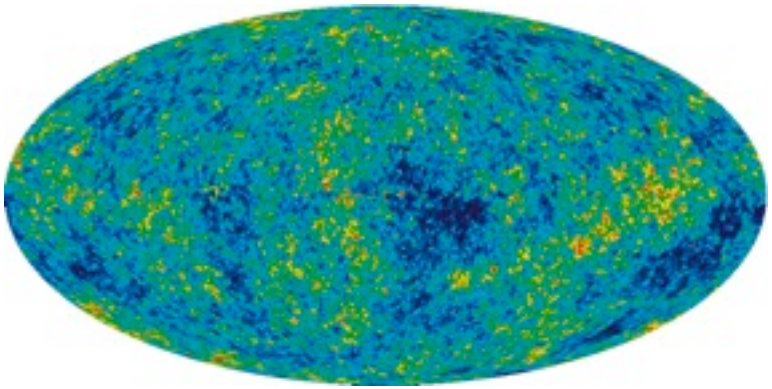
Why study non-Gaussianity (NG)?

1. NG presents a window to the very early universe. For example, NG can distinguish between physically distinct models of inflation.

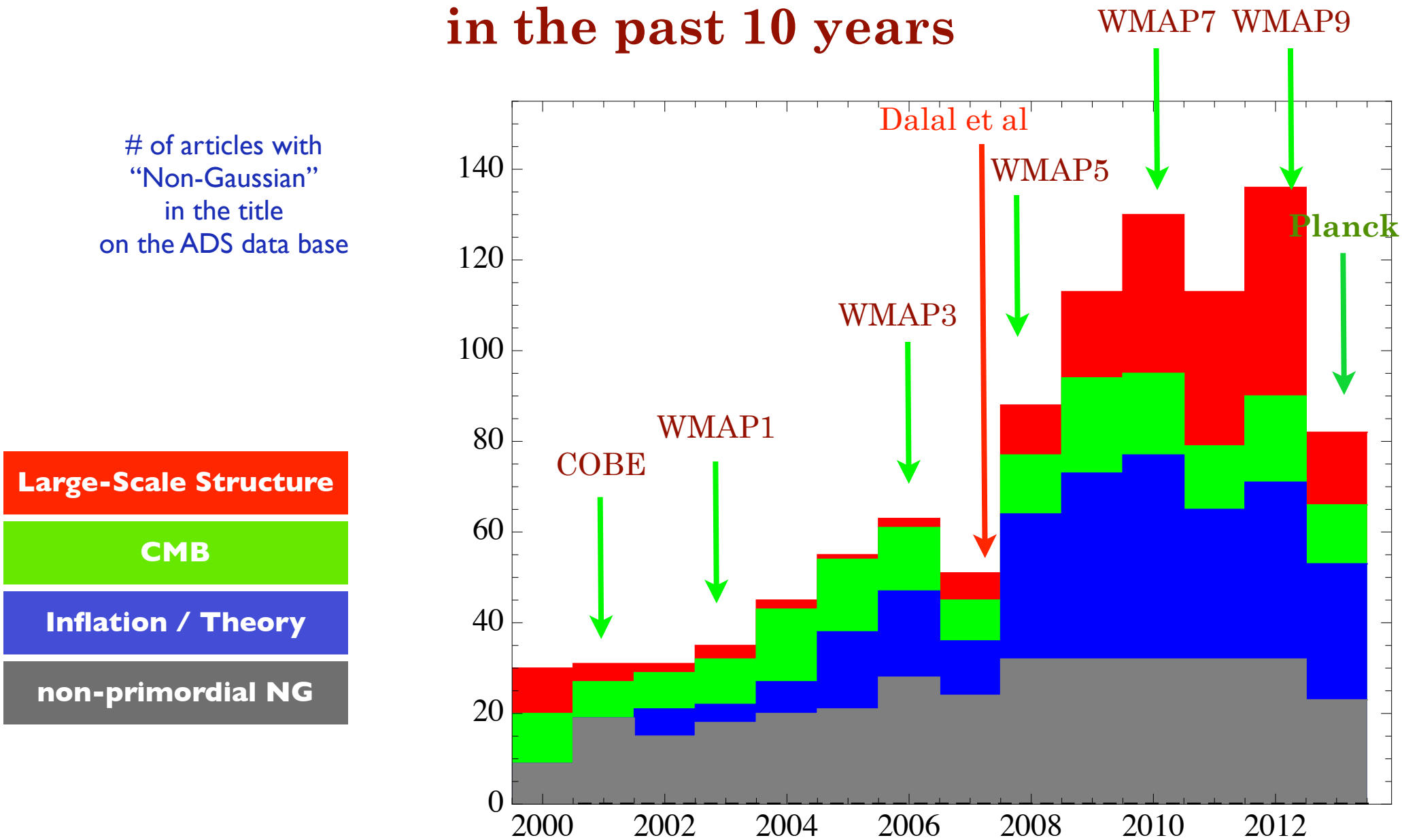
2. Conveniently, NG can be constrained/measured using CMB anisotropy maps and LSS. In particular, there is a rich set of observable quantities that are sensitive to primordial NG.





Non-Gaussianity papers in the past 10 years

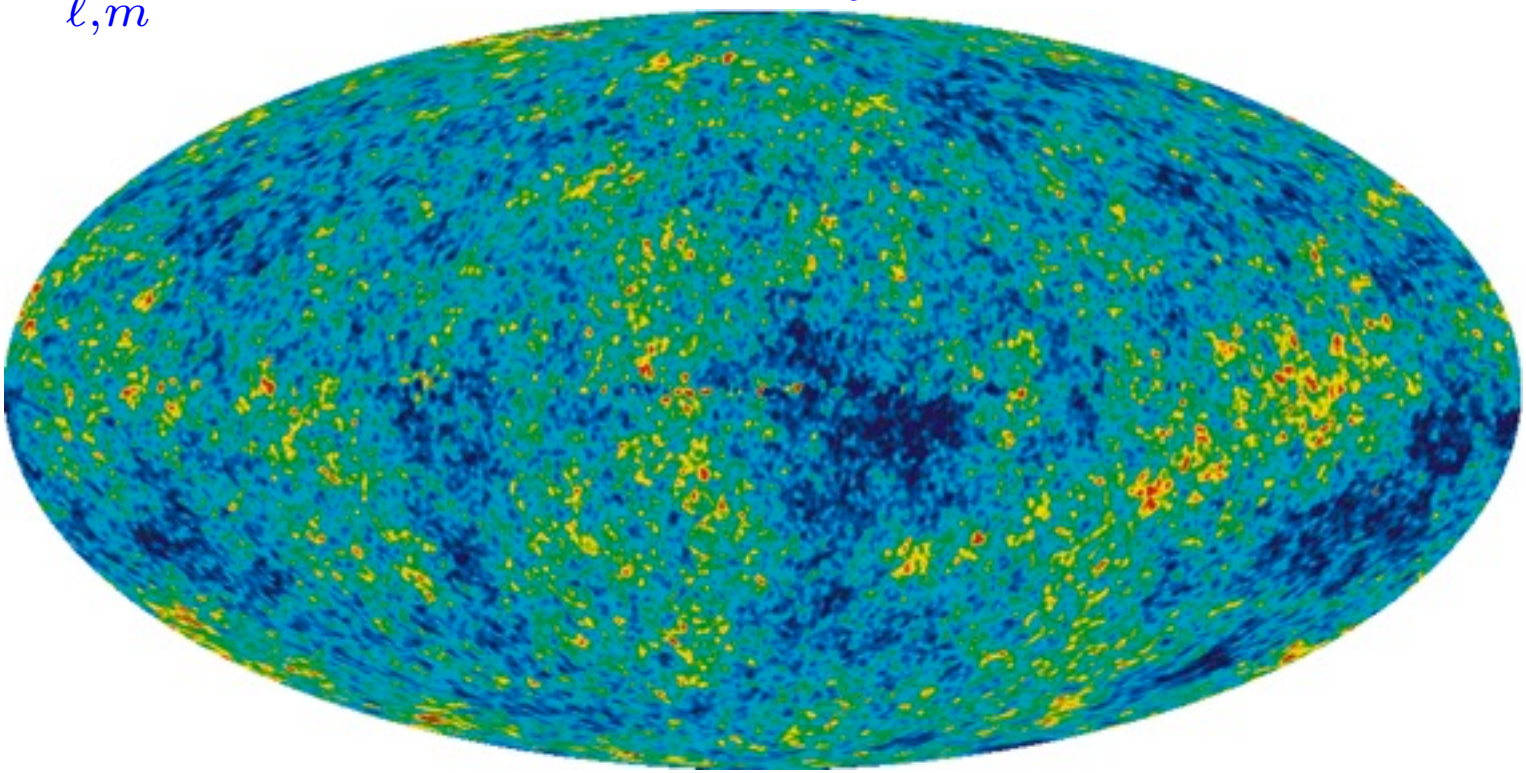
of articles with
"Non-Gaussian"
in the title
on the ADS data base



Produced by Emiliano Sefusatti

Initial conditions in the universe

$$\frac{\delta T}{T}(\theta,\phi) = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(\theta,\phi) \qquad \ell \simeq \frac{180^\circ}{\theta}$$



Generic inflationary predictions:

Statistical Isotropy:

$$\langle a_{\ell m}\, a_{\ell' m'} \rangle \equiv C_{\ell\ell' m m'} = C_\ell \delta_{\ell\ell'} \delta_{mm'}$$

- Nearly scale-invariant spectrum of density perturbations
- Background of gravity waves
- (Very nearly) gaussian initial conditions:

Gaussianity:

$$\langle a_{\ell m}\, a_{\ell' m'}\, a_{\ell'' m''} \rangle = 0$$

Standard Inflation, with...

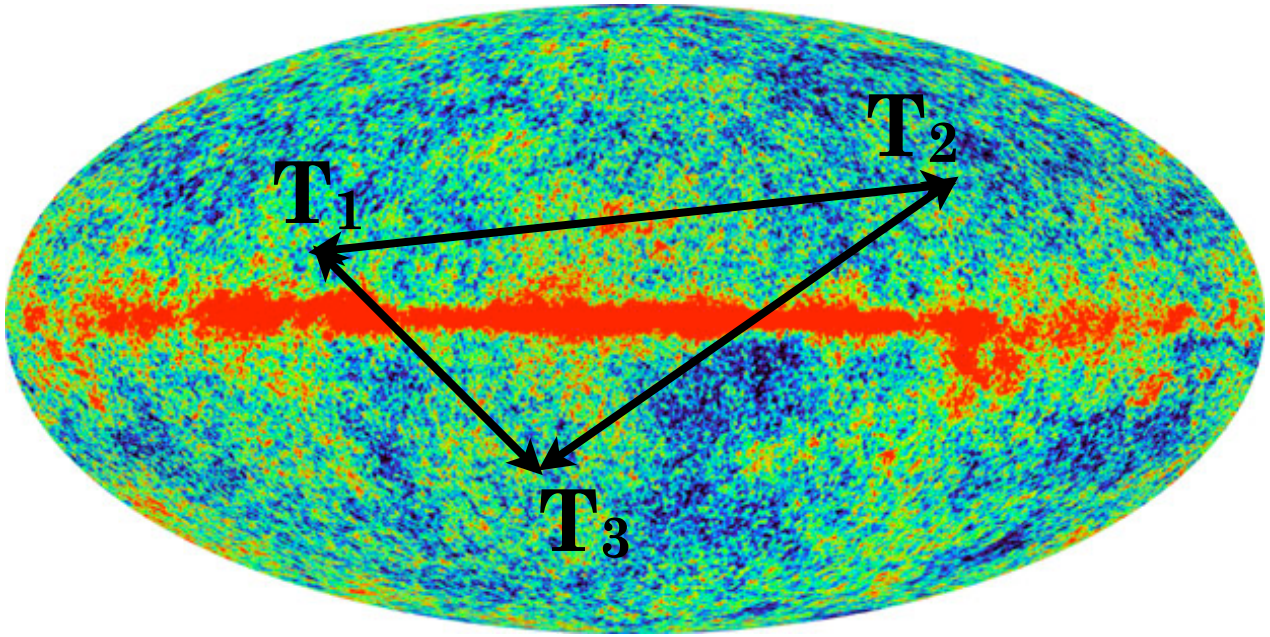
1. a single scalar field
2. the canonical kinetic term
3. always slow rolls
4. in Bunch-Davies vacuum
5. in Einstein gravity

produces **unobservable** NG

Therefore, measurement of nonzero NG would point to a **violation** of one of the assumptions above

e.g. Maldacena 2003, X. Chen, Adv. Astronomy, 2010; Komatsu et al, arXiv:0902.4759

NG from 3-point correlation function



Commonly used "local" model of NG

$$\Phi = \Phi_G + f_{\rm NL}\left(\Phi_G^2 - \langle \Phi_G^2 \rangle\right)$$

Salopek & Bond 1990; Verde et al 2000; Komatsu & Spergel 2001; Maldacena 2003

Then the 3-point function is related to $f_{\rm NL}$ via (in k-space)

$$B(k_1, k_2, k_3) \sim f_{\rm NL}\left[P(k_1)P(k_2) + \text{perm.}\right]$$

Recall: power spectrum

Define Fourier transform of density fluctuation:

$$\delta(\vec{r}) = \int \frac{d^3k}{(2\pi)^3}\, e^{-i\vec{k}\vec{r}}\, \delta_{\vec{k}}$$

Then the power spectrum P(k) is defined via

$$\boxed{\langle \delta_{\vec{k}_1}\, \delta_{\vec{k}_2^*} \rangle = (2\pi)^3\, \delta^{(3)}(\vec{k}_1 - \vec{k}_2)\, P(k)}$$

Sometimes it's nice to work in harmonic space

$$a_{\ell m} = 4\pi(-i)^\ell \int \frac{d^3k}{(2\pi)^3}\, T_\ell(k)\, \delta(\vec{k})\, Y_{\ell m}(\hat{k})$$

Then the angular power spectrum is defined as:

$$\boxed{\langle a_{\ell m} a_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'}\, C_\ell}$$

The bispectrum:

similar, but for 3-pt function

Fourier space:

$$\langle \delta_{\vec{k}_1}\, \delta_{\vec{k}_2}\, \delta_{\vec{k}_3} \rangle = (2\pi)^3\, \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)\, B(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

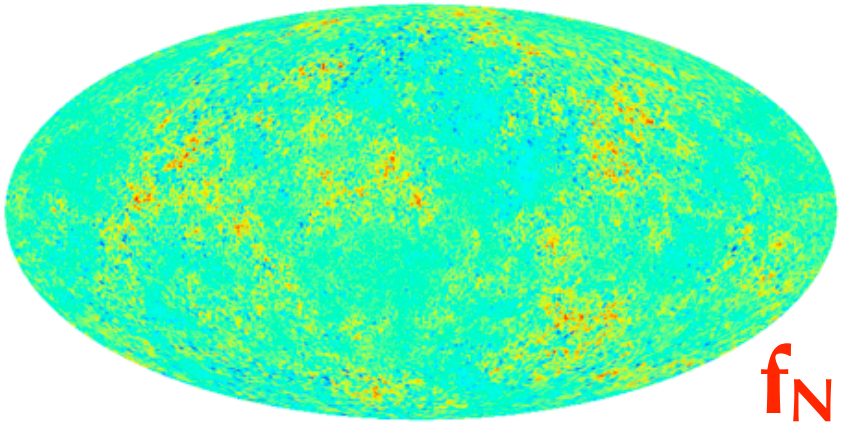
Harmonic space:

$$\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle \equiv B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3}$$

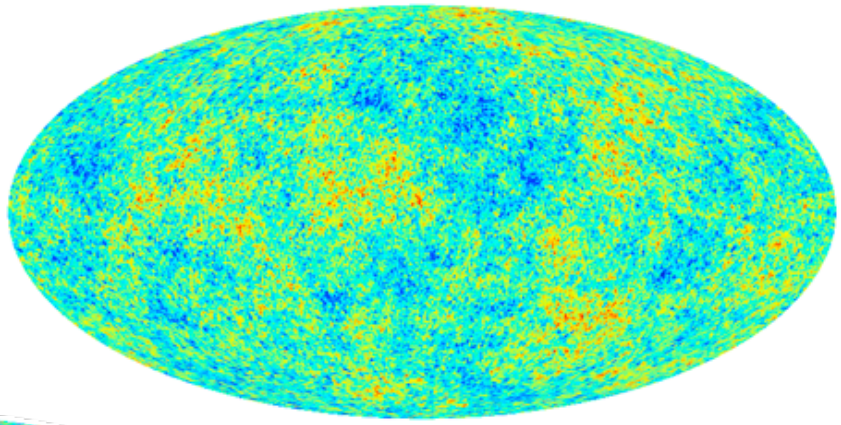
and the angle-averaged bispectrum is

$$\boxed{B_{\ell_1 \ell_2 \ell_3}} \equiv \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell+1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} \sum_{m_1 m_2 m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3}$$

nonzero when: $\max(l_i - l_j) \leq l_k \leq l_i + l_j$; $l_i + l_j + l_k$ is even

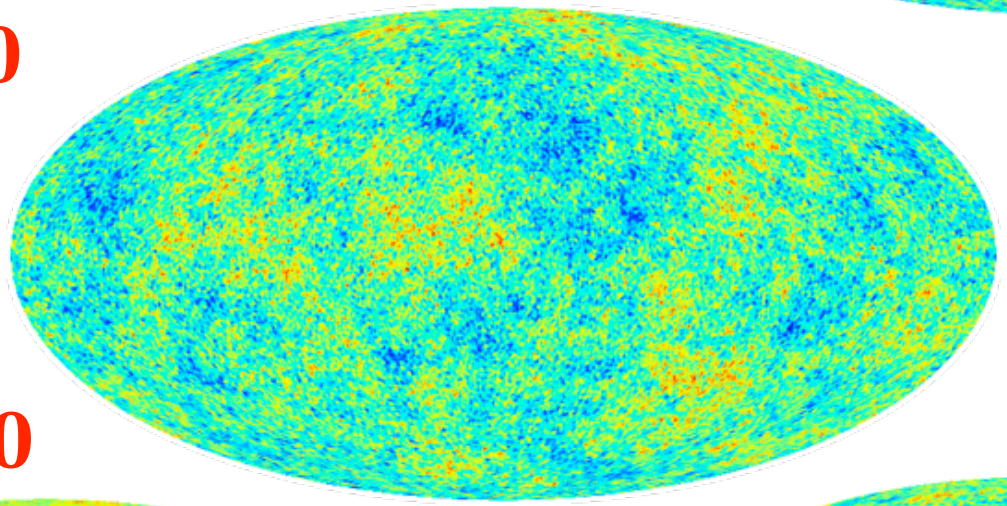


f_{NL}= -5000

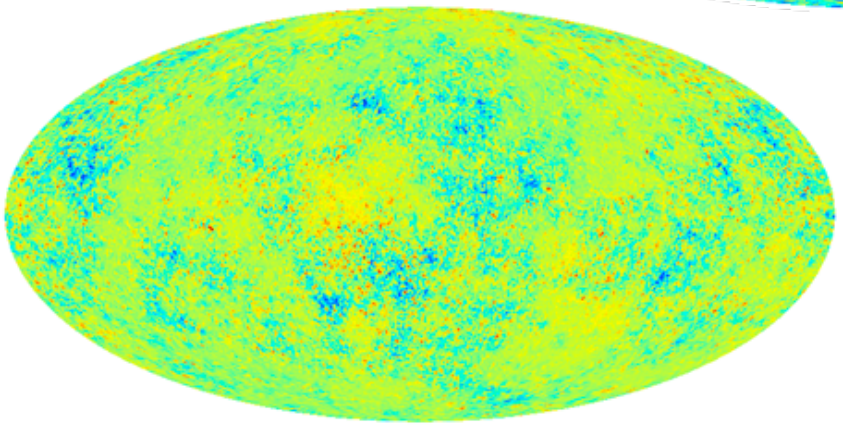


f_{NL}= -500

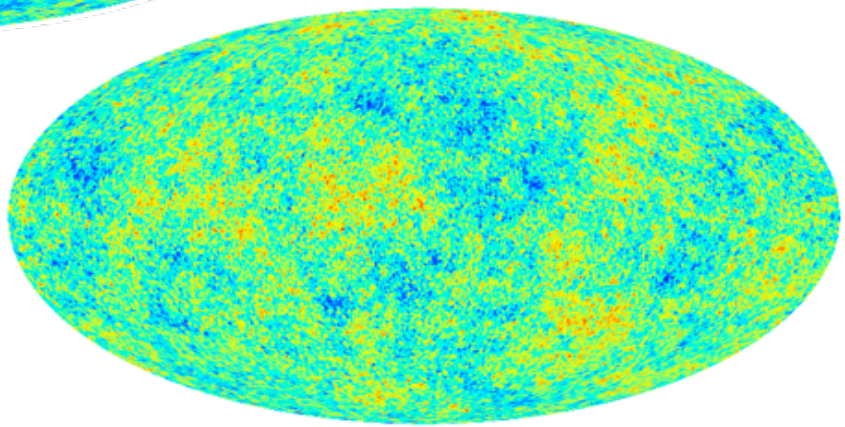
f_{NL}= 0



f_{NL}= +5000

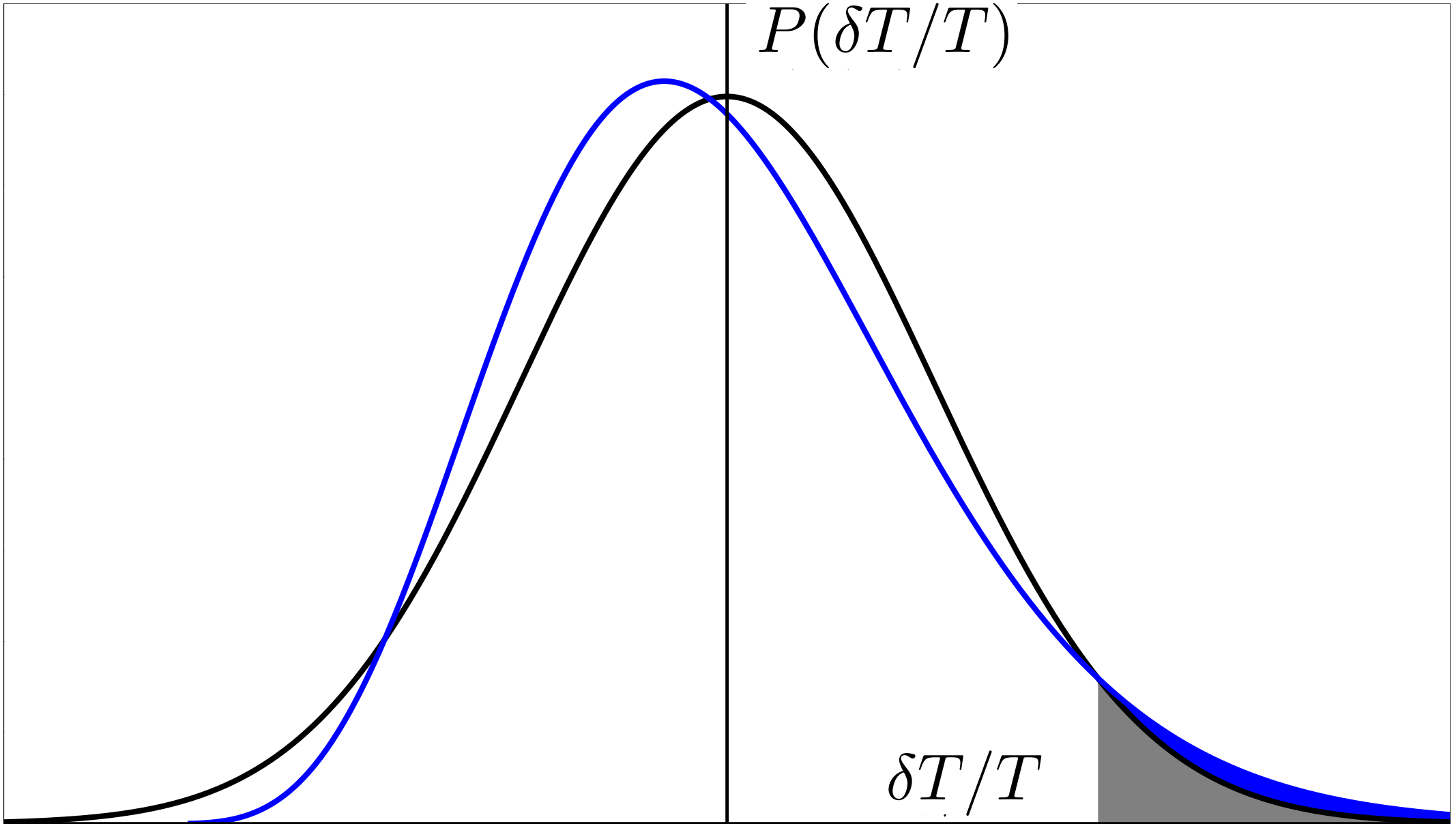


f_{NL}= +500



Using publicly available NG maps by Elsner & Wandelt

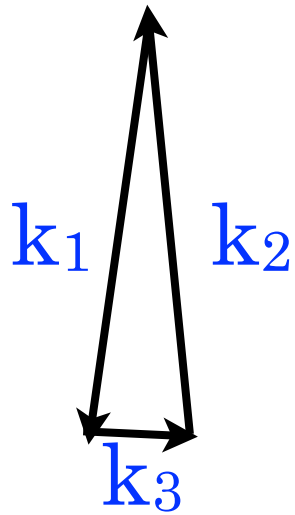
Current upper bound on NG is
~1000 times smaller than this:



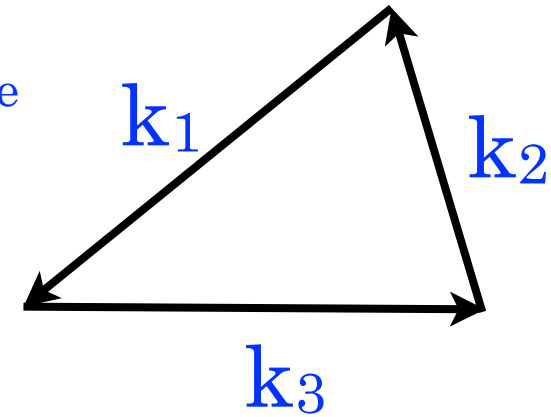
3-pt correlation function of CMB anisotropy
⇒ direct window into inflation

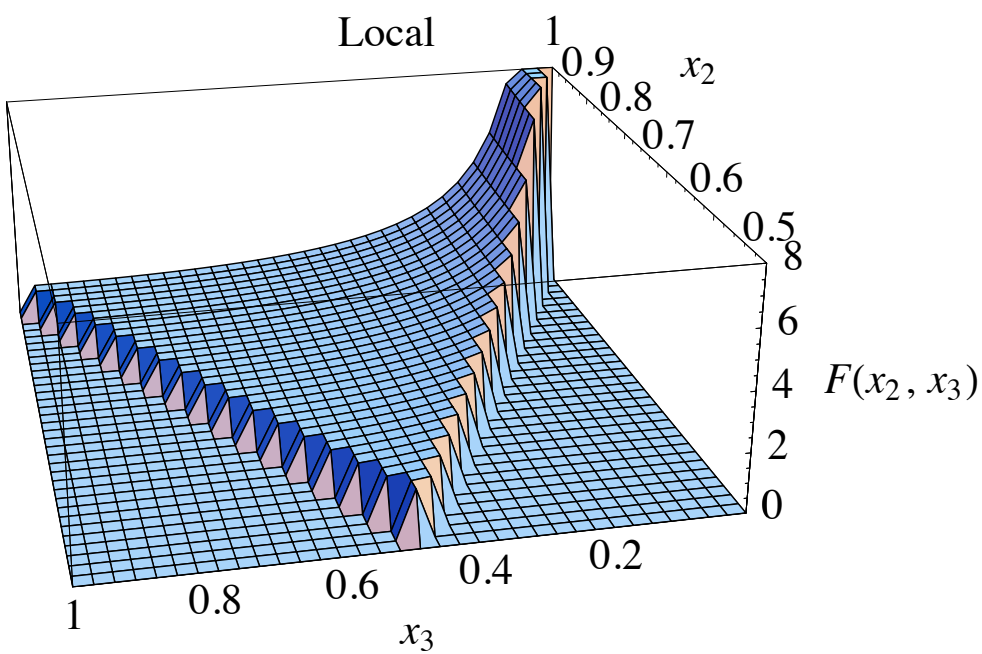
e.g. Luo & Schramm 1993

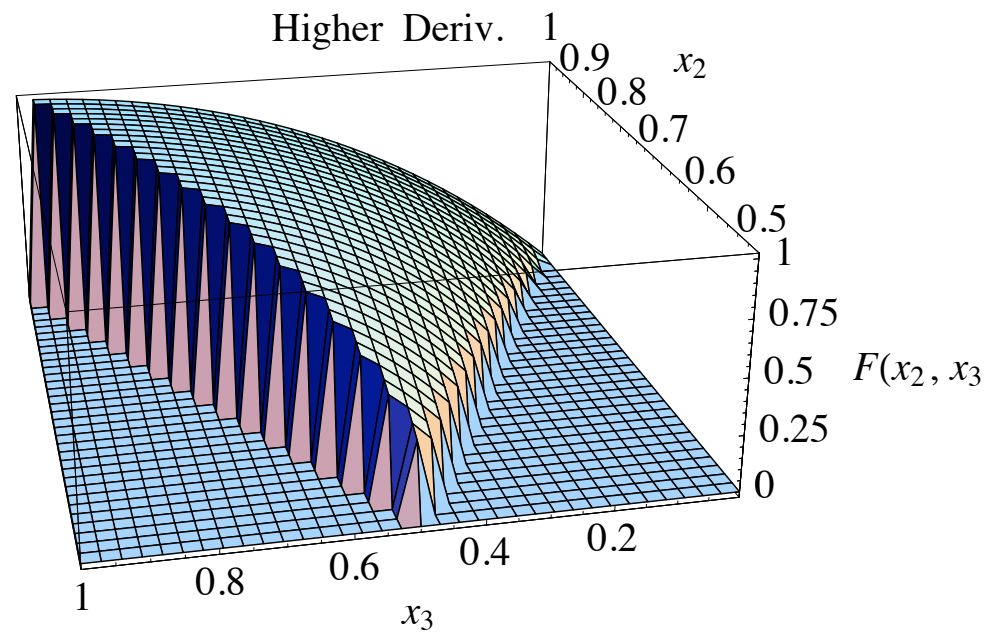
"local"
(eg. multi-field)



"equilateral"
(eg. higher-derivative action; interactions)







Babich, Creminelli & Zaldarriaga 2004

Brief history of NG measurements: 1990's

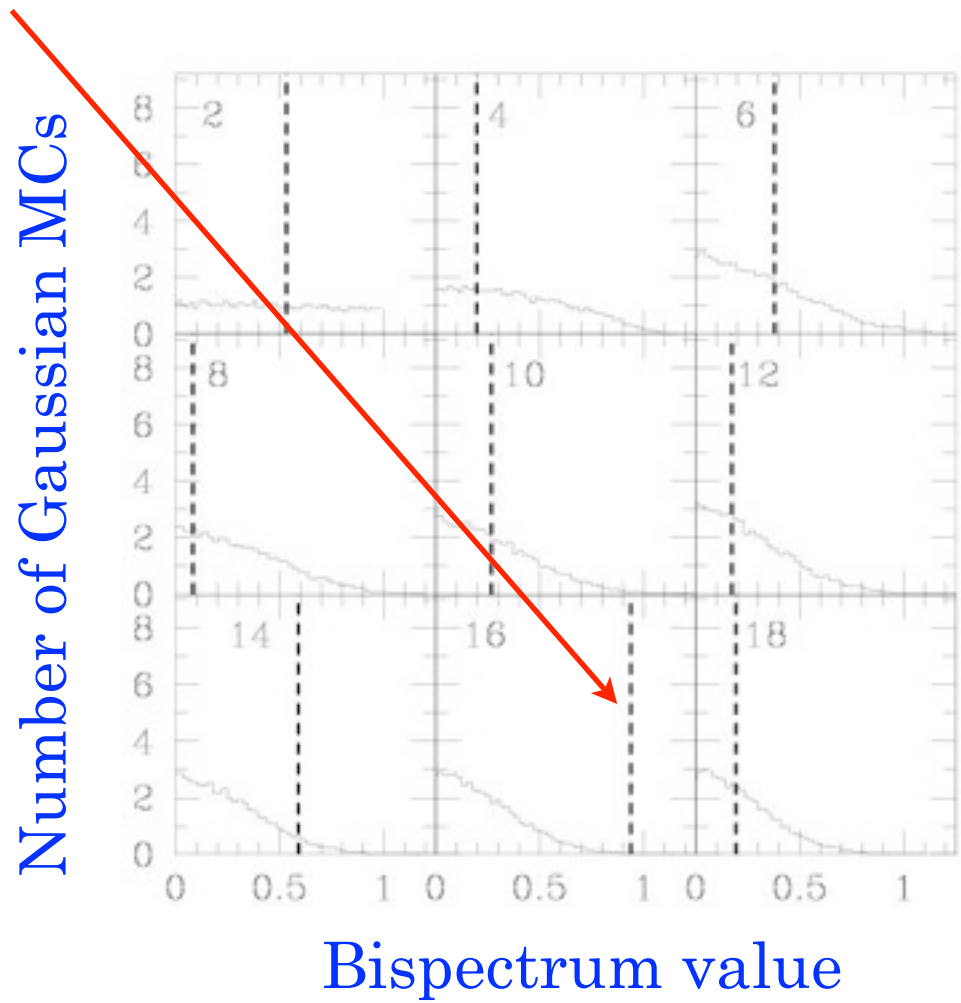
Early 1990s; COBE: Gaussian CMB sky (Kogut et al 1996)

$|f_{NL}| \lesssim 3000$ (Komatsu 2002)

1998; COBE: claim of NG at l=16 equilateral bispectrum
(Ferreira, Magueijo & Gorski 1998)

but explained by a known systematic effect!
(Banday, Zaroubi & Gorski 1999)

(and anyway isn't unexpected given all bispectrum configurations you can measure;
Komatsu 2002)



Brief history of NG measurements: 2000's

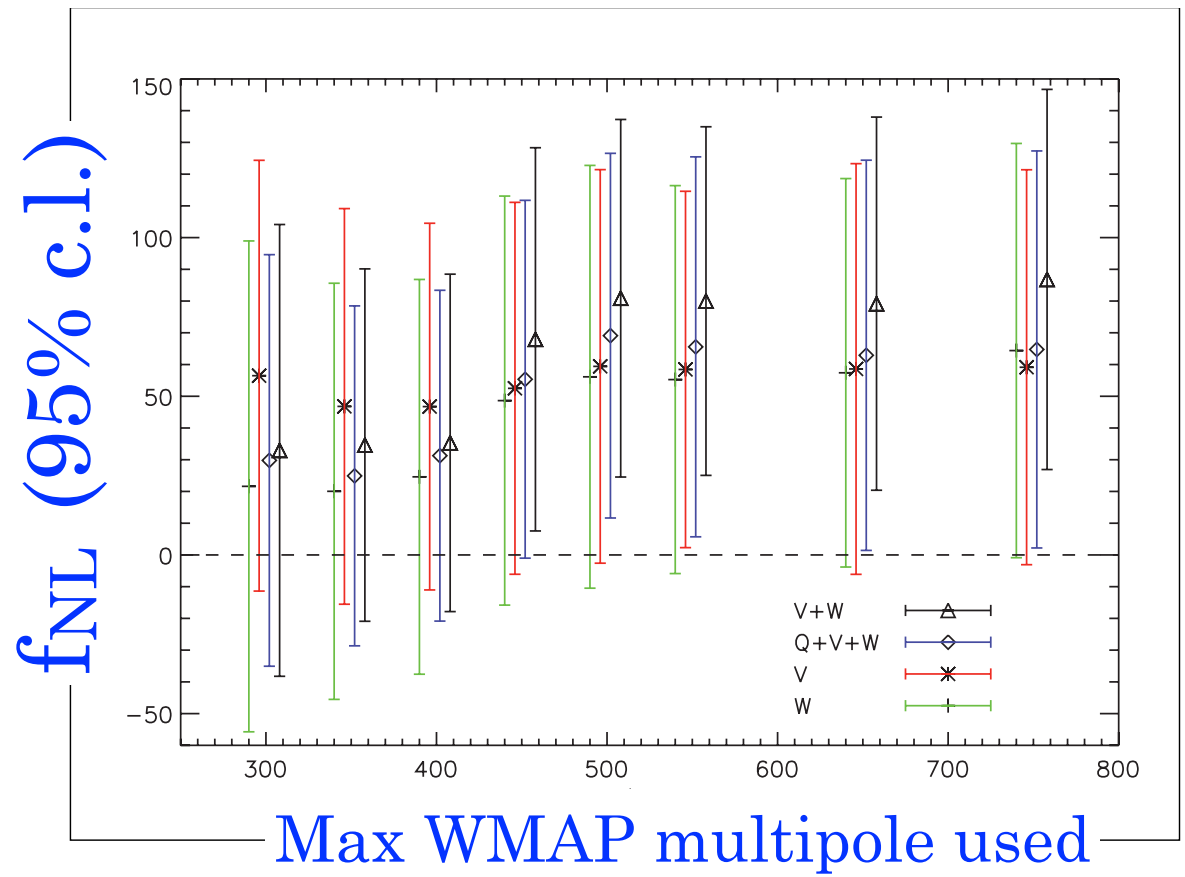
Pre-WMAP CMB: all is gaussian (e.g. MAXIMA; Wu et al 2001)

WMAP pre-2008: all is gaussian
(Komatsu et al. 2003; Creminelli, Senatore, Zaldarriaga & Tegmark 2007)

$-36 < f_{NL} < 100$ (95% CL)

Dec 2007, claim of NG in WMAP
(Yadav & Wandelt arXiv:0712.1148)

$27 < f_{NL} < 147$ (95% CL)



Constraints from WMAP

Band	Foreground[b]	$f_{NL}^{\rm local}$	$f_{NL}^{\rm equil}$	$f_{NL}^{\rm orthog}$	b_{src}
V+W	Raw	59 ± 21	33 ± 140	-199 ± 104	N/A
V+W	Clean	42 ± 21	29 ± 140	-198 ± 104	N/A
V+W	Marg.[c]	32 ± 21	26 ± 140	-202 ± 104	-0.08 ± 0.12
V	Marg.	43 ± 24	64 ± 150	-98 ± 115	0.32 ± 0.23
W	Marg.	39 ± 24	36 ± 154	-257 ± 117	-0.13 ± 0.19

Komatsu et al. 2010

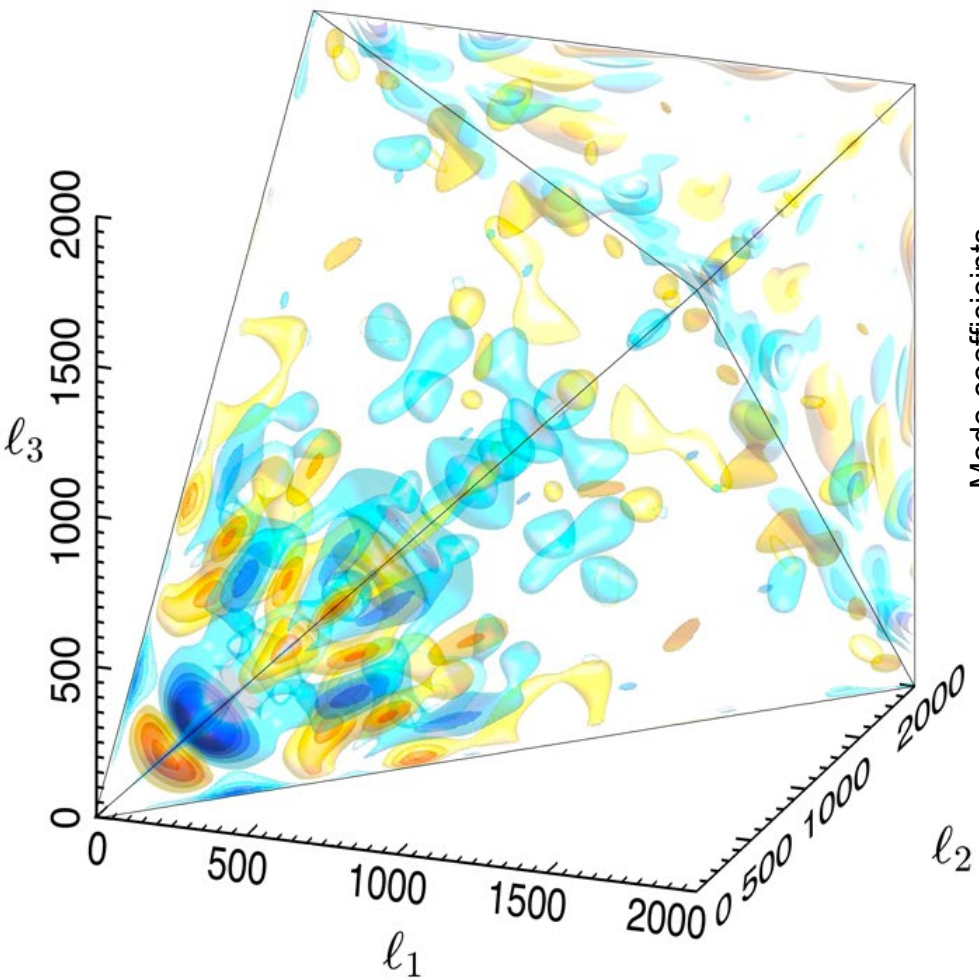
Constraints from Planck

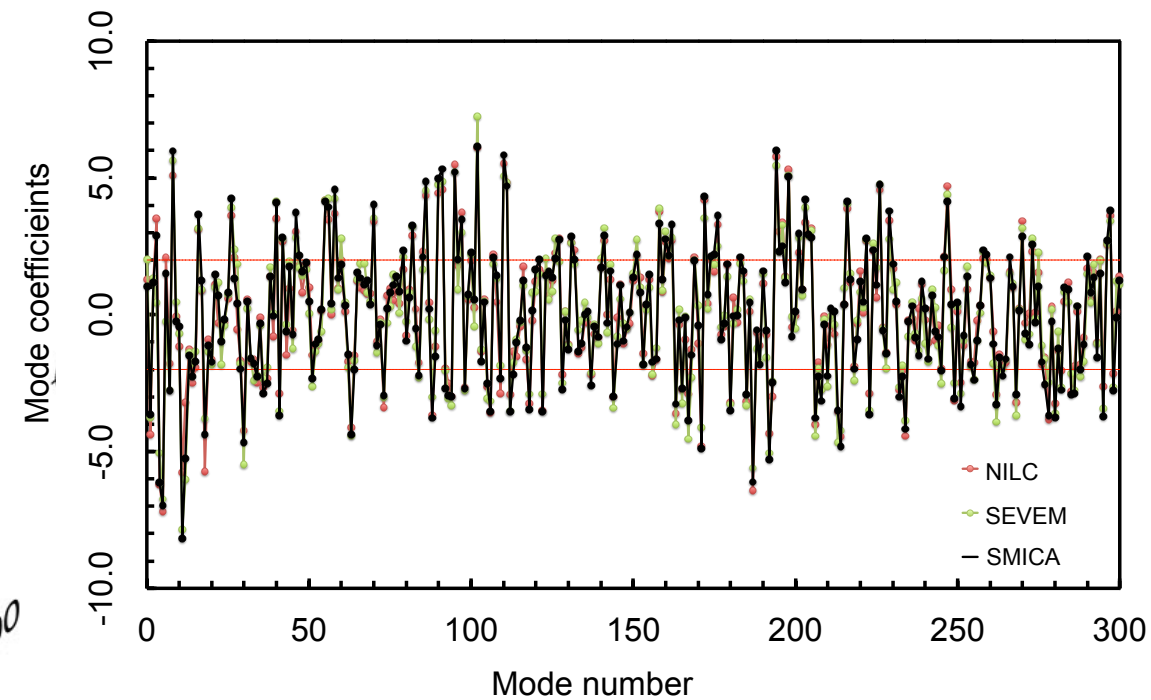
	ISW-lensing subtracted		
	KSW	Binned	Modal
SMICA			
Local	**2.7 ± 5.8**	2.2 ± 5.9	1.6 ± 6.0
Equilateral	**−42 ± 75**	−25 ± 73	−20 ± 77
Orthogonal	**−25 ± 39**	−17 ± 41	−14 ± 42
NILC			
Local	4.5 ± 5.8	3.6 ± 5.8	2.7 ± 6.0
Equilateral	−48 ± 76	−38 ± 73	−20 ± 78
Orthogonal	−53 ± 40	−41 ± 41	−37 ± 43
SEVEM			
Local	3.4 ± 5.9	3.2 ± 6.2	2.6 ± 6.0
Equilateral	−36 ± 76	−25 ± 73	−13 ± 78
Orthogonal	−14 ± 40	−9 ± 42	−2 ± 42
C-R			
Local	6.4 ± 6.0	5.5 ± 5.9	5.1 ± 5.9
Equilateral	−62 ± 79	−55 ± 74	−32 ± 78
Orthogonal	−57 ± 42	−41 ± 42	−42 ± 42

Planck collaboration XXIV, 2013

Constraints from Planck: modal expansion

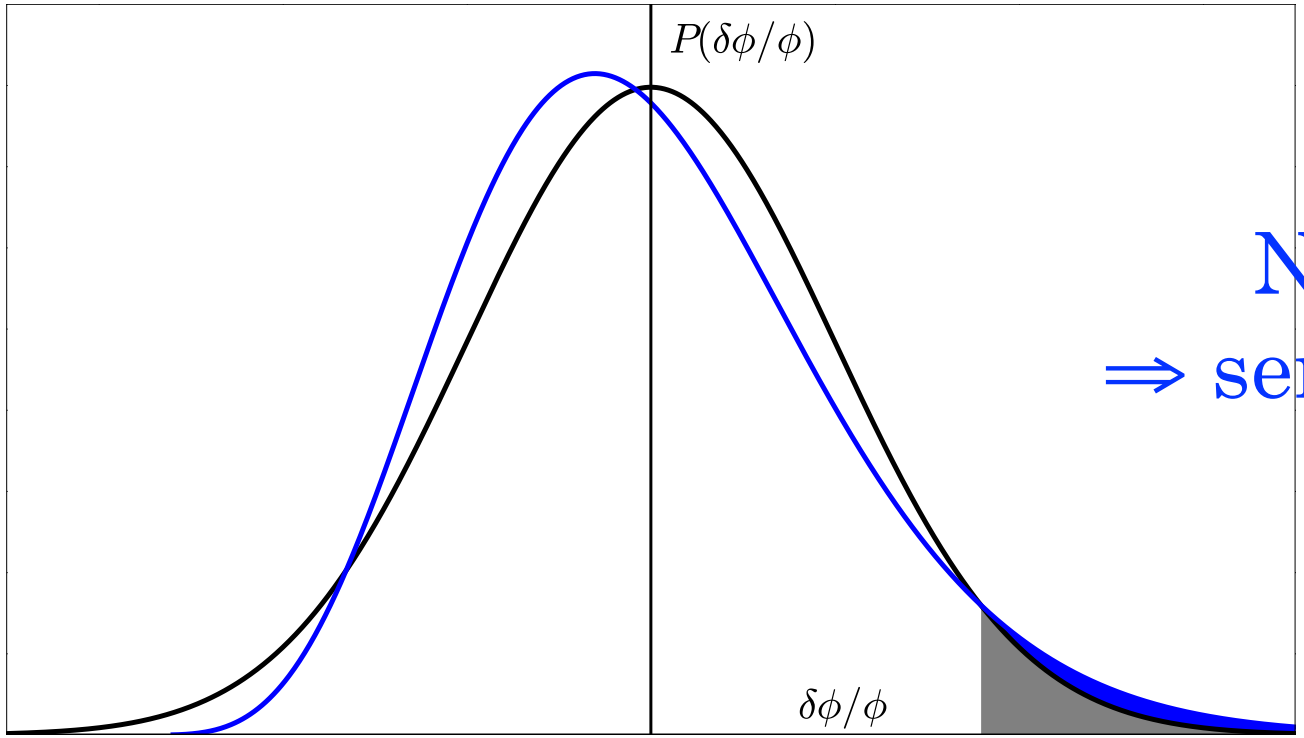
$$B(k_1, k_2, k_3) = \sum_{p,r,s} \alpha_{prs}\, q_p(k_1) q_r(k_2) q_s(k_3)$$





Planck collaboration XXIV, 2013

Galaxy cluster counts' sensitivity to NG



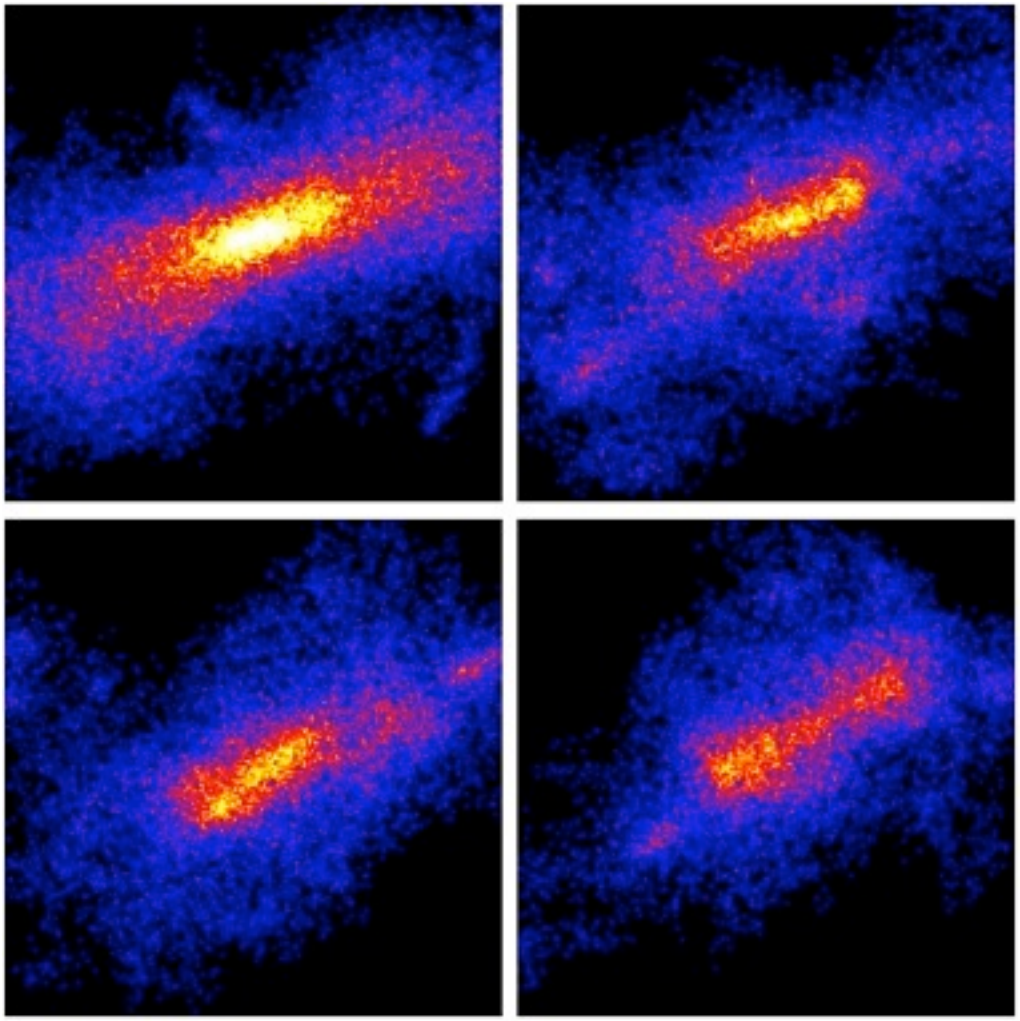
NG initial PDF
$\Rightarrow$ sensitivity to counts
"on the tail"

Lots of effort in the community to calibrate
the non-Gaussian mass function -
dn/dlnM(M, z) - of DM halos
(analytic extensions of Press-Schechter + simulations)

DM halo gets more massive with fNL>0 (and v.v.)

f_{NL}=+5000
M=1.2 10^{16} $M_\odot$

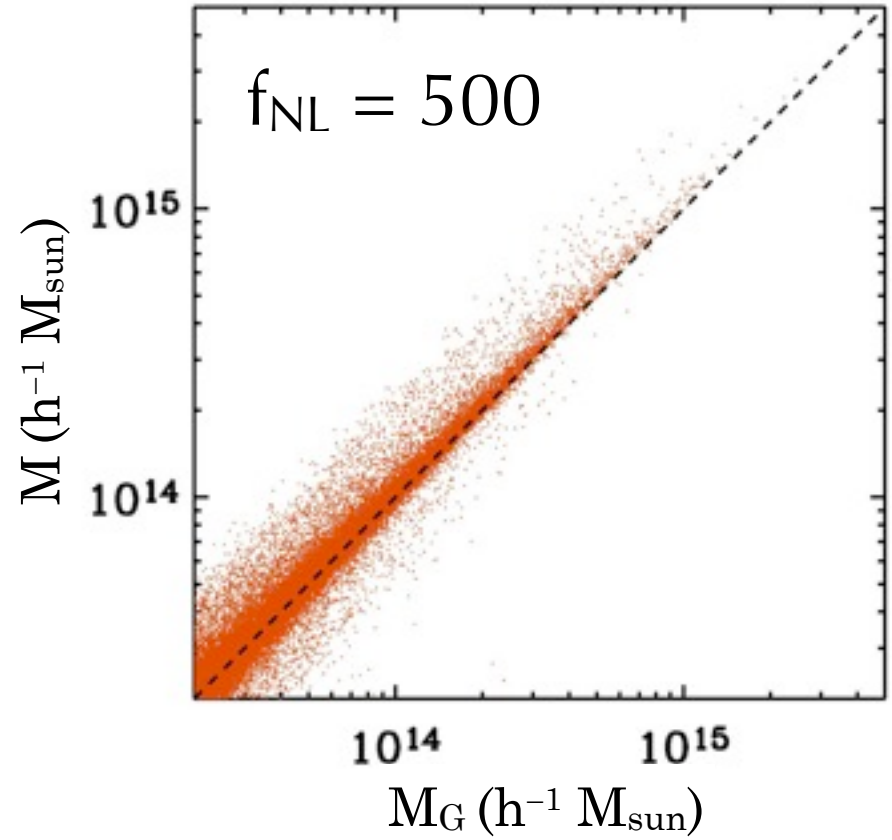
f_{NL}=+500
M=5.9 10^{15} $M_\odot$



f_{NL}=0
M=5.1 10^{15} $M_\odot$

f_{NL}=-500
M=4.3 10^{15} $M_\odot$

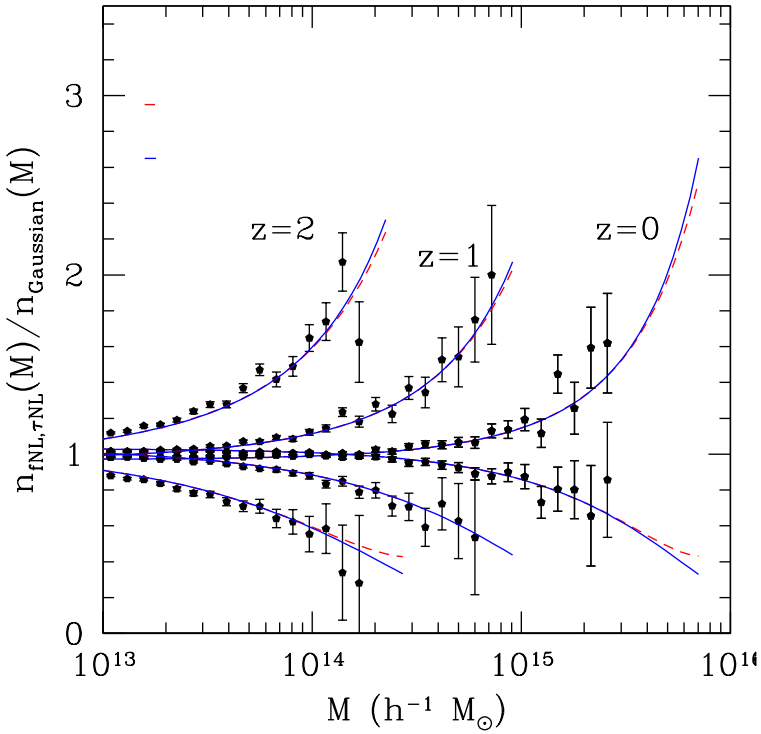
Mapping between M_G and $M \equiv M_{NG}$:



⇒ NG mass function:

$$\frac{dN}{dM} = \int \frac{dP(M|M_G)}{dM} \frac{dN}{dM_G} dM_G$$

Dalal, Doré, Huterer & Shirokov 2008



NG/Gaussian mass function ratios: for fixed M, more sensitivity at higher redshift

Smith & LoVerde 2011; Pillepich, Porciani and Hahn 2009; many others going back to 1990s

Unfortunately, cluster counts are **weakly** sensitive to NG

e.g. $\sigma(f_{NL})$=145 forecasted from SDSS (Sefusatti, Vale, Kadota & Frieman 2007)
e.g. $\sigma(f_{NL})$=450 measured from SPT (Williamson et al 2010)

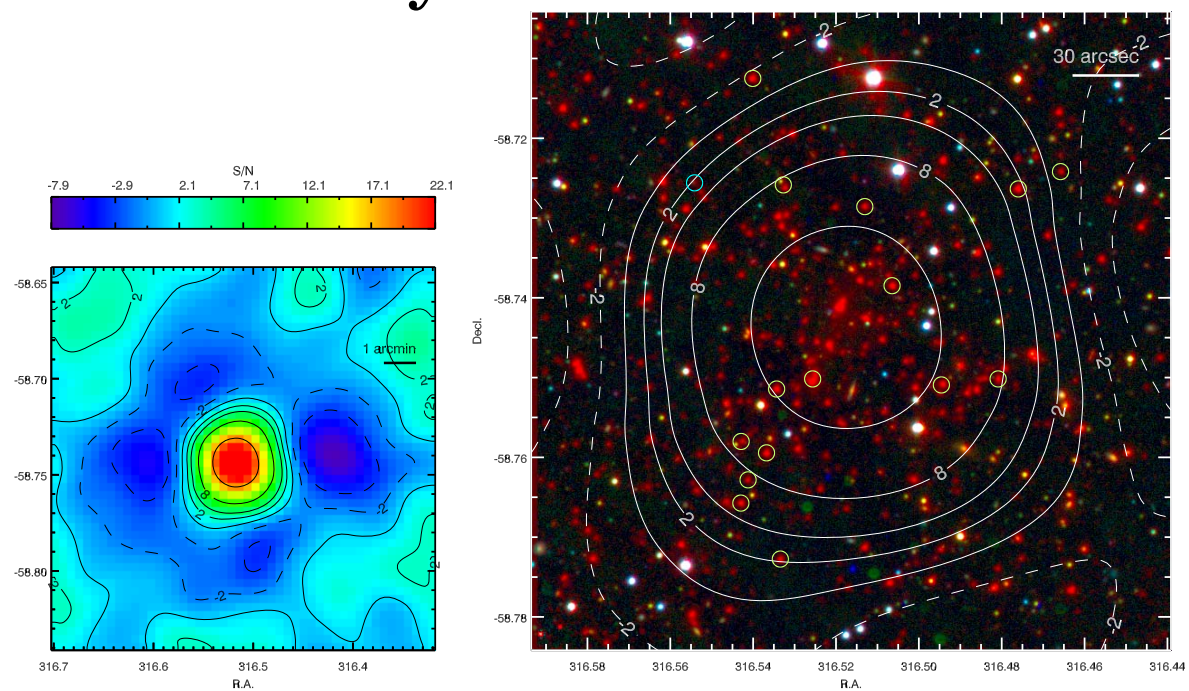
Nevertheless:

- cluster abundance is sensitive to ALL non-Gaussianity
- (large) amount of (local model) NG can boost the number of 'pink elephant' clusters

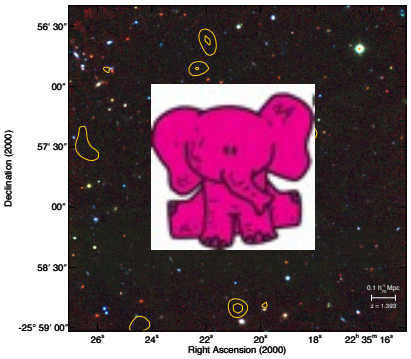
High-z, high-M - "pink elephant" - clusters of galaxies

- SPT-CL J0546-5045: z=1.067, $M \approx (8.0 \pm 1.0) \cdot 10^{14}\, M_{sun}$
- XMMU J2235.3-2557: z=1.39, $M \approx (8.5 \pm 1.7) \cdot 10^{14}\, M_{sun}$
- SPT-CL J2106-8544: z=1.132, $M \approx (1.3 \pm 0.2) \cdot 10^{15}\, M_{sun}$

Some authors have claimed the existence of these clusters is in conflict with LCDM, but can be explained with (huge; $f_{NL} \sim 500$) **non-Gaussianity**



Hoyle, Jimenez & Verde (2011);
Cayon, Gordon & Silk (2011);
Holz & Perlmutter 2011



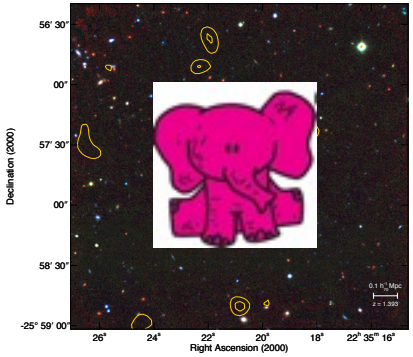
Are the pink elephants in conflict with LCDM?!

4 things to account for:

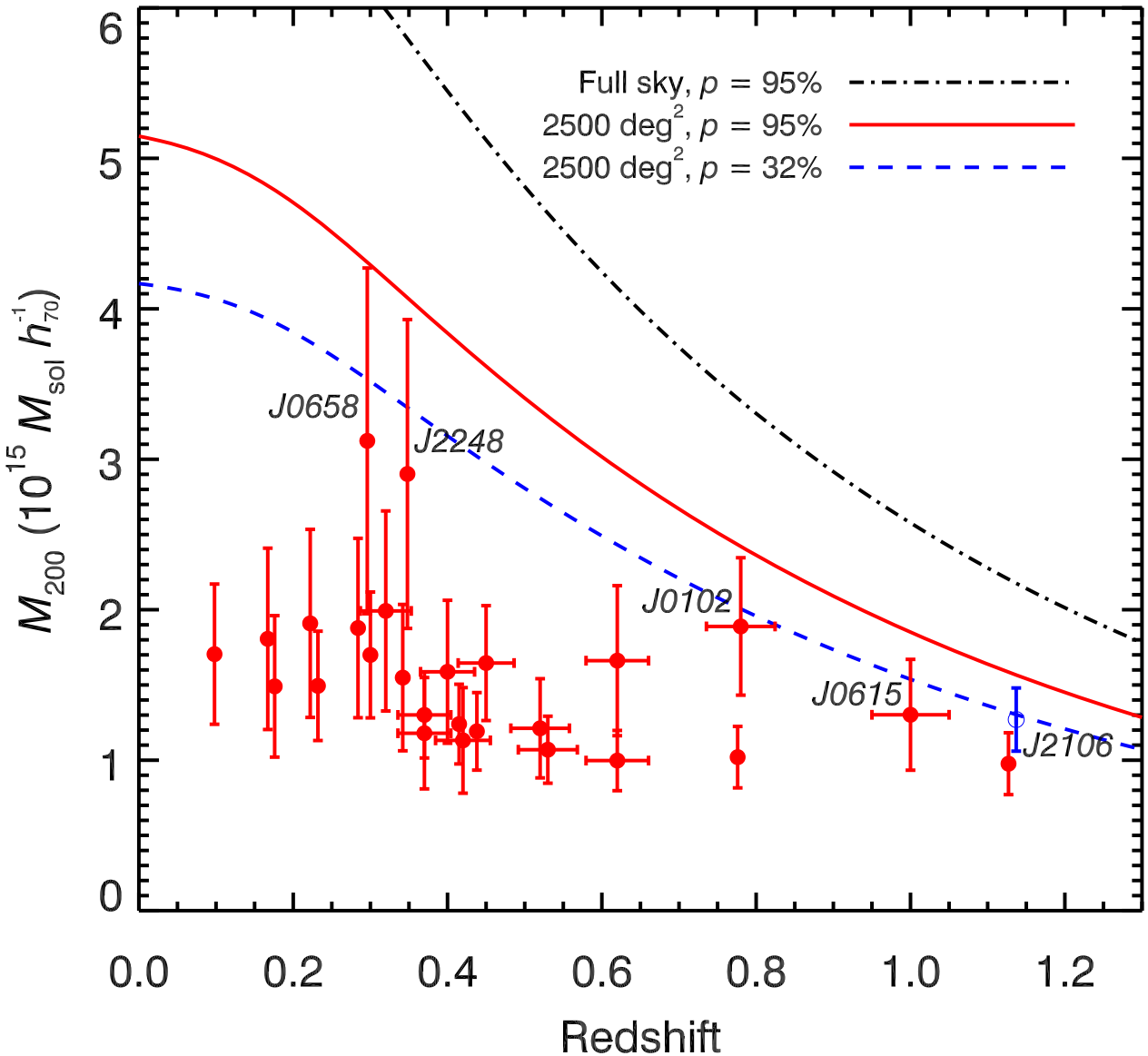
1. **Sample variance** - the Poisson noise in counting rare objects in a finite volume
2. **Parameter variance** - uncertainty due to fact that current data allow cosmological parameters to take a range of values
3. **Eddington bias** - mass measurement error will preferentially 'scatter' the cluster into higher mass
4. **Survey sky coverage** - needs to be fairly assessed

N.B. If a cluster rules out LCDM, it will rule out quintessence too!

Mortonson, Hu & Huterer 2011



No conflict - for now.



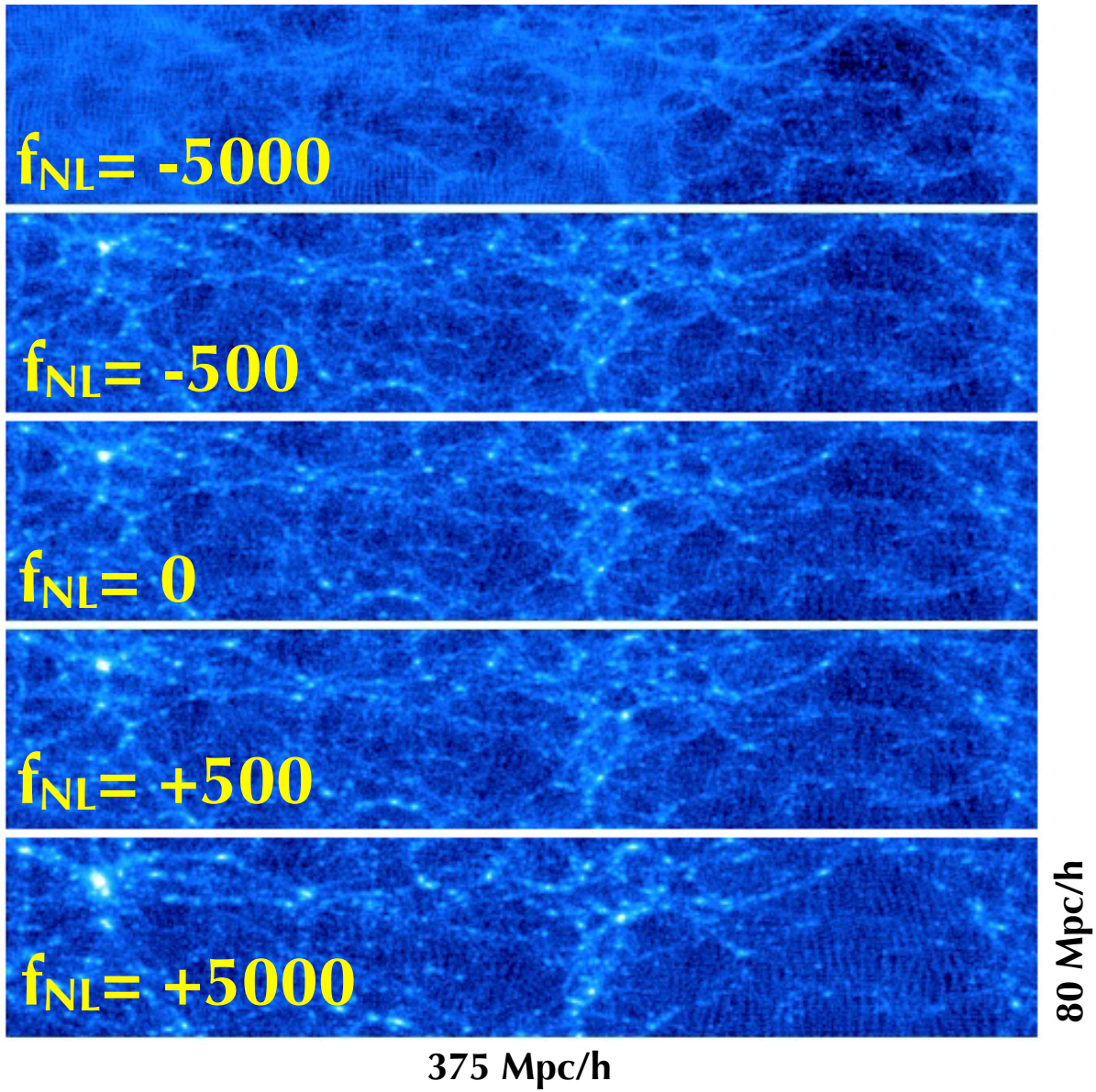
Foley et al 2011 (SPT team);
Mortonson, Hu & Huterer 2011

Next Frontier: **Large-Scale Structure**

	CMB	LSS
dimension	2D	3D
# modes	$\propto l_{max}^2$	$\propto k_{max}^3$
systematics & selection func.	relatively clean	relatively messy
temporal evol.	no	yes
can slice in	λ only	λ, M, bias...

Effects of primordial NG on the bias of virialized objects

Simulations with non-Gaussianity (f_{NL})



- Under-dense region evolution decrease with f_{NL}
- Over-dense region evolution increase with f_{NL}

- Same initial conditions, different f_{NL}
- Slice through a box in a simulation $N_{part}=512^3$, L=800 Mpc/h

Dalal, Doré, Huterer & Shirokov 2008

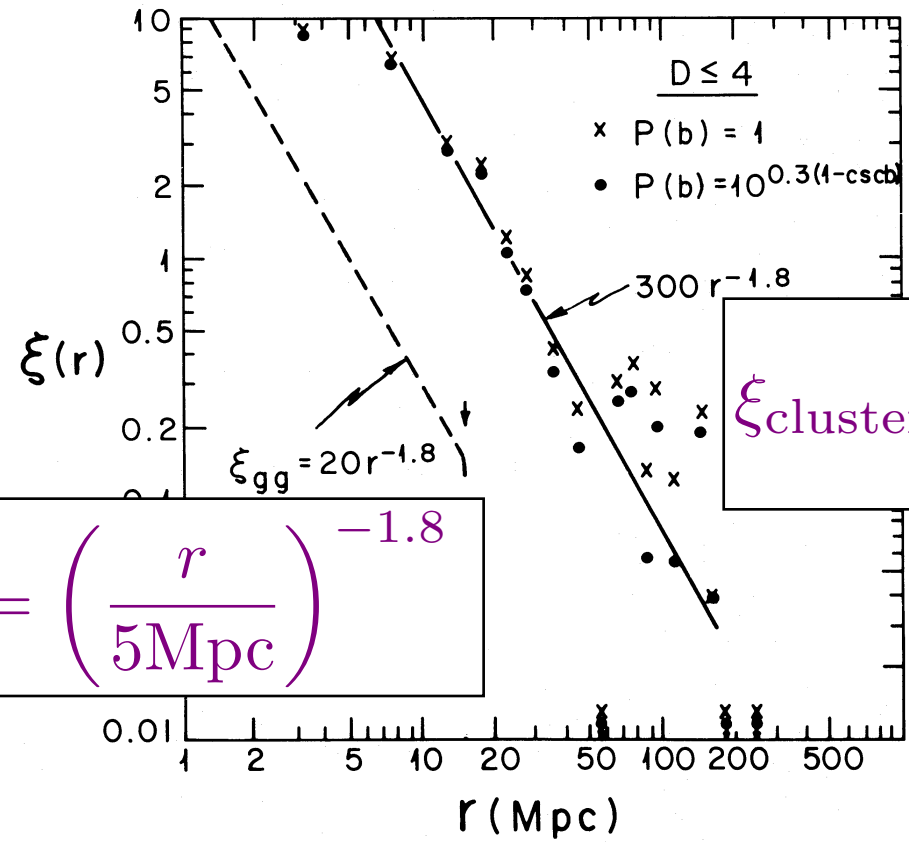
Does galaxy/halo bias depend on NG?

$$\text{bias} \equiv \frac{\text{clustering of galaxies}}{\text{clustering of dark matter}} = \frac{\left(\frac{\delta\rho}{\rho}\right)_{\text{halos}}}{\left(\frac{\delta\rho}{\rho}\right)_{\text{DM}}}$$

cosmologists measure

usually nuisance parameter(s)

theory predicts



$$\xi_{\text{clusters}}(r) = \left(\frac{r}{25\text{Mpc}}\right)^{-1.8}$$

$$\xi_{\text{galaxies}}(r) = \left(\frac{r}{5\text{Mpc}}\right)^{-1.8}$$

Bahcall & Soneira 1983

Bias of dark matter halos

$$P_h(k,z) = \boxed{b^2(k,z)}\, P_{\rm DM}(k,z)$$

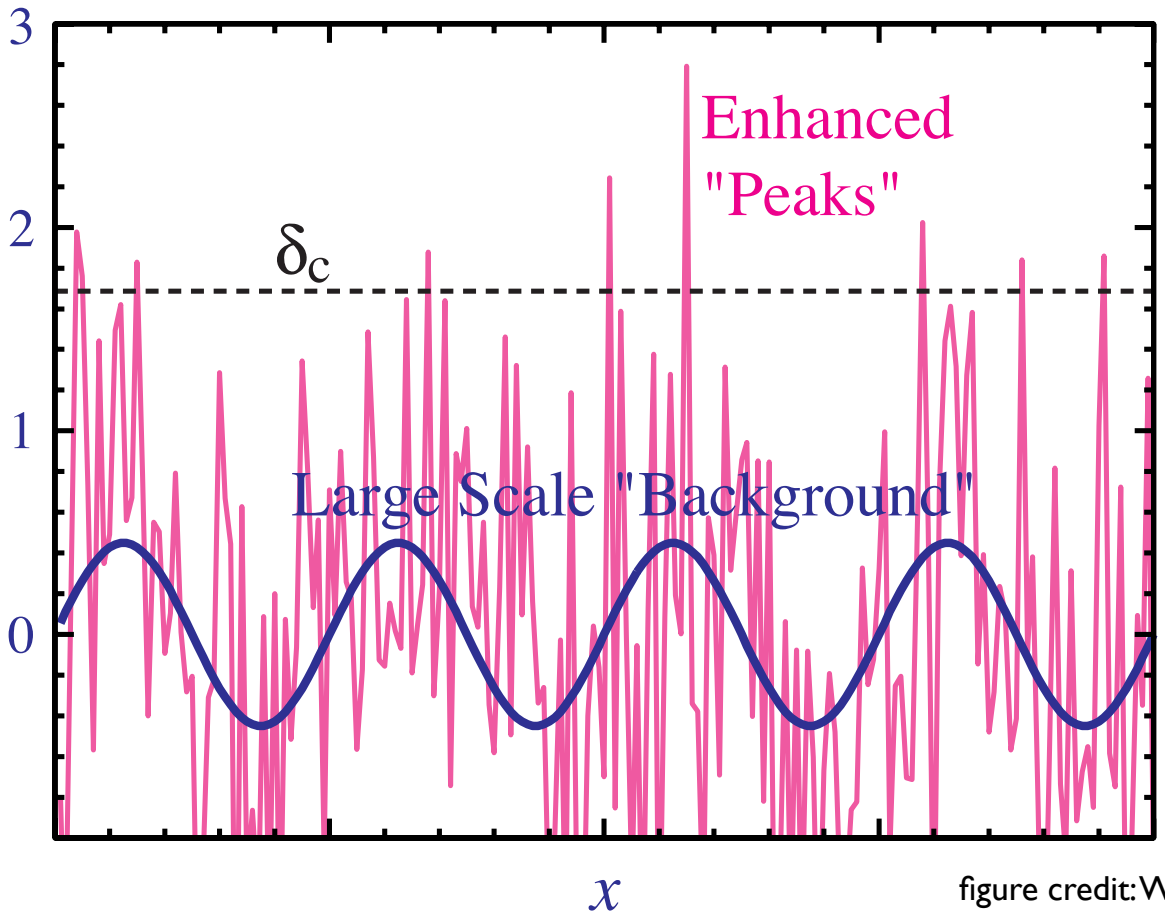
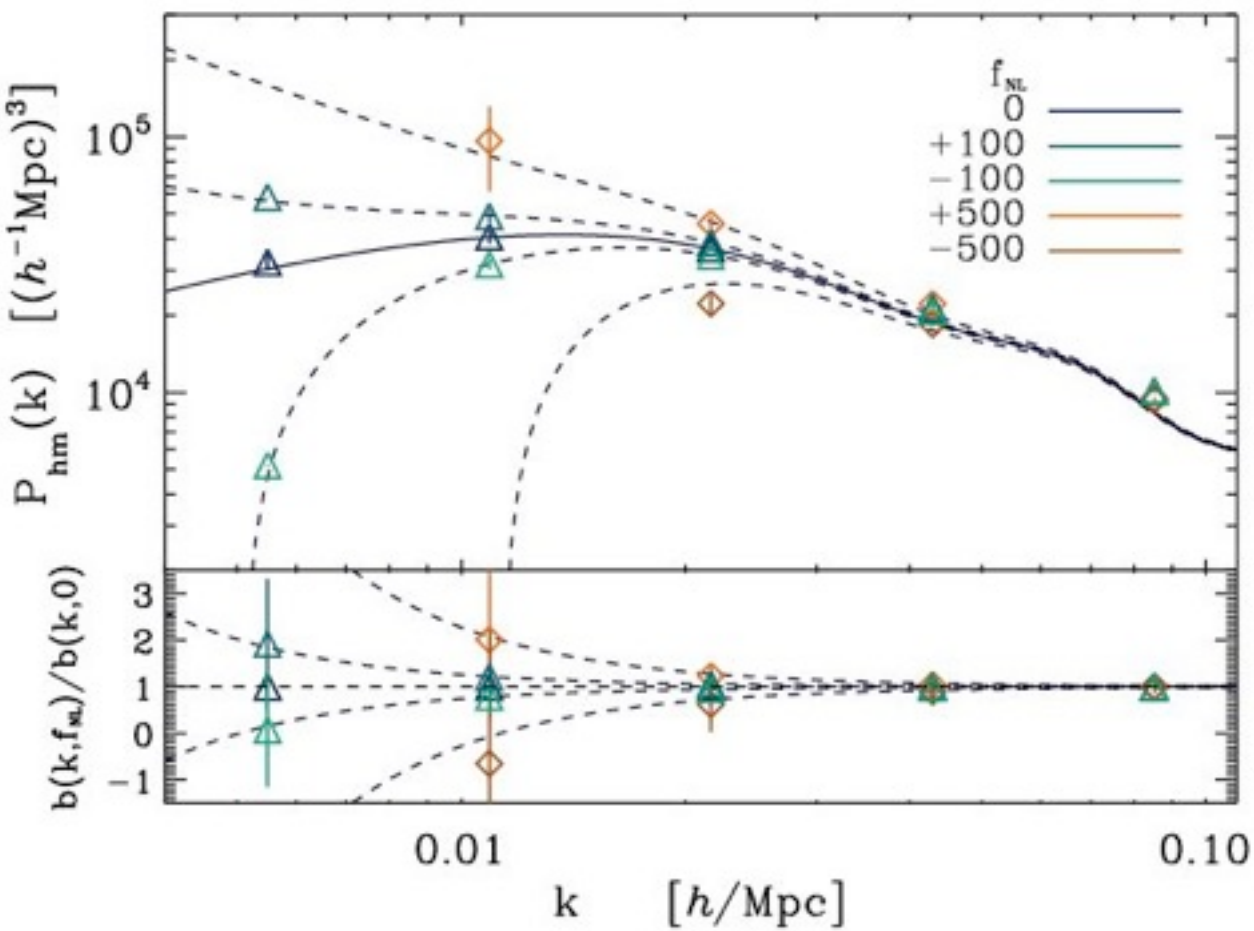


figure credit: Wayne Hu

Simulations and theory both say: large-scale bias is scale-independent
(theorem if halo abundance is function of local density
and if the short and long modes are uncorrelated)

Scale dependence of NG halo bias



$$b(k) = b_G + f_{NL}\,\frac{\mathrm{const}}{k^2}$$

Verified using a variety of theoretical derivations and numerical simulations.

Dalal, Doré, Huterer & Shirokov 2008

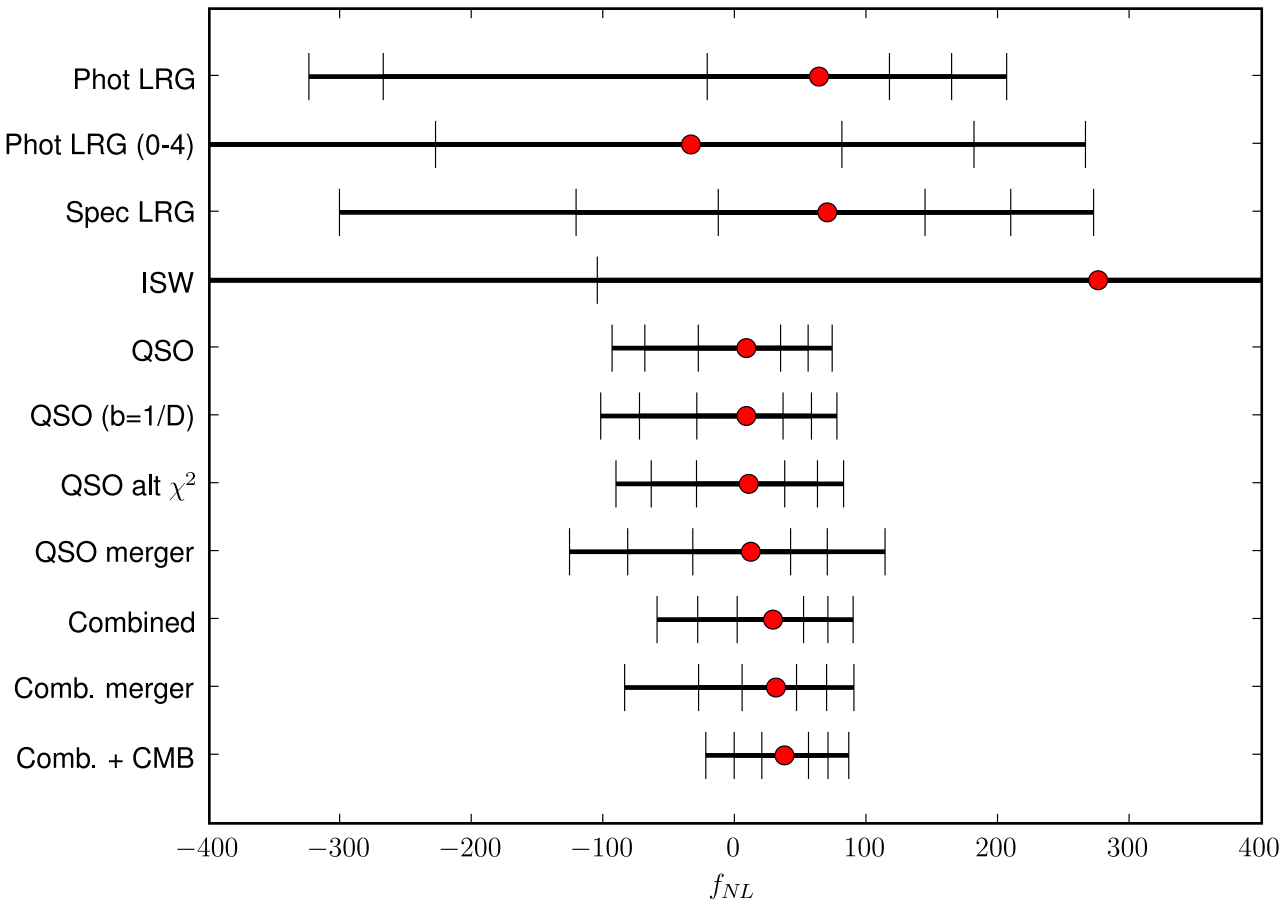
$$\Delta b(k) = f_{\rm NL}(b_G - 1)\,\delta_c\,\frac{3\,\Omega_M H_0^2}{T(k)D(a)k^2}$$

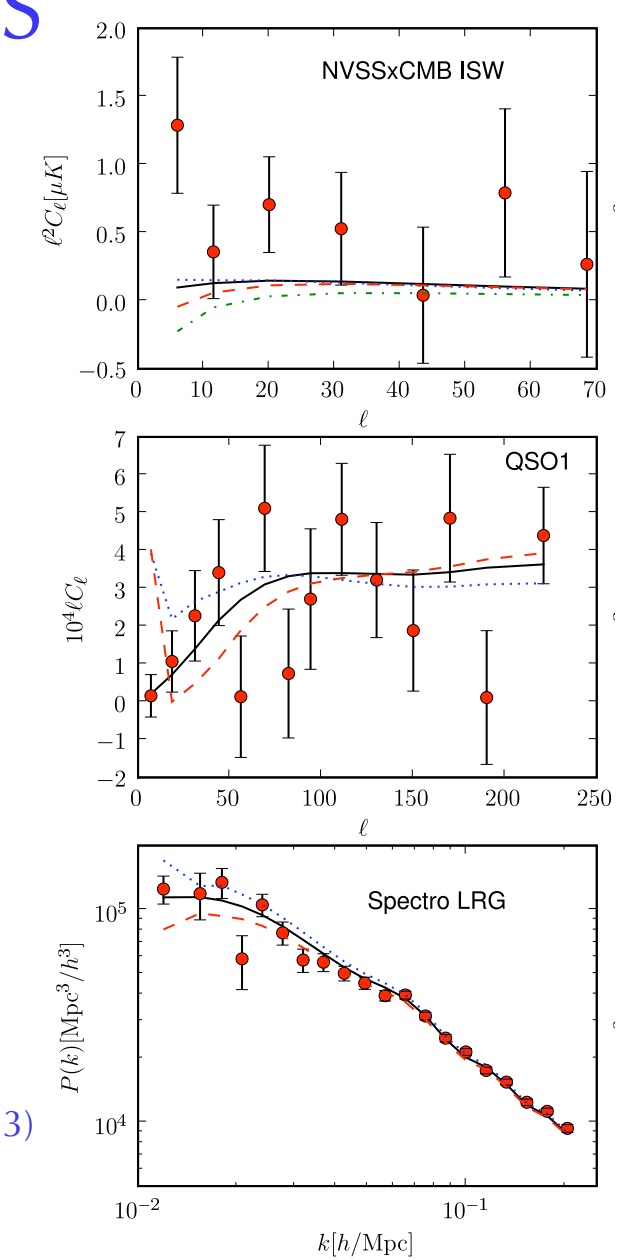
Implications:

- Unique $1/k^2$ scaling of bias; no free parameters
- Distinct from effect of all other cosmo parameters
- Straightforwardly measured (g-g, g-T,...)
- Derived theoretically several different ways
- Extensively tested with numerical simulations; good agreement found
- In general, LSS can probe: $\Delta b(k) \propto \begin{cases} k^{-2} \text{ (local)} \\ k^{-1} \text{ (folded)} \\ k^{0} \text{ (equilateral)} \\ k^{-\alpha} \text{ (generic); } 0 \leq \alpha \leq 3 \end{cases}$

Dalal et al.; Matarrese & Verde; Slosar et al; Afshordi & Tolley; Desjacques et al; Giannantonio & Porciani; Grossi et al; McDonald; ....

Constraints from **current** data: SDSS



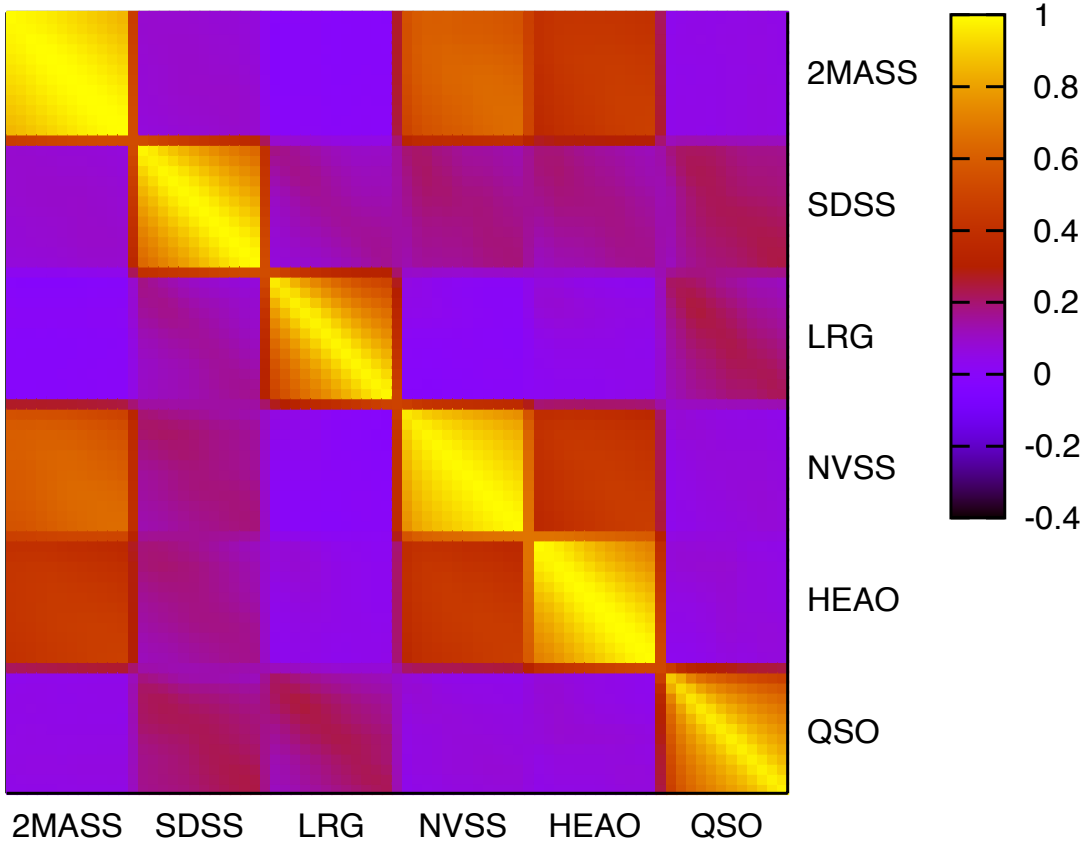


f_{NL} = 8 +/- 30 (68%, QSO)

f_{NL} = 23 +/- 23 (68%, all)

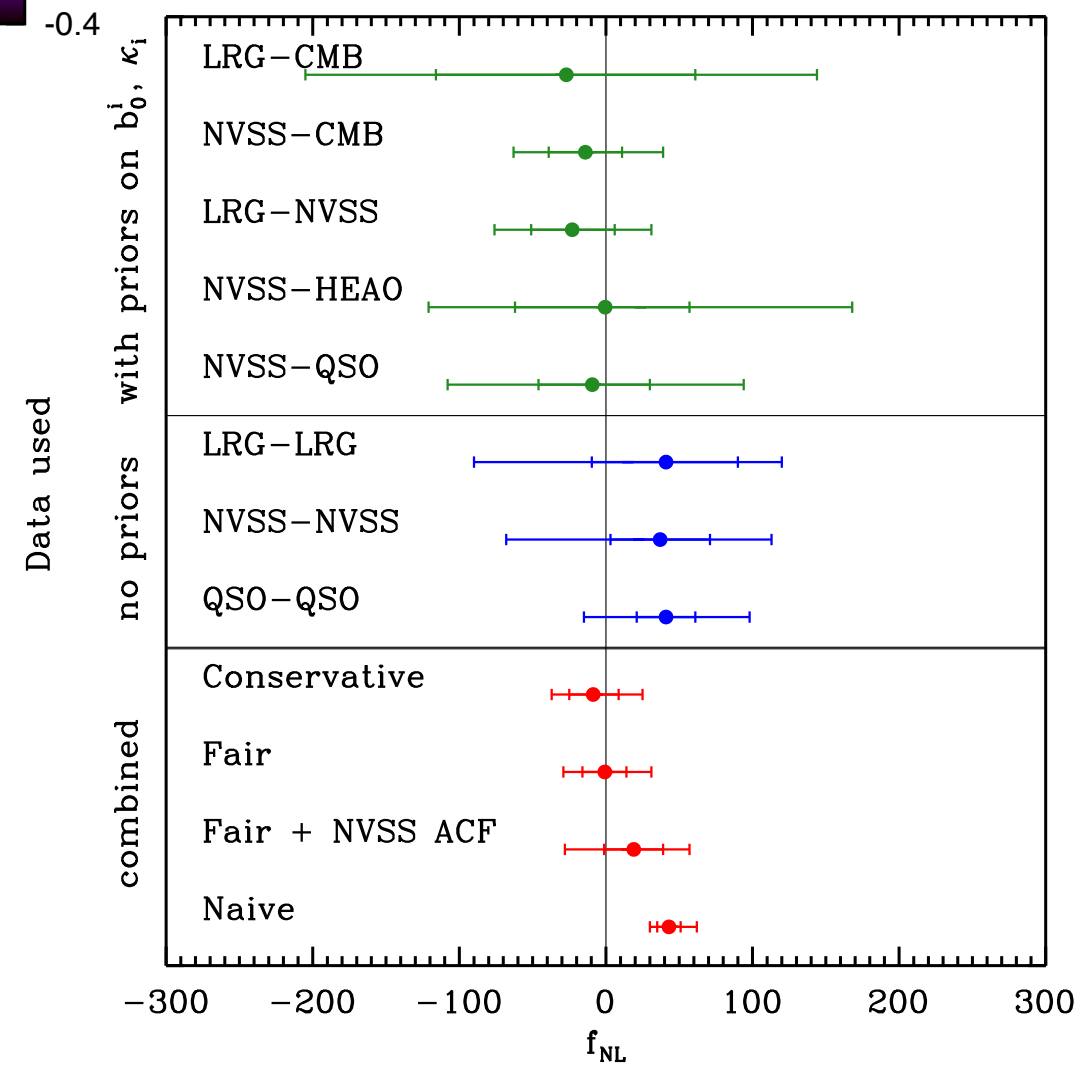
Slosar et al. 2008
(also Giannantonio et al 2013)

Future data forecasts for LSS: $\sigma(f_{NL}) \approx O(\text{few})$
(at least?) as good as, and highly complementary, to Planck CMB



Covariance matrix

Final constraints:



Giannantonio et al. 2013

Nongaussianity form clustering of galaxy **clusters**

NG can survive marginalization over numerous systematic effects

e.g:

- relation of mass of cluster and its observable quantity (T, flux, etc)
- redshift evolution of bias

Dark Energy Survey cluster forecasts

Marginalized errors—Full Covariance										
Nuisance parameters		Counts			Covariance			Counts + Covariance		
Halo bias	M_{obs}	$\sigma(\Omega_{DE})$	$\sigma(w)$	$\sigma(f_{NL})$	$\sigma(\Omega_{DE})$	$\sigma(w)$	$\sigma(f_{NL})$	$\sigma(\Omega_{DE})$	$\sigma(w)$	$\sigma(f_{NL})$
Marginalized	Marginalized	∞	∞	∞	∞	∞	∞	0.069	0.23	**6.0**
Known	Marginalized	0.097	0.33	$\mathbf{2.1 \times 10^3}$	0.13	0.43	**12**	0.065	0.22	**5.4**
Marginalized	Known	∞	∞	∞	0.099	0.34	**7.0**	0.0036	0.014	**3.8**
Known	Known	0.0051	0.023	**94**	0.042	0.13	**5.1**	0.0036	0.014	**1.8**

Counts mainly probe DE parameters

Covariance (= cluster P(k)) mainly probes f_{NL}

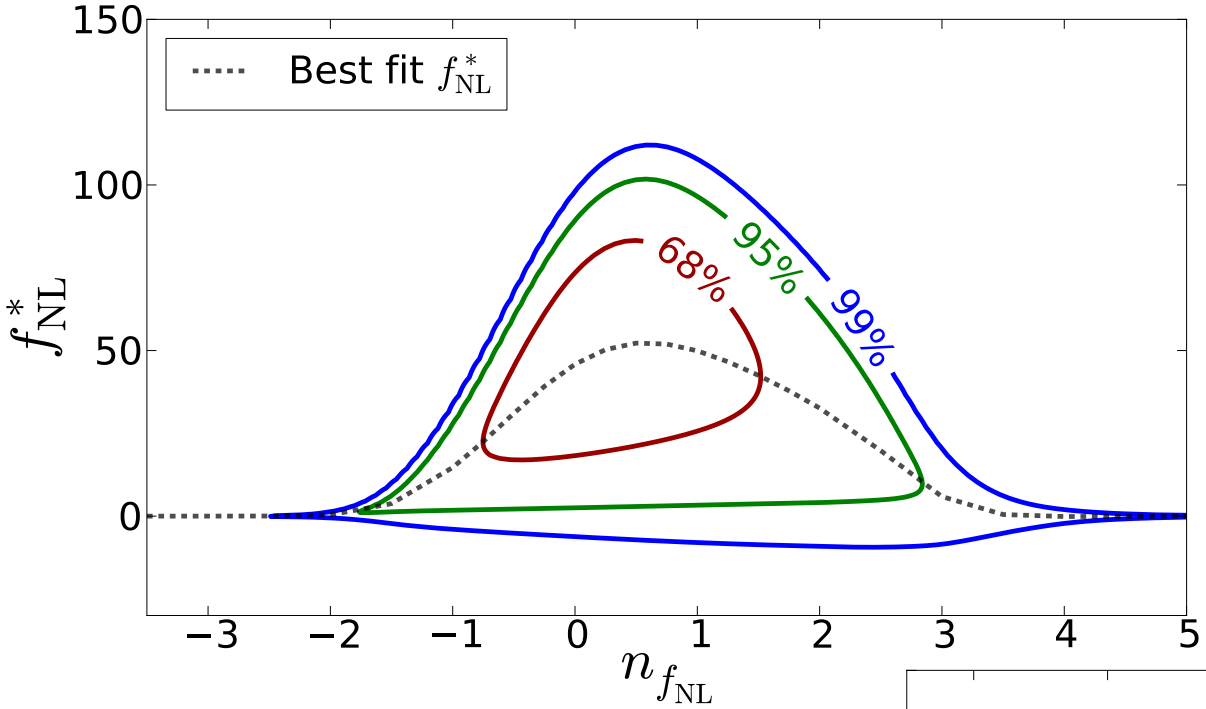
Combining them, you get both DE and f_{NL}

Cunha, Huterer & Doré 2010

More general NG models: beyond f_{NL}

First constraints on the running of NG

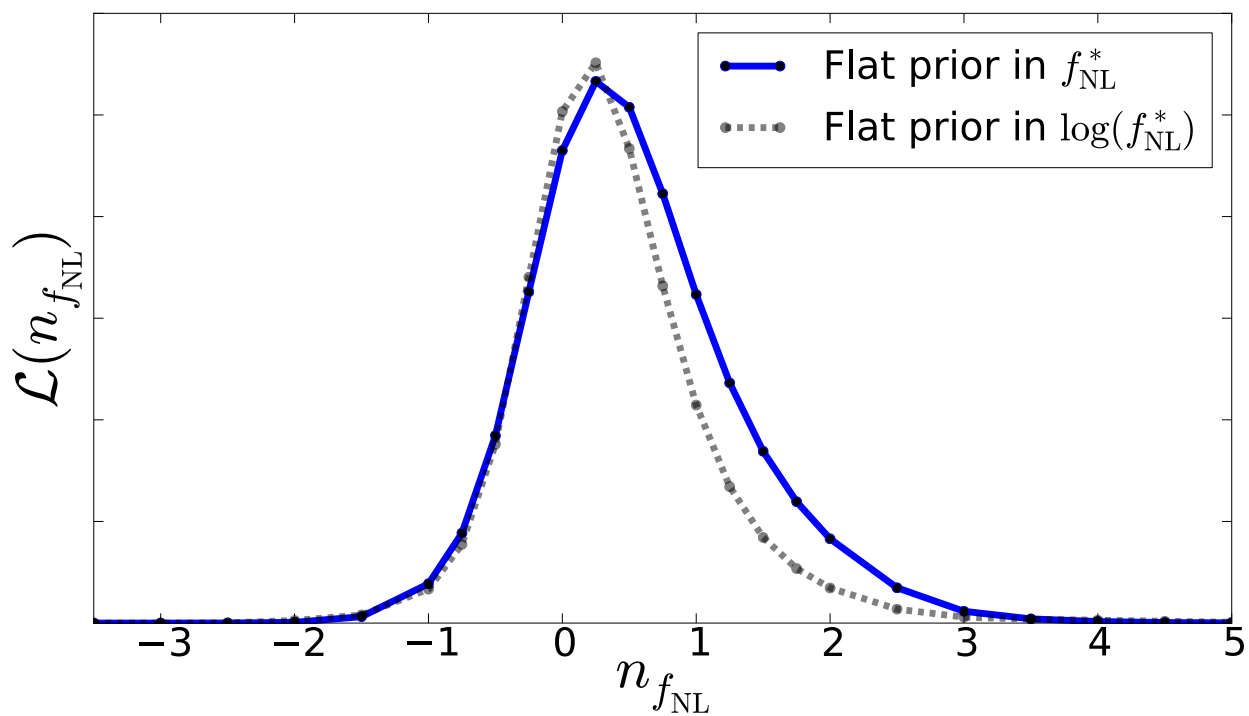
WMAP7 data, modified KSW estimator



$$f_{\rm NL}(k) = f_{\rm NL}^* \left(\frac{k}{k_*}\right)^{n_{f_{\rm NL}}}$$

$$n_{f_{\rm NL}} = 0.3^{+1.9}_{-1.2}$$

at 95% CL

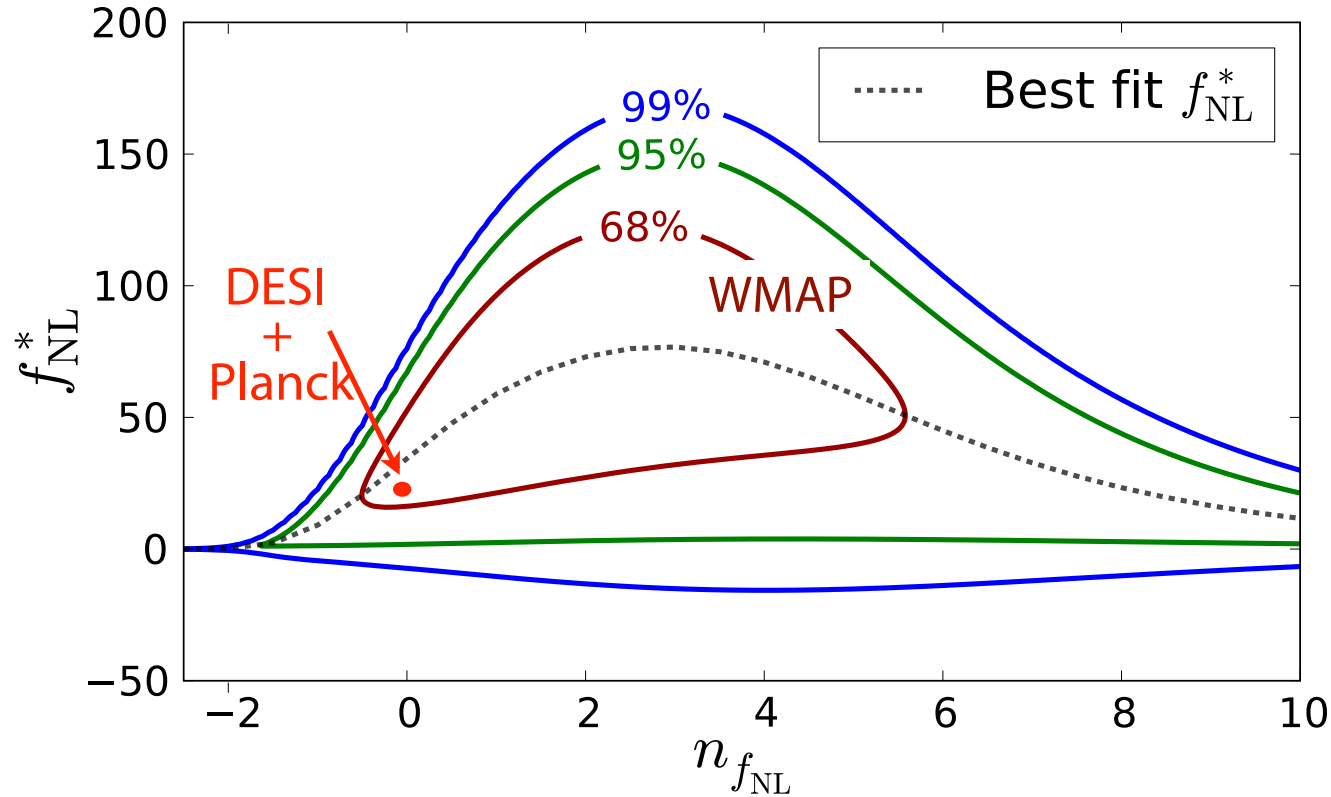


Becker & Huterer, PRL 2012

Forecasts for $f_{NL}(k)$

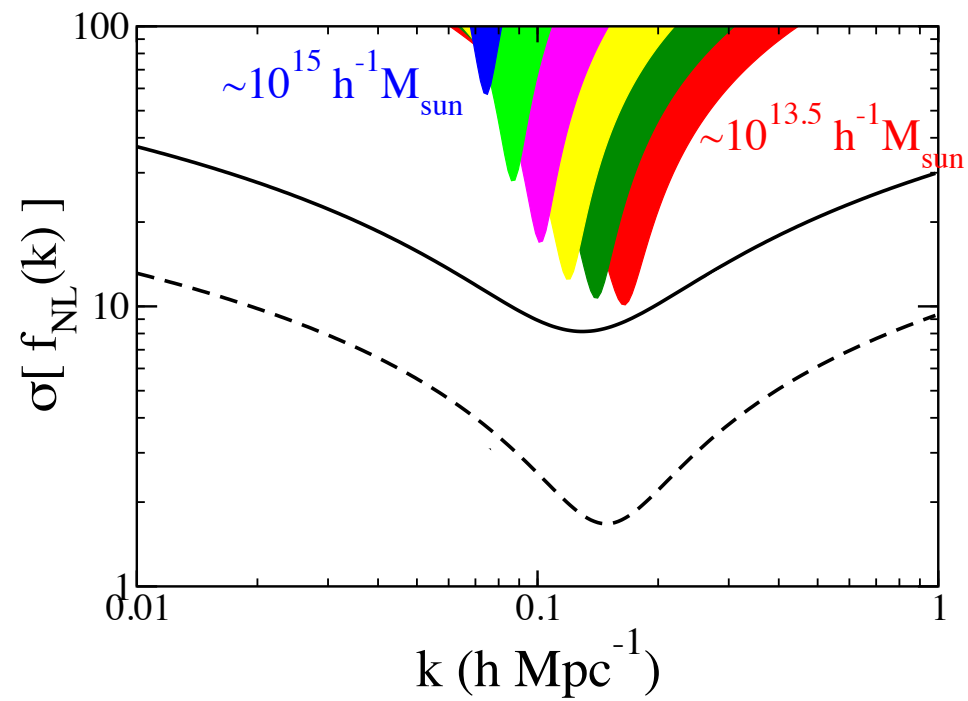
$$f_{\rm NL}(k) = f^*_{\rm NL}\left(\frac{k}{k_*}\right)^{n_{f_{\rm NL}}}$$

Becker, Huterer & Kadota, 2012



Notably:

Halos of mass M probe NG on scale $k \sim M^{-1/3}$:



Shandera, Dalal & Huterer 2012

Systematic Errors:
(photometric) calibration errors

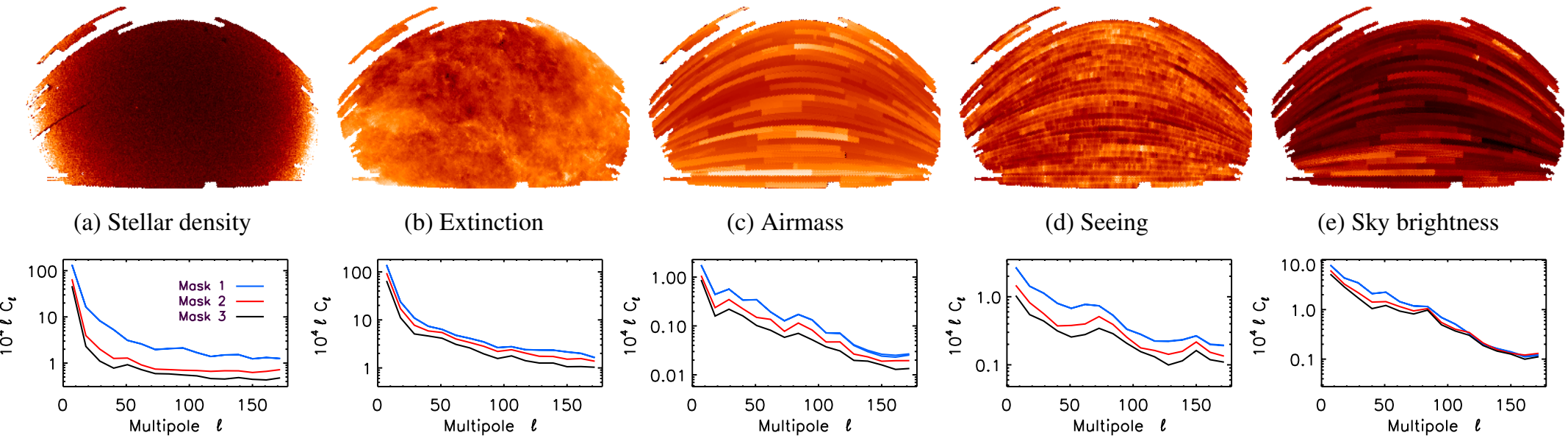
c.f.
"Calibration and Standardization of Large Surveys and Missions in A&A" workshop
Fermilab, 16-19 April 2012

For the NG measurements, photo-z but also: (photometric) calibration errors

- **Detector sensitivity**: sensitivity of the pixels on the camera vary along the focal plane. Sensitivity of a given pixel can change with time.
- **Observing conditions**: spatial and temporal variations.
- **Bright objects**: The light from foreground bright stars and galaxies affects the sky subtraction procedure, which impairs the surveys' completeness near bright objects.
- **Dust extinction**: Dust in the Milky Way absorbs light from the distant galaxies.
- **Star-galaxy separation**: In photometric surveys, faint stars can be erroneously included in the galaxy sample. Conversely, galaxies are sometimes misclassified as stars and culled from the sample. Remember, stars are *not* randomly distributed across the sky.
- **Deblending**: Galaxy images can overlap, and it can be difficult to cleanly separate photometric and spectroscopic measurements for the blended objects.

Carlos Cunha

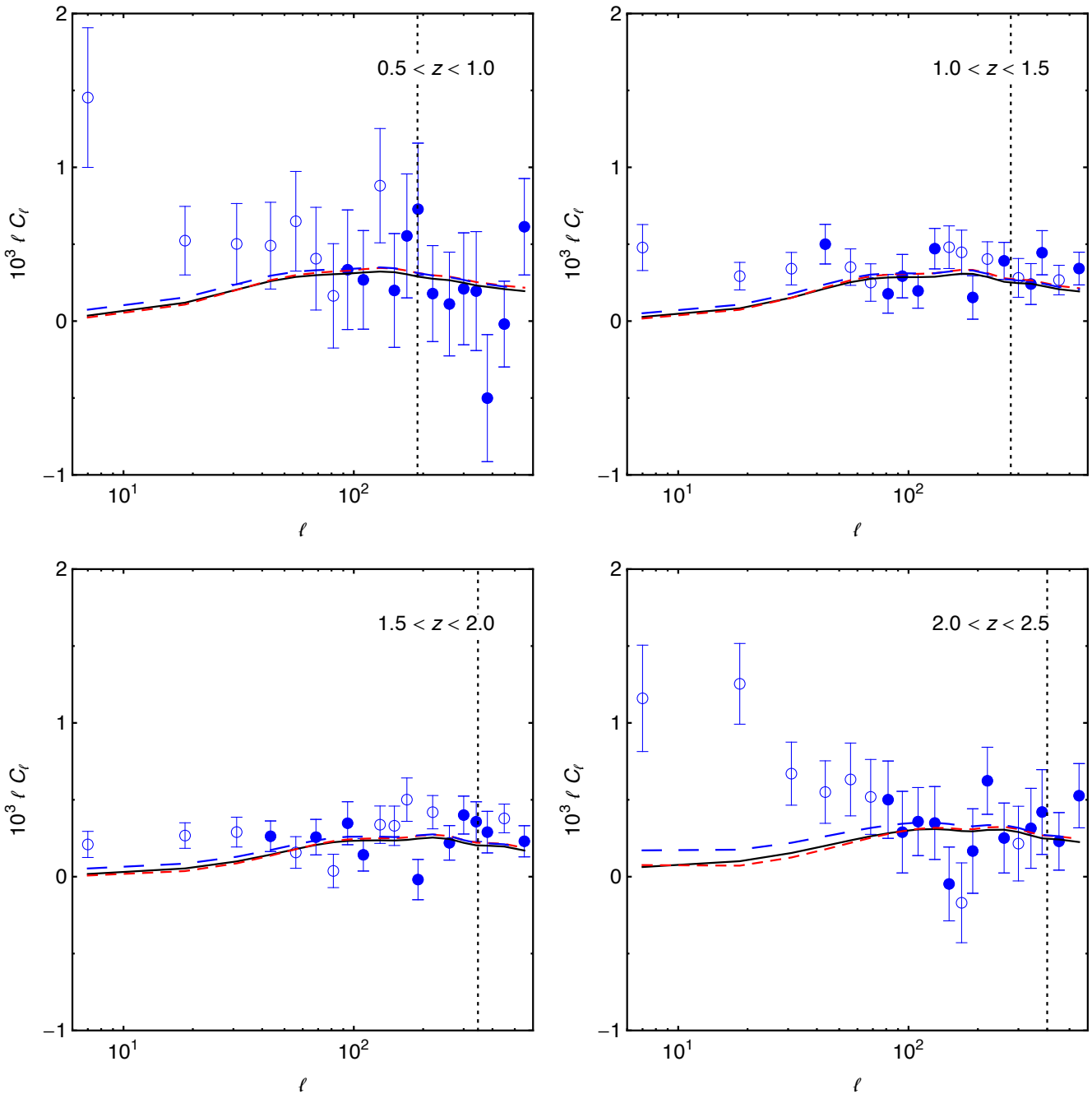
LSS calibration errors: example maps, power spectra



- dominate on large angular scales
- can be measured, removed using same or other data

Leistedt et al 2013

Calibration errors in SDSS DR8 power spectra



QSO power spectra from SDSS;
open circle points not used since they may be systematics-contaminated!

Similar results for LRGs (not shown)

Agarwal, Ho & Shandera, arXiv:1311.2606

How do the **most generic calibration errors** look (in the power spectrum)?

How do they affect NG (and DE) parameters?

MNRAS **432,** 2945–2961 (2013)
Advance Access publication 2013 May 11
doi:10.1093/mnras/stt653

Calibration errors unleashed: effects on cosmological parameters and requirements for large-scale structure surveys

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[2] *Kavli Institute for Particle Astrophysics and Cosmology, 452 Lomita Mall, Stanford University, Stanford, CA 94305, USA*
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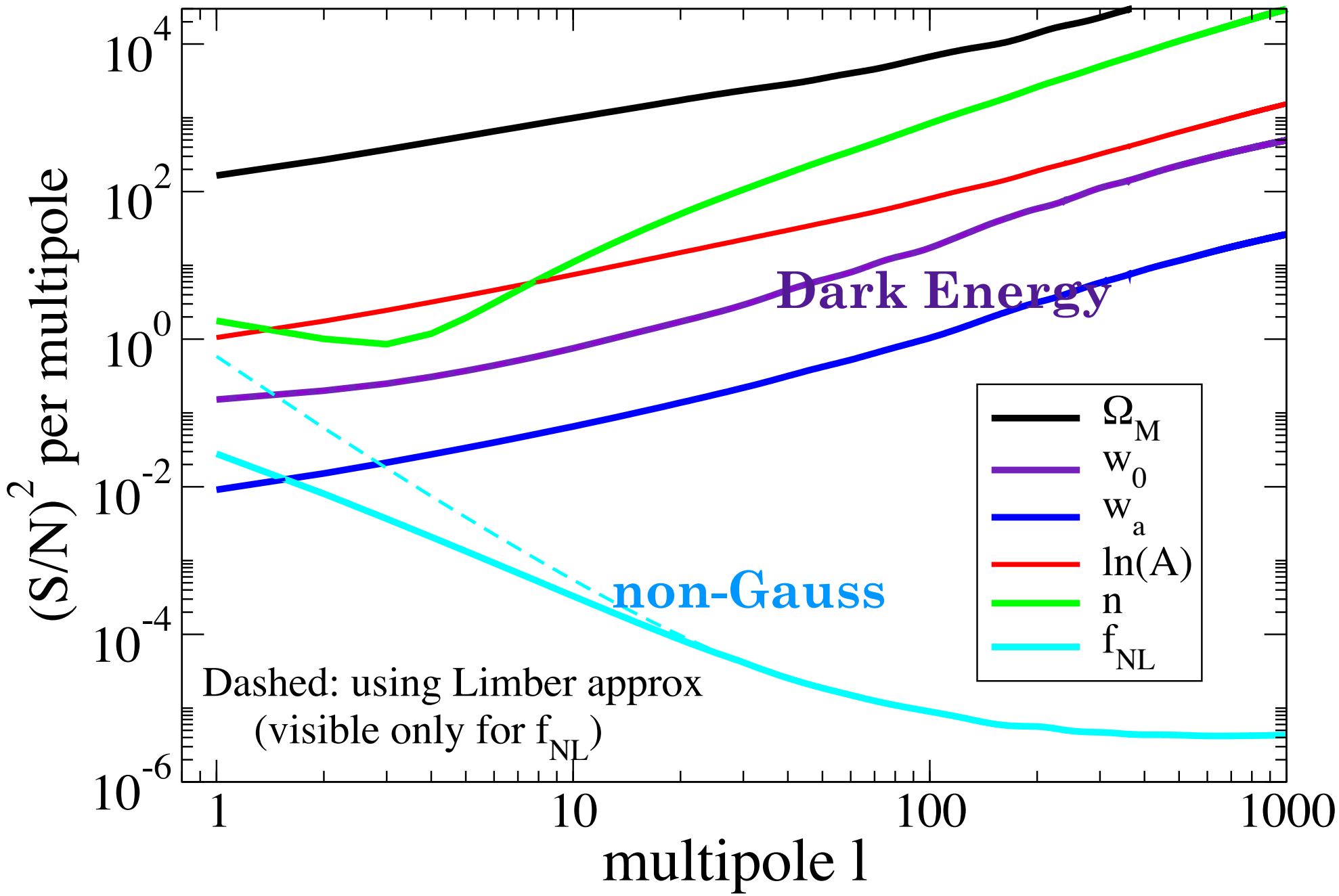
Accepted 2013 April 15. Received 2013 April 12; in original form 2012 November 26

Related works: Pullen & Hirata 2012, Leistedt et al 2013, Agarwal et al 2013

Non-Gaussianity constraints are special: they come from large angular/spatial scales



How do calibration errors affect the measured galaxy angular power spectrum?

$t_{\boxed{A}m}$ – observed galaxy field
$c_{\boxed{A}m}$ – calibration (systematics) field
$C_{\boxed{A}}$ – true galaxy clustering power

Final result for the **observed** power spectrum is:

$$\langle t_{\ell m} t^*_{\ell' m'} \rangle = \frac{1}{(1+\epsilon)^2} \left\{ \underbrace{\delta_{mm'} \delta_{\ell\ell'} C_\ell}_{\text{isotropic}} + \underbrace{\left[U^{\ell\ell'}_{mm'} C_{\ell'} + (U^{\ell\ell'}_{mm'})^* C_\ell \right] + \sum_{\ell_2 m_2} U^{\ell_2 \ell}_{m_2 m} (U^{\ell_2 \ell'}_{m_2 m'})^* C_{\ell_2} + c_{\ell m} c^*_{\ell' m'}}_{\text{breaks statistical isotropy}} \right\}$$

Cancels effects of calibration monopole

True power

Calibration (biases)

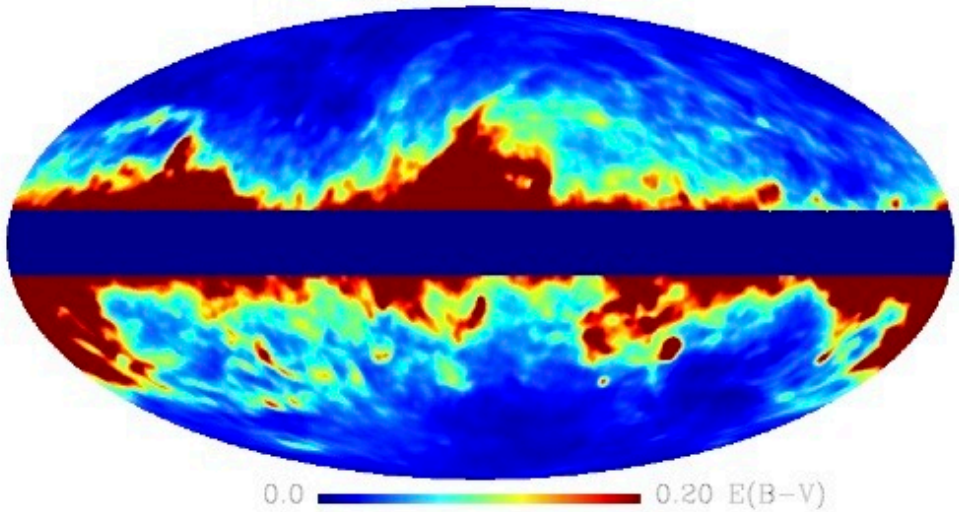
where $U^{\ell_2 \ell}_{m_2 m} \equiv \sum_{\ell_1 m_1} c_{\ell_1 m_1} R^{\ell_1 \ell_2 \ell}_{m_1 m_2 m}$

$$R^{\ell_1 \ell_2 \ell}_{m_1 m_2 m} \equiv (-1)^m \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell+1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell \\ m_1 & m_2 & -m \end{pmatrix}$$

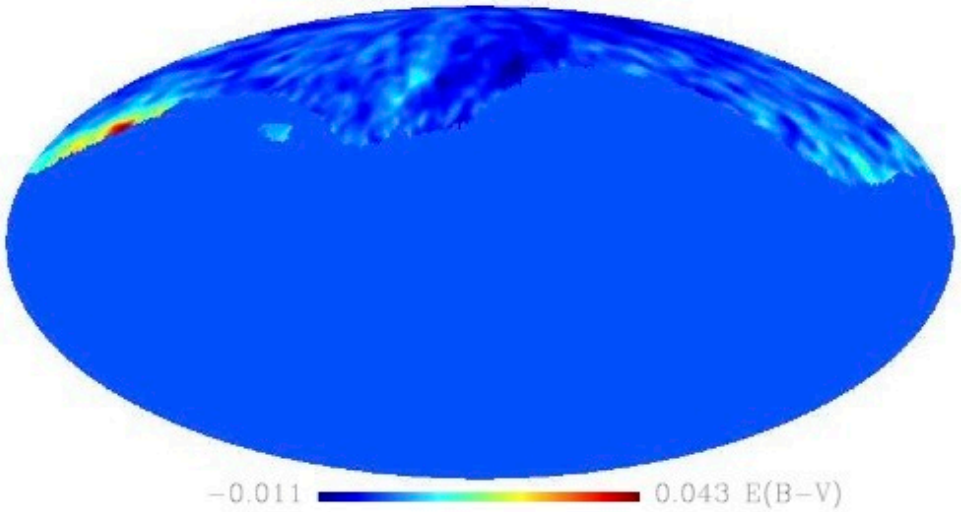
Huterer, Cunha & Fang, 2013

Calibration bias: Worked Example 1

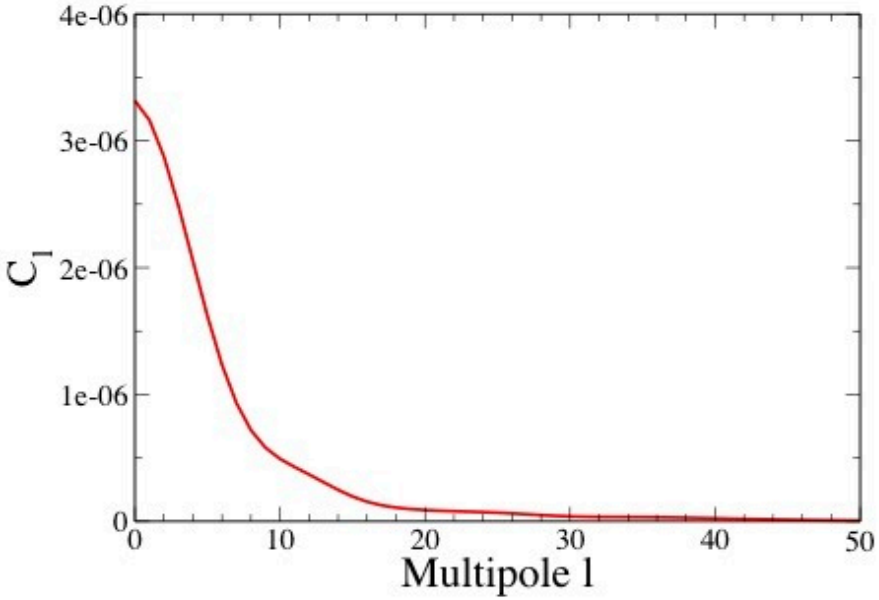
SFD dust map



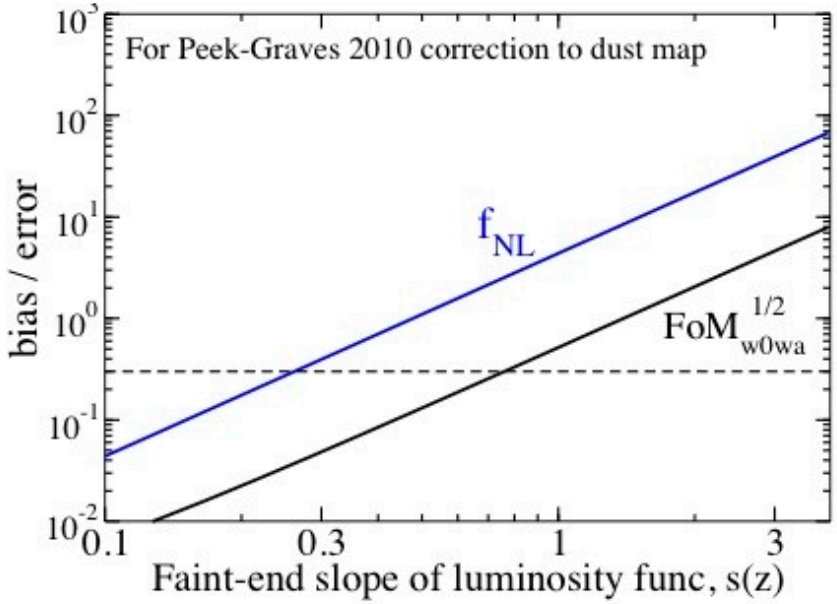
PG10 corrections to map



angular power of corrections



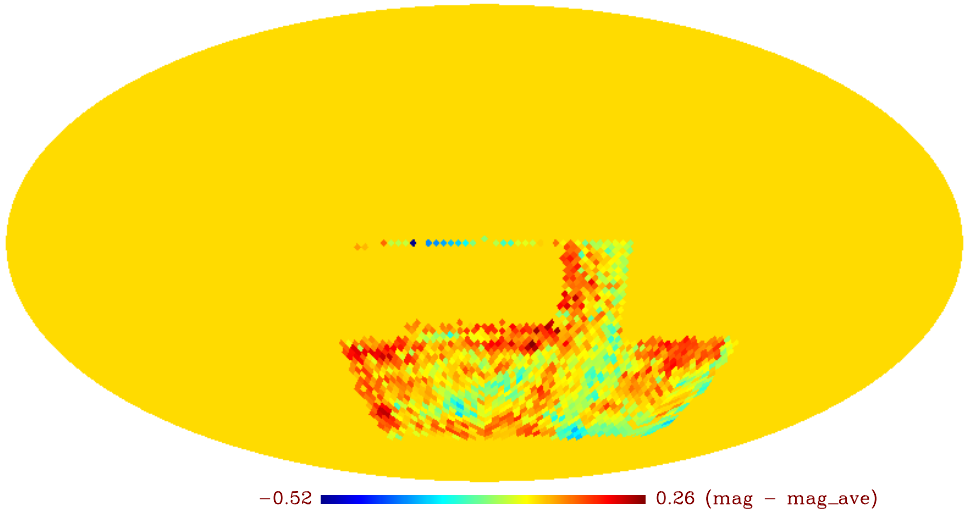
bias/error in cosmology

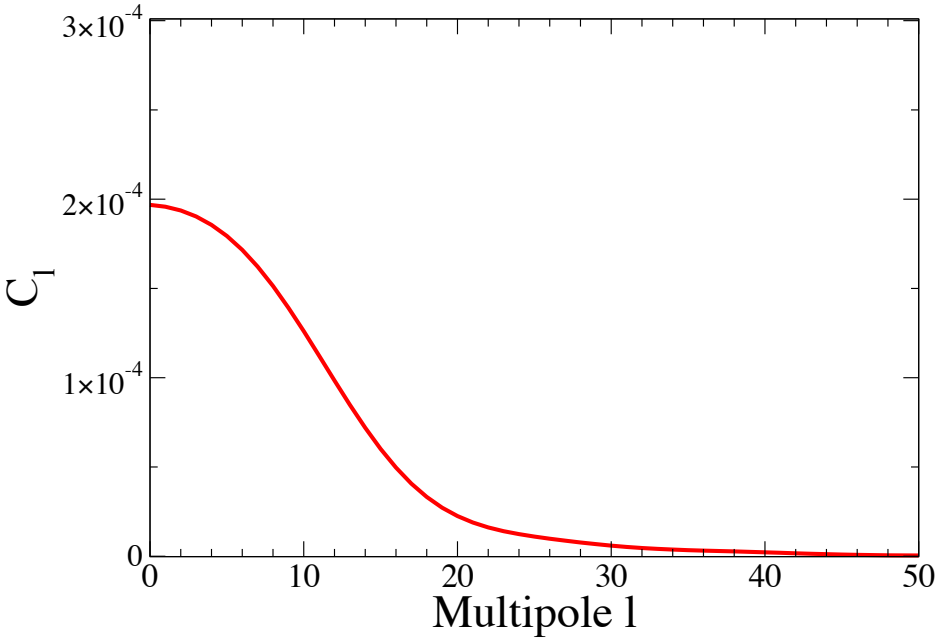


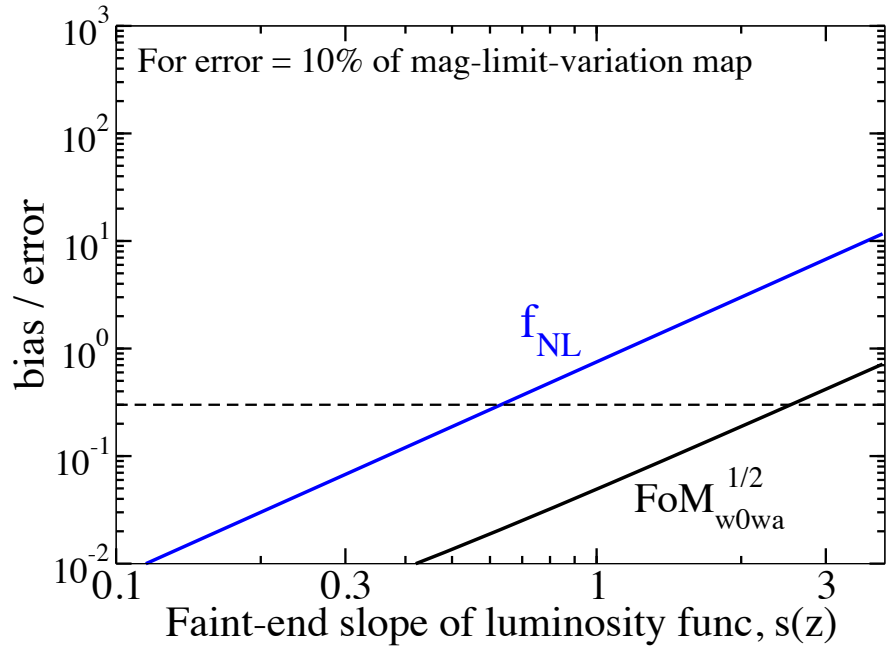
Huterer, Cunha & Fang 2013

Calibration bias: Worked Example 2

DES magnitude limit (J. Annis)

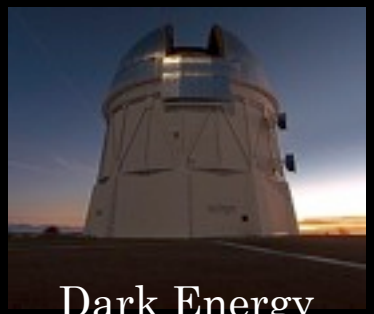






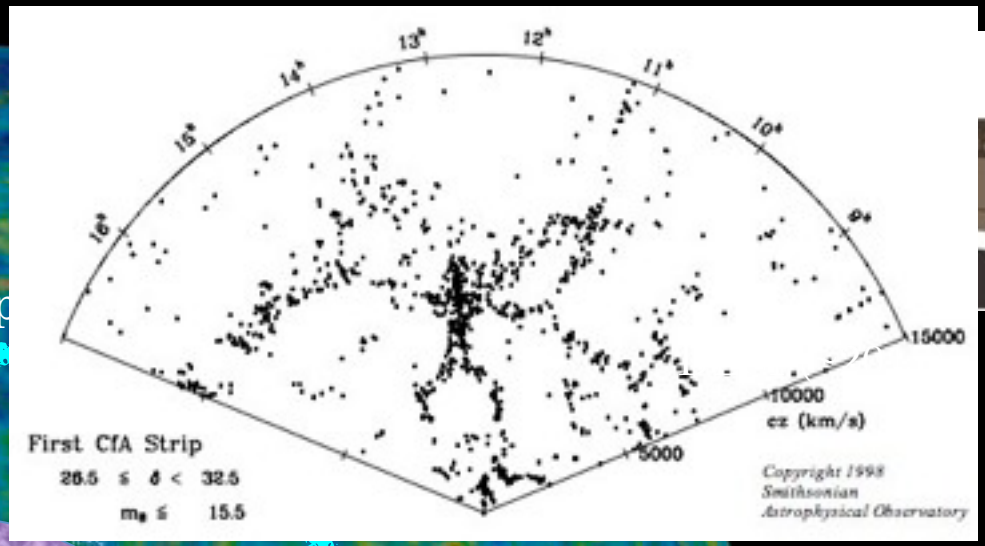
Huterer, Cunha & Fang 2013

Conclusions



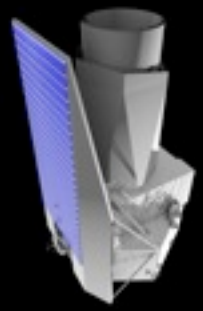
Dark Energy Survey (2012)

▲Harvard-Cfa survey (1980s)

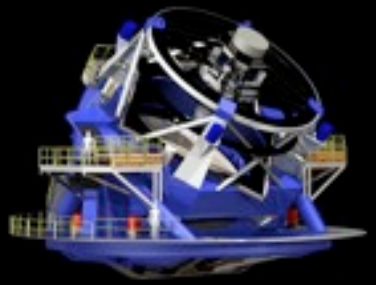


21cm mapp

Euclid and WFIRST (~202X)



LSST (~2018)



Challenges for NG/LSS program

... and approximate current status

- Motivate NG models ✓ (single-field, multiple fields, self-int)
- Utilize a variety of observables in LSS and CMB to get at NG ✓
- Develop fast, near-optimal estimators to extract NG from the CMB ✓ and LSS ✓✗
- Develop theory to relate NG models to LSS observables ✓✗ (messy; still need to check with sims)
- Develop theory to use LSS info from 1, 2 pt function of halos ✓ and galaxies/QSO ✓✗ (both with concerns)
- Control the systematic errors, esp large-scale LSS ✓✗
- Use galaxy bispectrum ✗ and weak lensing bispectrum ✗✗ to get at primordial NG [eg f_{NL}^{equil}]

Advances in Astronomy special issue on

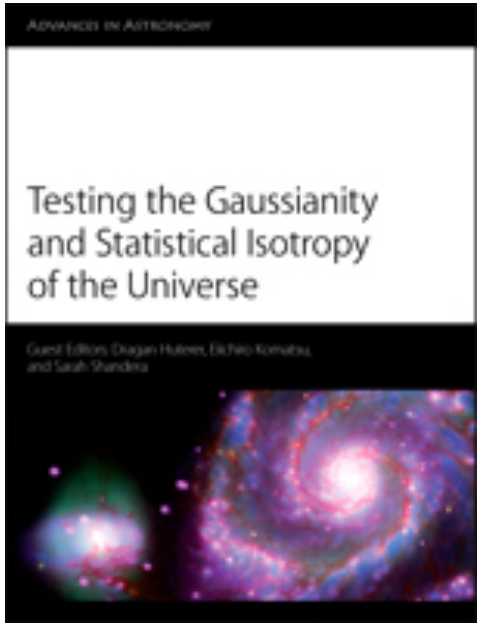
"Testing the Gaussianity and Statistical Isotropy of the Universe"

http://www.hindawi.com/journals/aa/2010/si.gsiu/

15 review articles (all also on arXiv)

Testing the Gaussianity and Statistical Isotropy of the Universe

Guest Editors: Dragan Huterer, Eiichiro Komatsu, and Sarah Shandera



Non-Gaussianity from Large-Scale Structure Surveys, Licia Verde
Volume 2010 (2010), Article ID 768675, 15 pages

Non-Gaussianity and Statistical Anisotropy from Vector Field Populated Inflationary Models, Emanuela Dimastrogiovanni, Nicola Bartolo, Sabino Matarrese, and Antonio Riotto
Volume 2010 (2010), Article ID 752670, 21 pages

Cosmic Strings and Their Induced Non-Gaussianities in the Cosmic Microwave Background,