Interferometers for Cosmic Microwave Background Polarization Measurements

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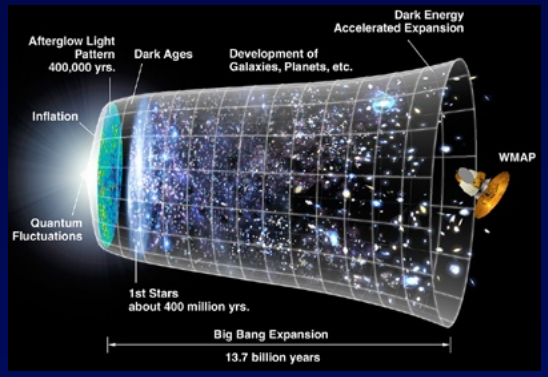
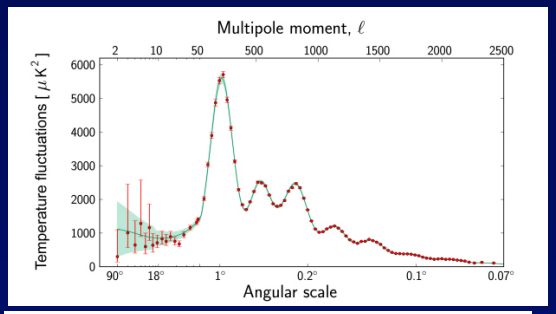
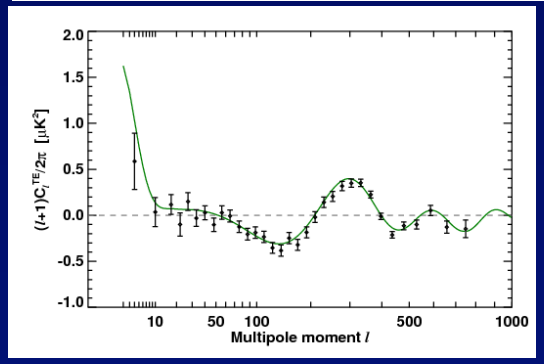
Why the CMB matters

- One of the main reasons for the emergence of a well-tested "standard model" in cosmology.
- Probes large scales, early times (linear perturbations).
- CMB angular power spectrum matches predictions remarkably well.
  - » Hundreds of data points fit with ~6 parameters.
  - » Fit is "bumpy" – unlikely to get a good fit by chance.







The standard model

- (Nearly) homogeneous & isotropic spacetime
- Governed by GR.
- Contents:
  - » Baryons, cold dark matter, Λ-like dark energy, photons, neutrinos.
- Initial perturbations:
  - » Small, Gaussian, adiabatic, with a nearly scale-invariant power spectrum.

Consistency among different measurements

- CMB
- Sne Ia
- Galaxy clusters
- Galaxy power spectrum
- Galaxy velocities
- Light element abundances
- Globular cluster ages
- H_0 measurements
- …

H_0 = 70 (km/s)/Mpc

Age of Universe = 14 Gyr

Oldest stars = 12-13 Gyr.

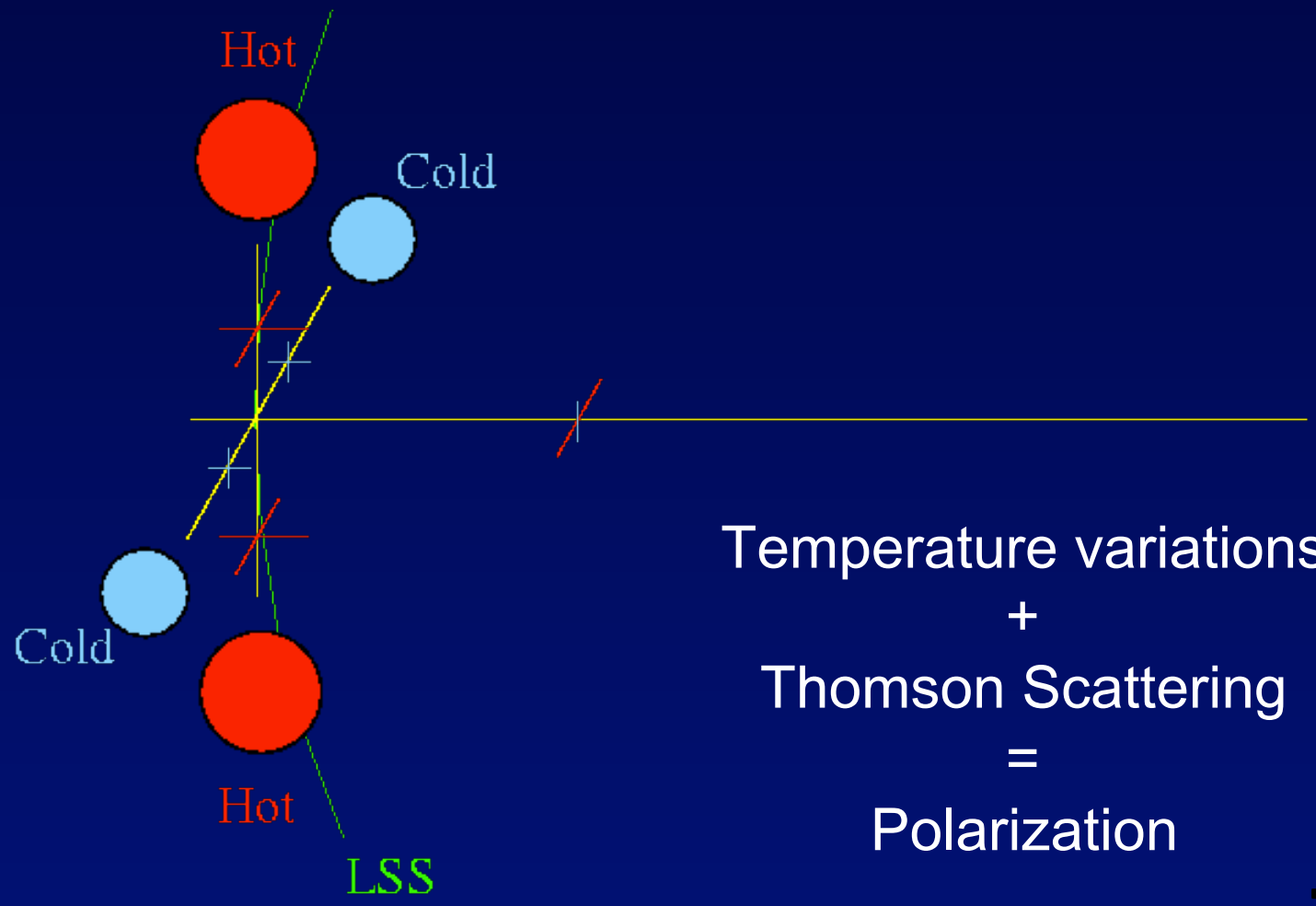
Dark energy density ≈ 0.7.

Matter density ≈ 0.2.

Baryon density: 0.04.

Acoustic waves in the early Universe

CMB Polarization

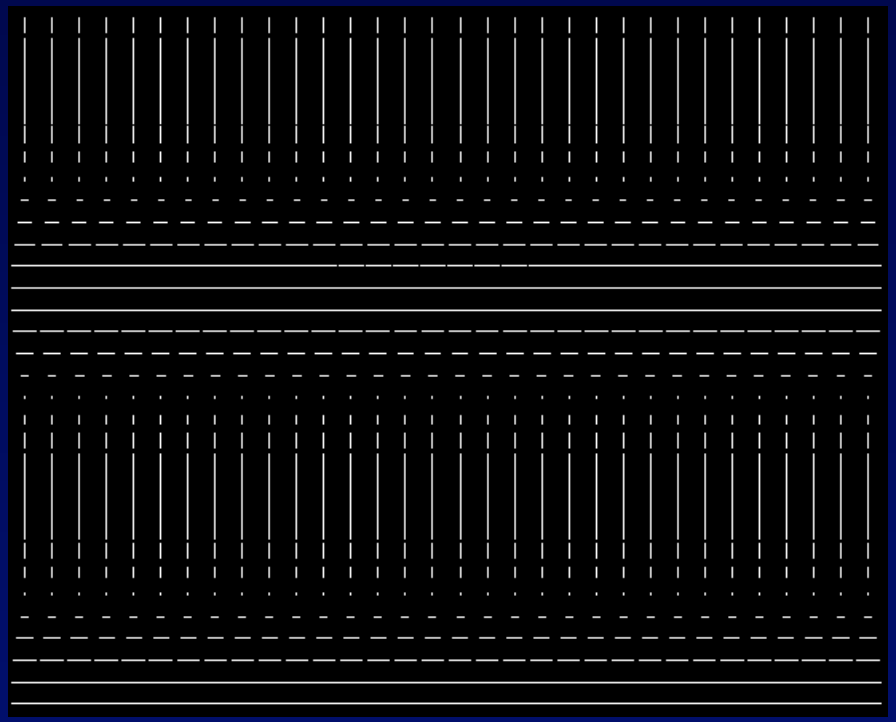


Temperature variations
+
Thomson Scattering
=
Polarization

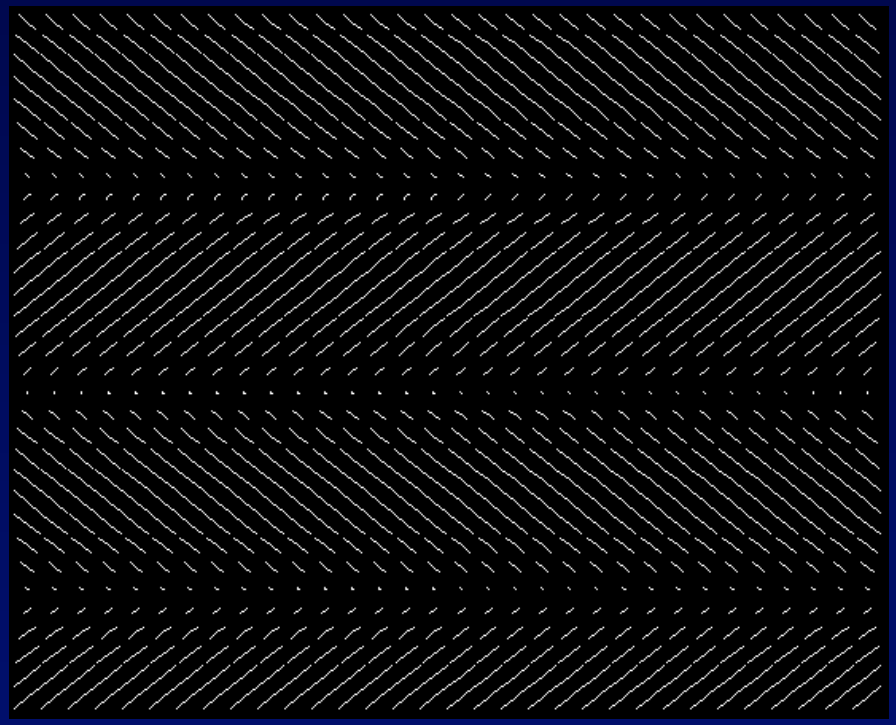
E and B Components

- Two different kinds of polarization patterns:
  - » E component: scalar (no handedness).
  - » B component: pseudoscalar (handedness).
  - » (Analogy: curl-free, divergence-free vector fields)
- Gravitational waves during inflation should produce B; ordinary density perturbations can't (to linear order).
- Detection of B: Strong confirmation of inflation!
- But … non-detection doesn't falsify inflation
  - » Signal is detectable only if $E_{infl} > 10^{15}$ GeV (close to existing upper limits).

E and B Components

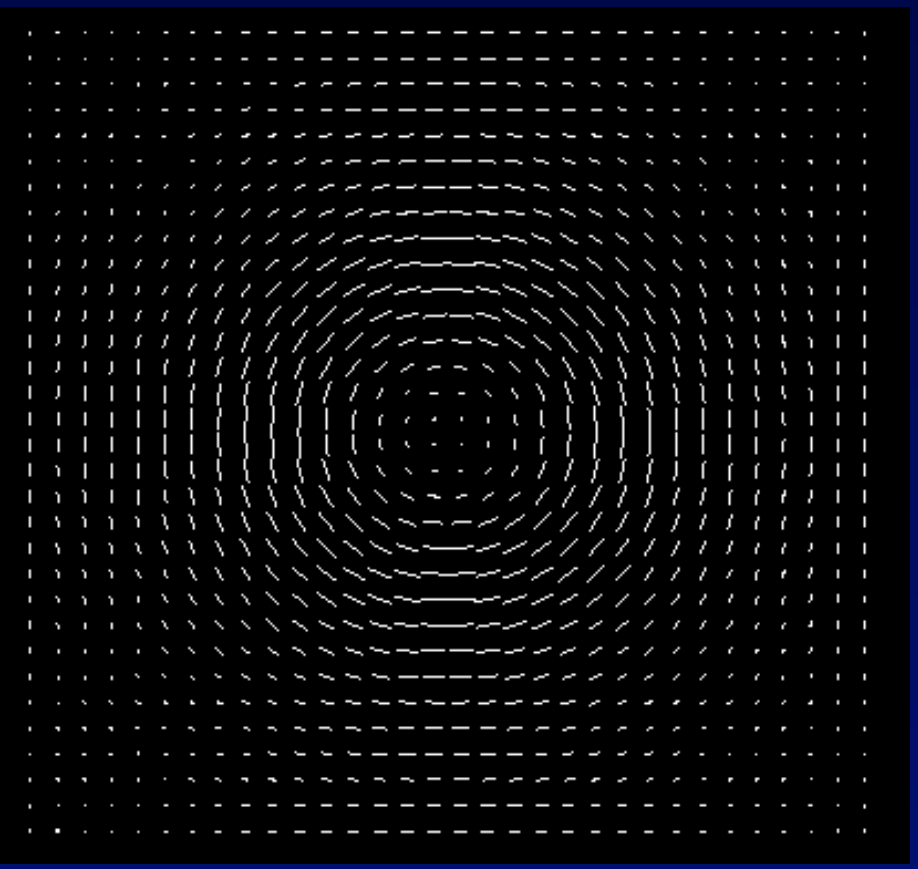


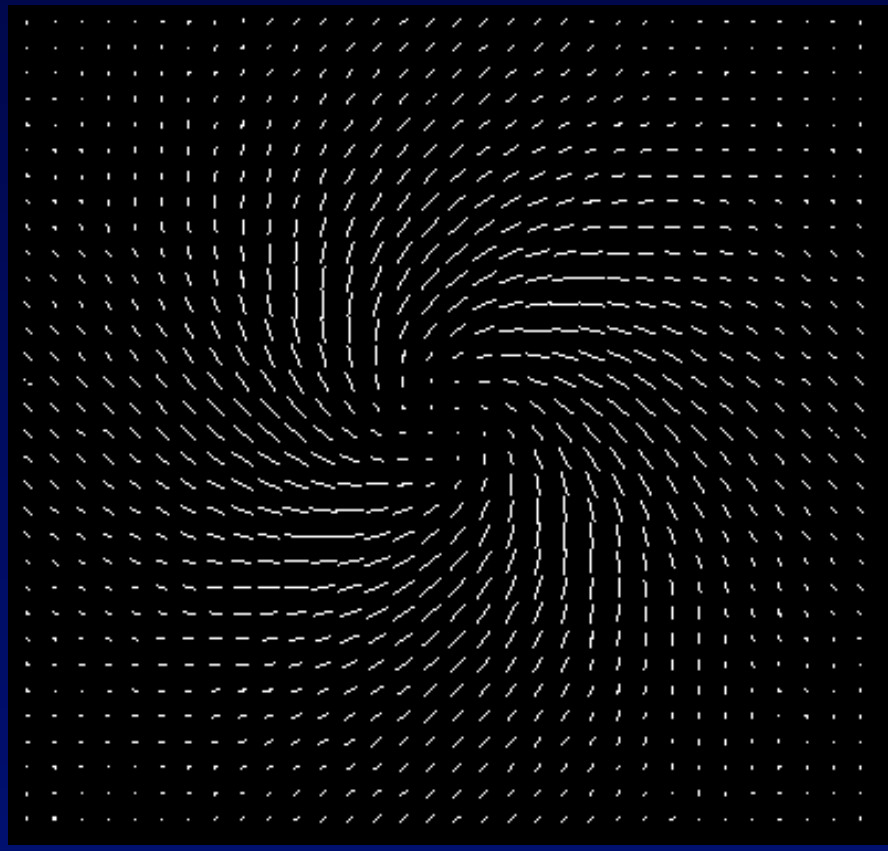
Polarization, k in “+” orientation



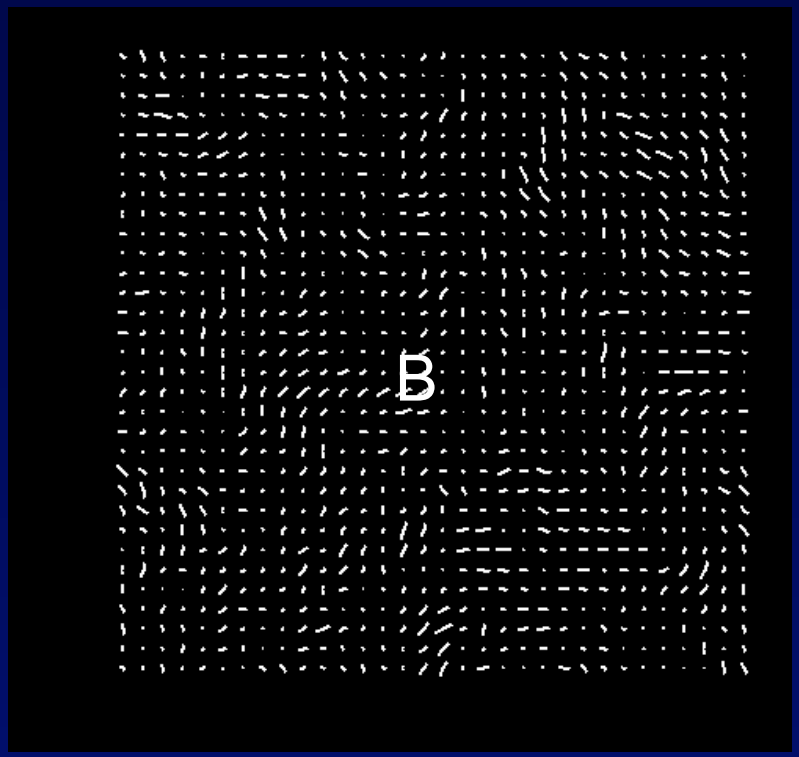
Polarization, k in “x” orientation

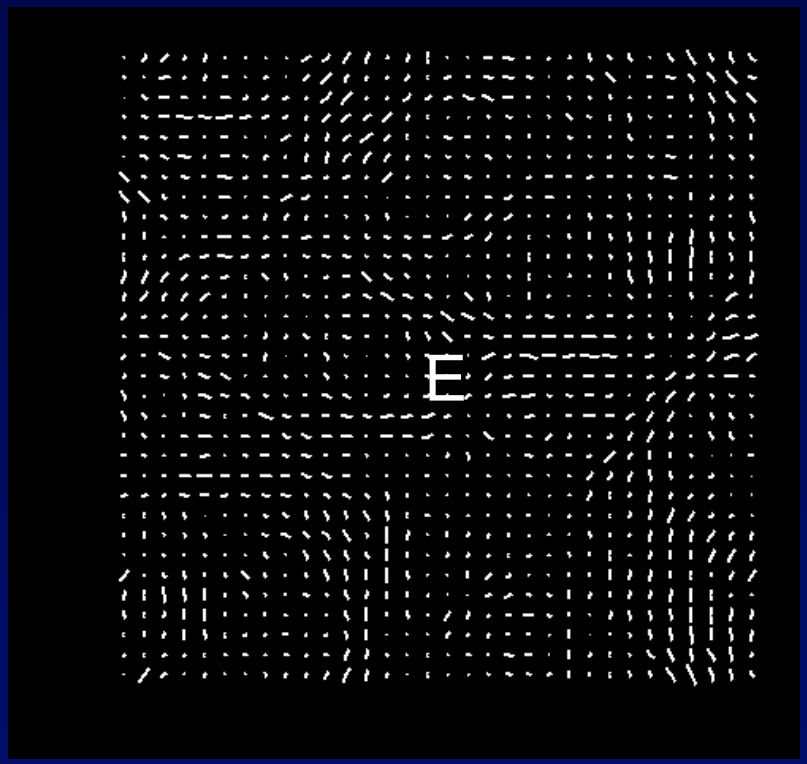
E and B Components



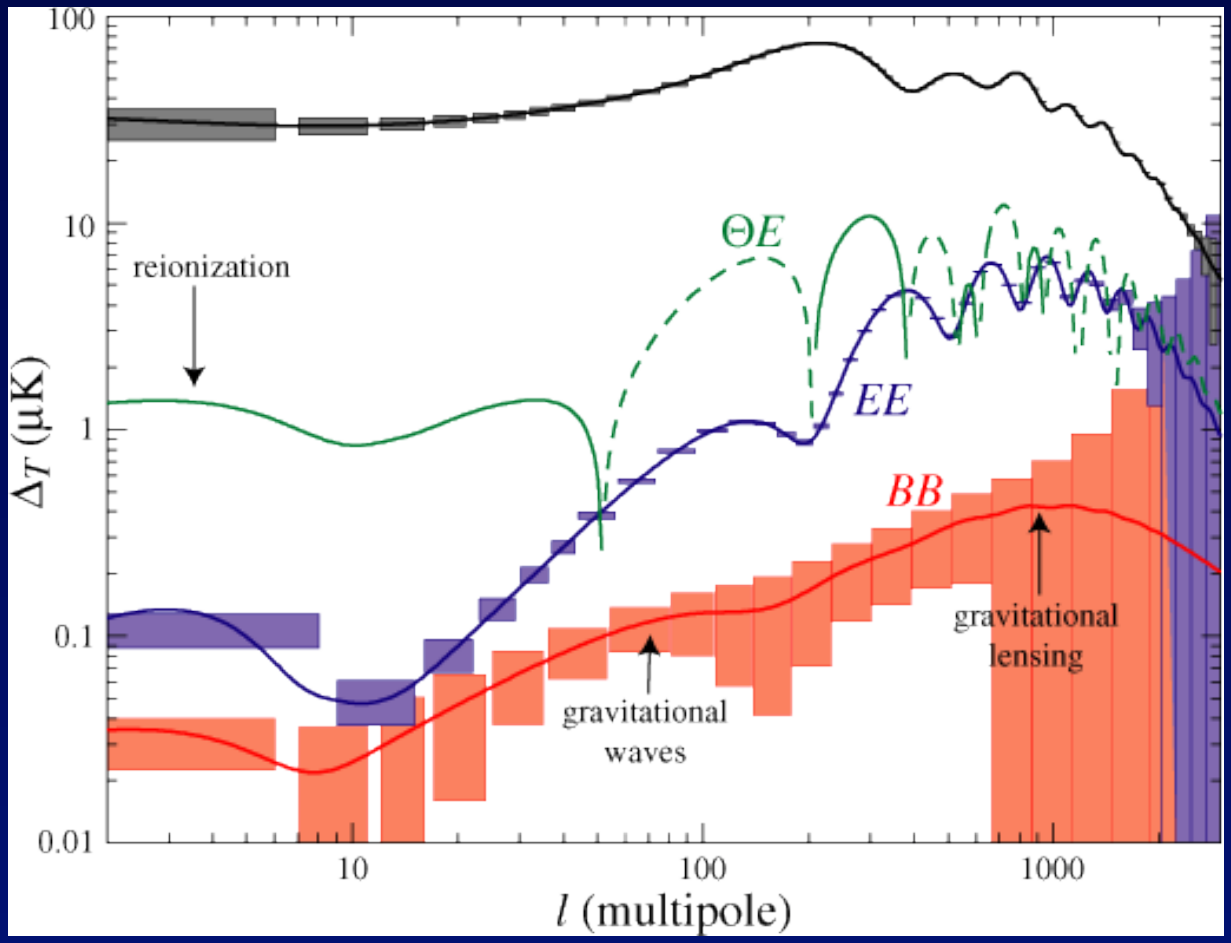


Quiz



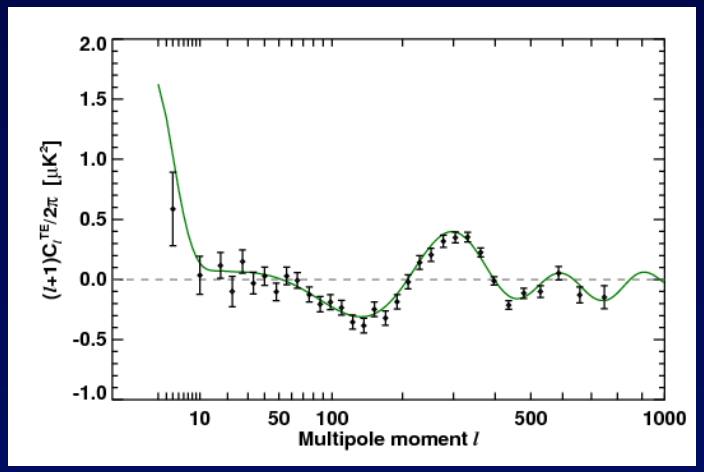


Predictions

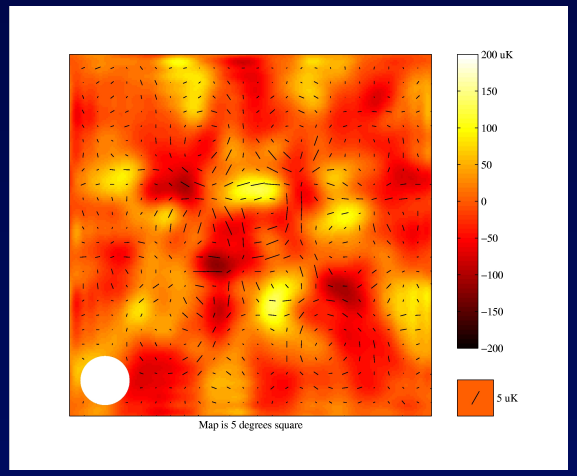


(Hu & Dodelson '02)

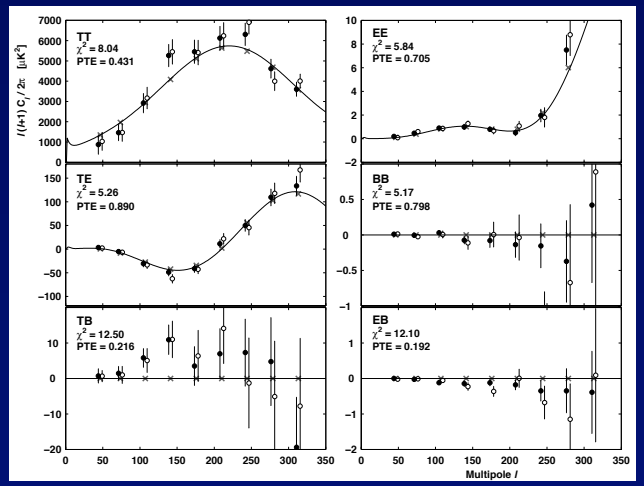
Multiple polarization detections



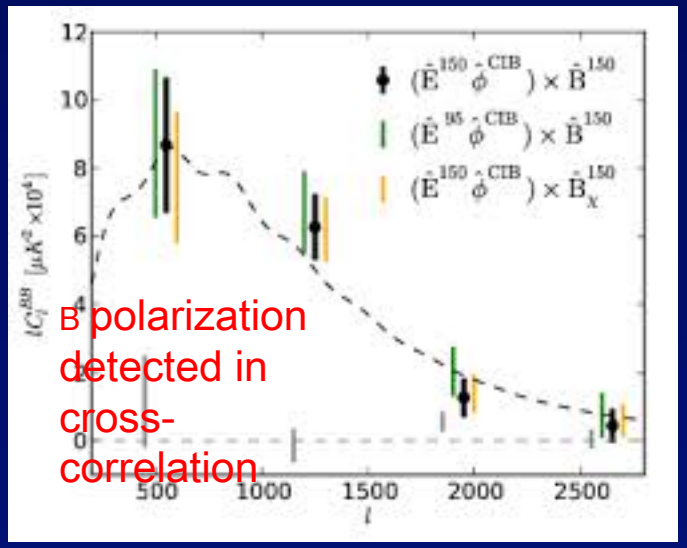
WMAP



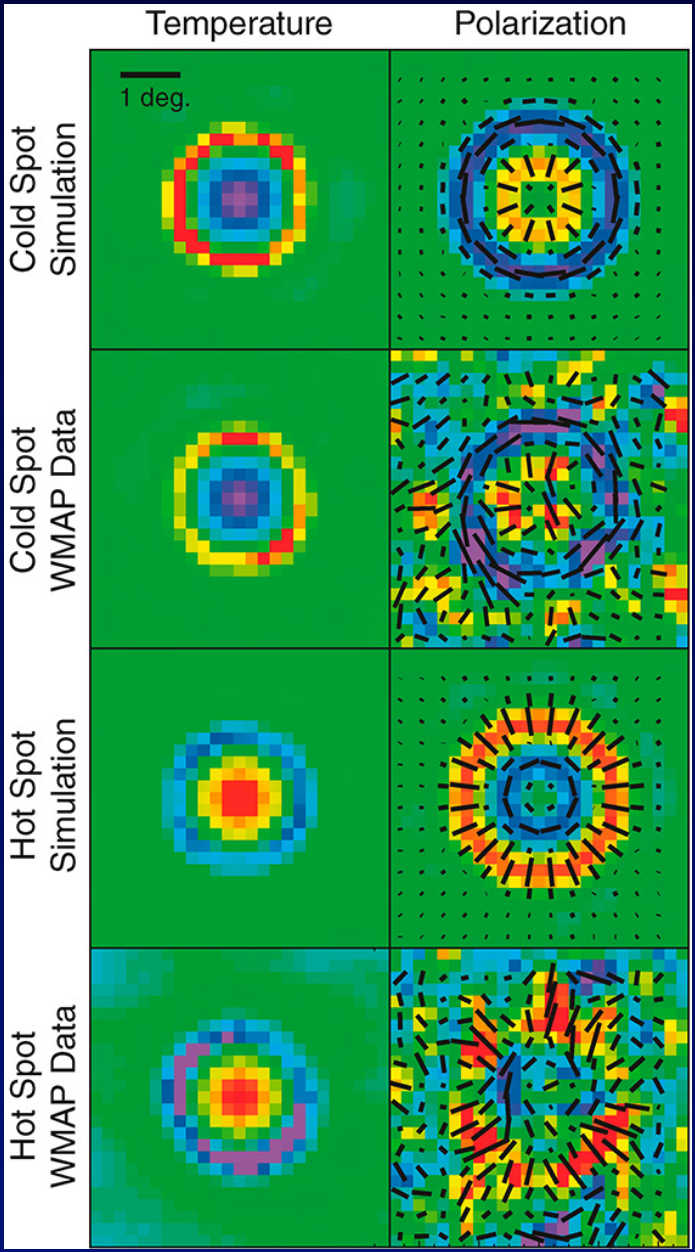
DASI



QUIET



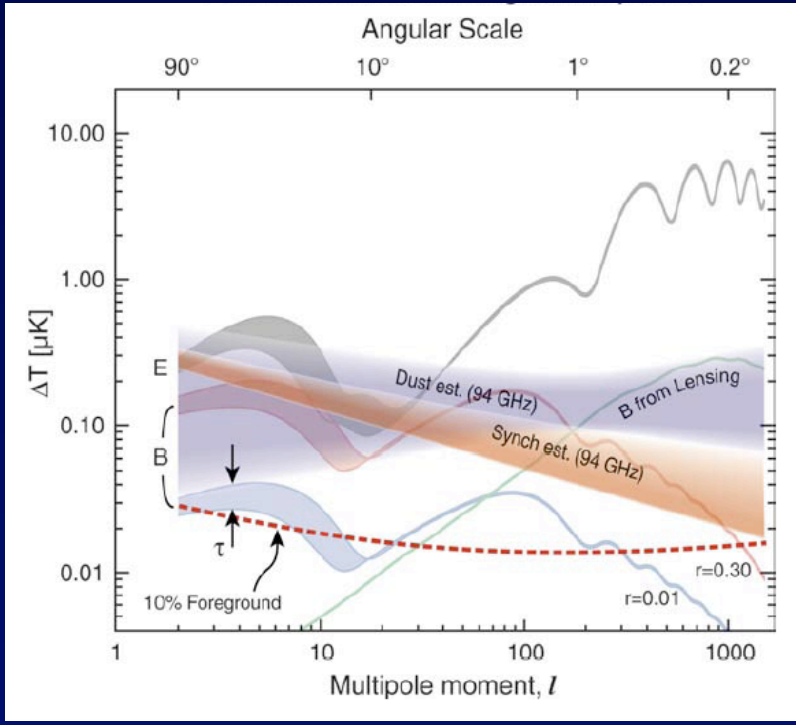
SPTPOL



(Komatsu et al. 2010)

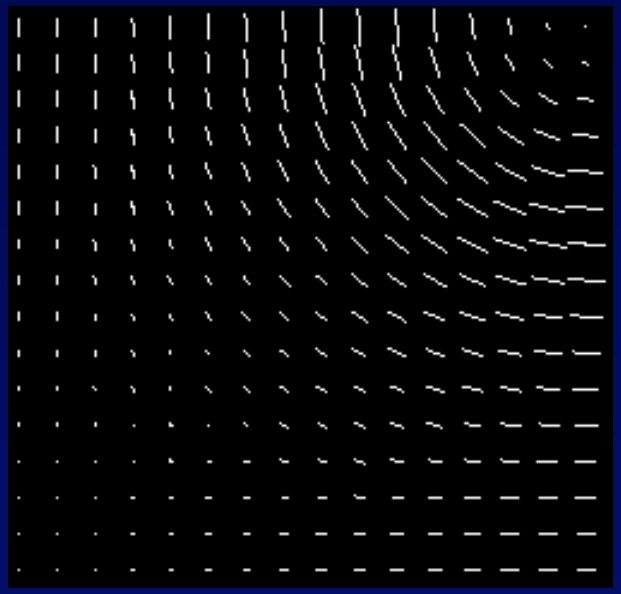
Characterizing B modes will be hard.

- Small signal (~ 100 nK).
  - » ~10^3 detectors for ~1 year to reach required sensitivity.
- Need to avoid contamination from much larger E,T signals.
  - » Incomplete sky coverage ➔ E/B mixing.
  - » Control of systematic errors is crucial.
- Inflation signal on large angular scales – hard to see from ground.
- *Foregrounds??*



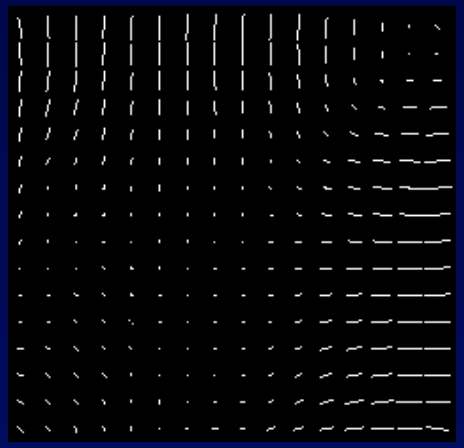
Bock et al. '06 (Weiss report)

Ambiguity



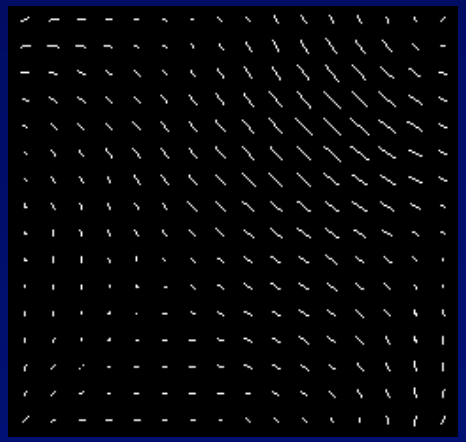
(Quadrant of E spot)

=



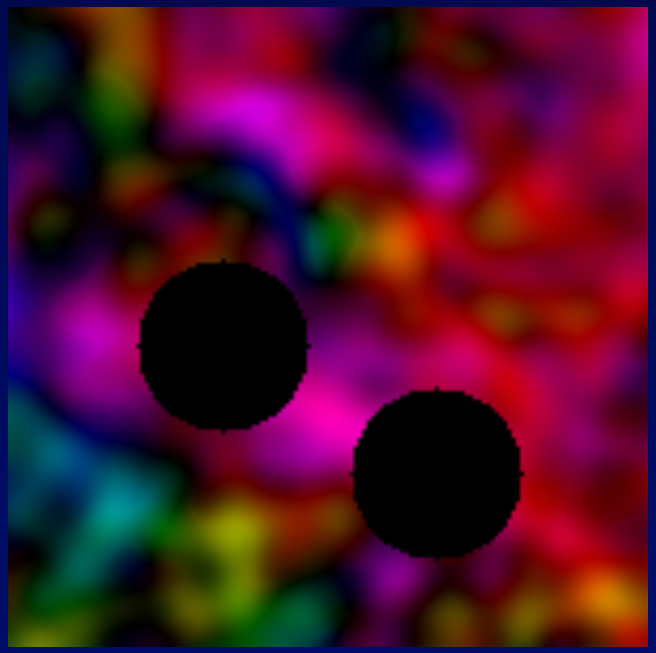
(E)

+



(B)

E



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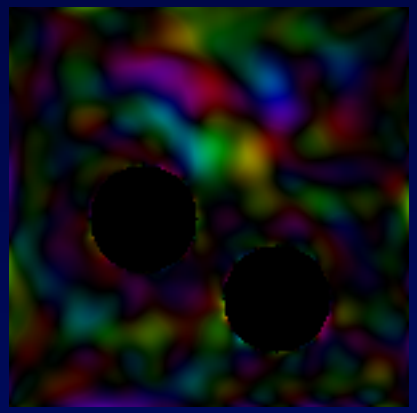
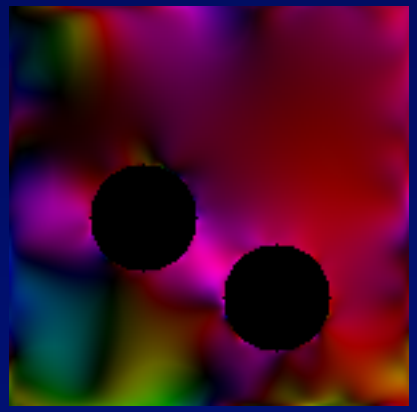


Pure E

+



Ambiguous

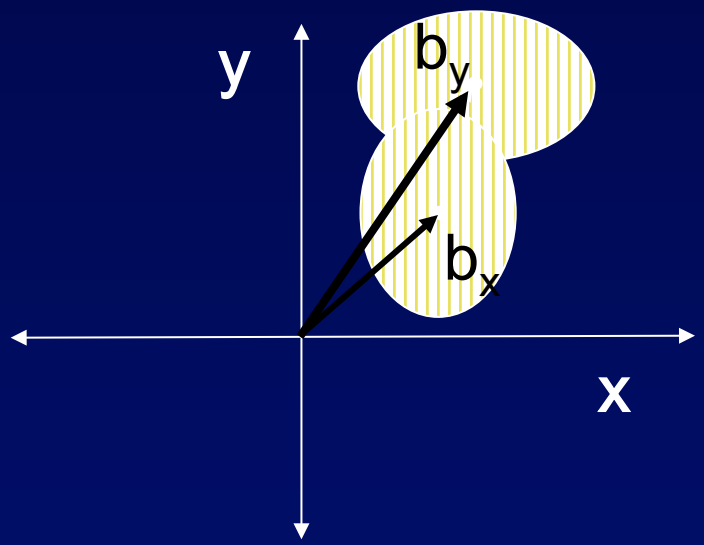
(Bunn 2011)

Interferometry?

- Usual way of measuring polarization involves differencing:

$$|E_x|^2 - |E_y|^2 \propto (I+Q) - (I-Q) \propto Q$$

- Calibration, pointing, beam errors ➔ leakage of *I* into *Q*.
- Interferometer: Visibilities measure just a given Stokes parameter.



$$V_U(\vec{u}) = \int d^2r\, E_x(\vec{r}) E_y^*(\vec{r}) A(\vec{r}) e^{2\pi i \vec{u}\cdot\vec{r}}$$

May not be better, but definitely different (and underexplored).

E/B separation in interferometry

Heuristically:

- E/B separation is unambiguous in Fourier space.
- Visibilities live in Fourier space (or its vicinity).

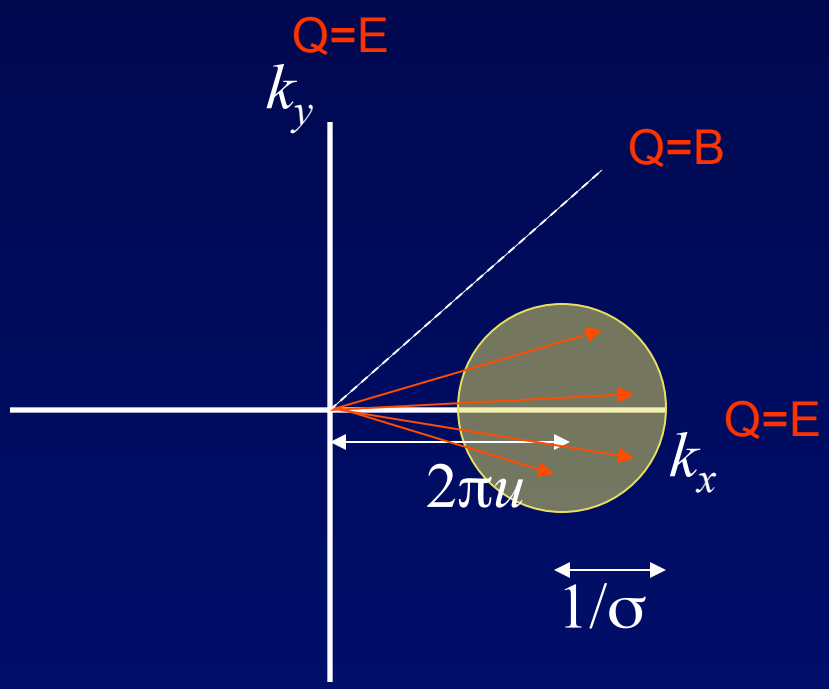
➔ Maybe they're better.

Not so fast:

- Most information is in short baselines

➔ Beam convolution mixes Fourier modes.

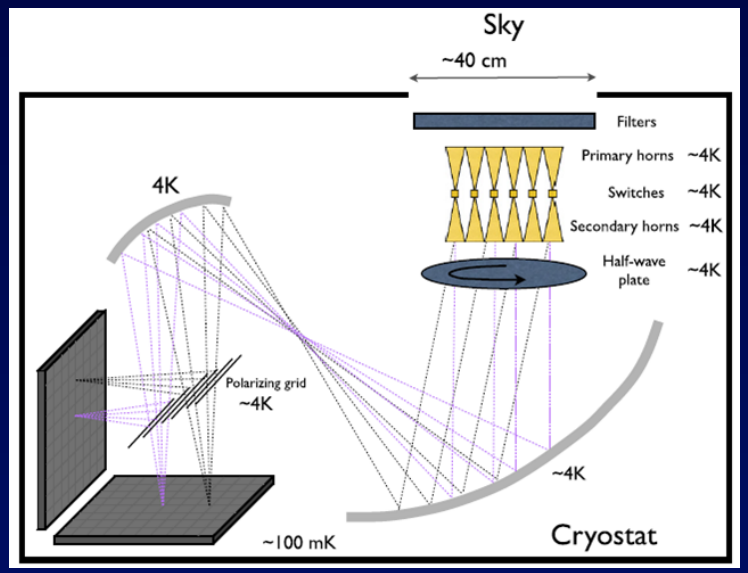
- A simple analytic approach gives rough guidance about the effects of E/B mixing, but you need simulations for detailed characterization.

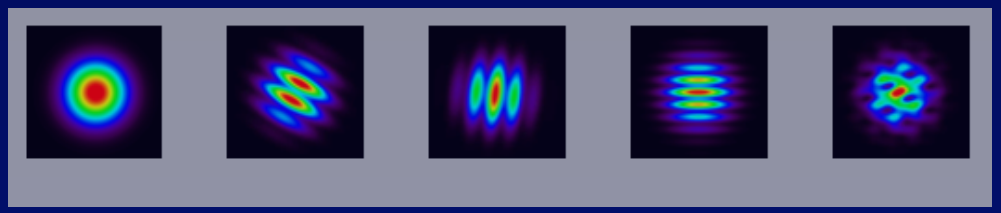


$$\text{Mixing} \sim \sin^2 2\theta$$
$$\theta = 2\pi u \sigma$$

QUBIC

- Proposed bolometric interferometer.
- Feedhorns look directly at the sky; quasioptical (Fizeau) combiner produces "dirty image" on focal plane. Can solve for visibilities or think of it as an imager with a dirty beam.
- May combine senstivities of bolometers with systematic error control of interferometry.
- Dome C, Antarctica, 2017.





(Battistelli et al. 2010, Ghribi et al. 2013)

France (APC, Toulouse, Orsay), Italy (Rome, Milan, Siena), UK (Manchester), Ireland (Maynooth), India (Bangalore) USA (Brown, Wisconsin, Richmond),

Potential advantages

- HWP located before telescope → Cross-polarization not modulated.
- Primary beam determined by feedhorns → small sidelobes.
- Synthesized beam can be thoroughly characterized ("autocalibration").
  - » Switch off various combinations of horns.
  - » Get a system of equations that can be solved for all gains, couplings, etc.
- Regular lattice of horns → Redundantly-measured visibilities, coherently summed. (Sensitivity comparable to imager.)
- At the very least, errors will be different than with a standard imager.

Characterizing systematic errors

- A simple analytic approach gives a rough idea of how various sorts of error cause "leakage" from strong signals to weak (T → E → B) (Bunn 2007).
- Exact results require simulation.
- We* have simulated a QUBIC-like experiment:
  - » Generate visibilities from Gaussian skies.
  - » Estimate power spectra two ways:
    - Maximum likelihood
    - Gibbs sampling
  - » Determine power spectrum errors caused by various errors.

$$\Delta C^{XY} = \left\langle (C^{XY}_{\rm actual} - C^{XY}_{\rm ideal})^2 \right\rangle^{1/2}$$

Who's "we"?

- Brown University (Greg Tucker, Ata Karakci, Andrei Korotkov)
- University of Wisconsin (Peter Timbie, Le Zhang)
- UIUC / IAP (Ben Wandelt, Paul Sutter)
- U. Richmond (me).

As usual, the students and postdocs did all the actual work.

(Karakci et al. 2013ab, Zhang et al. 2013, Sutter et al. 2013)

Formalism

Signal in an antenna

2x2 matrix-valued antenna pattern

Incoming electric field

$$\boldsymbol{\epsilon}_{\rm out} = \int d^2\hat{\mathbf{r}}\, \mathbf{A}(\hat{\mathbf{r}}) \cdot \boldsymbol{\epsilon}_{\rm in}(\hat{\mathbf{r}}) e^{i(\mathbf{k}\cdot\boldsymbol{\xi} - \omega t)}$$

$$\langle \boldsymbol{\epsilon}_{{\rm in}\,i}(\hat{\mathbf{r}}_1)\boldsymbol{\epsilon}_{{\rm in}\,j}(\hat{\mathbf{r}}_2)^* \rangle = S_{ij}\delta(\hat{\mathbf{r}}_1 - \hat{\mathbf{r}}_2)$$

$\boldsymbol{\epsilon}_{\rm in}$,$\boldsymbol{\epsilon}_{\rm out}$ can be defined in either linear or circular bases.

Only one component of $\boldsymbol{\epsilon}_{\rm out}$ may be measured in a given experiment.

$$\mathbf{S}_{\rm lin} = \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix}. \qquad \boldsymbol{\epsilon} = \begin{pmatrix} \epsilon_x \\ \epsilon_y \end{pmatrix}.$$

$$\mathbf{S}_{\rm circ} = \begin{pmatrix} I+V & Q+iU \\ Q-iU & I-V \end{pmatrix}. \qquad \boldsymbol{\epsilon} = \begin{pmatrix} \epsilon_R \\ \epsilon_L \end{pmatrix}.$$

Visibilities

2 x 2 matrix of visibilities from interfering antennas *j,k:*

$$\mathbf{V}^{(jk)} = \int d^2\hat{\mathbf{r}}\, \mathbf{A}^{(j)}(\hat{\mathbf{r}}) \cdot \mathbf{S}(\hat{\mathbf{r}}) \cdot \mathbf{A}^{(k)\dagger}(\hat{\mathbf{r}})\; e^{-2\pi i \mathbf{u}_{jk}\cdot\hat{\mathbf{r}}}$$

In an ideal experiment, measured visibilities can be converted into Stokes visibilities:

$$V_Q = \frac{1}{2}(V_{XX} - V_{YY}) = \frac{1}{2}(V_{RL} + V_{LR})$$

$$V_U = \frac{1}{2}(V_{XY} + V_{YX}) = \frac{1}{2i}(V_{RL} - V_{LR})$$

- Circular experiment: V_Q, V_U measured simultaneously.
- Linear experiment: Only one Stokes visibility typically measured per baseline. (V_U in coordinates aligned with baseline).

Modeling errors

$$\mathbf{A}(\hat{\mathbf{r}}) = \mathbf{J}_i \cdot \mathbf{A}_s(\hat{\mathbf{r}})$$

Error-free experiment: $\mathbf{J}_i = \mathbf{1}, \quad \mathbf{A}_s(\hat{\mathbf{r}}) = A(\hat{\mathbf{r}})\mathbf{1}$

"Instrument errors":

Gain

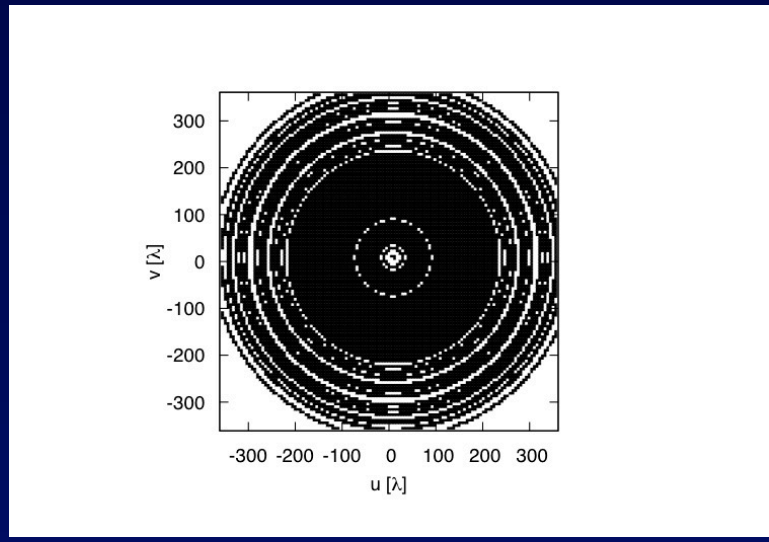
Coupling

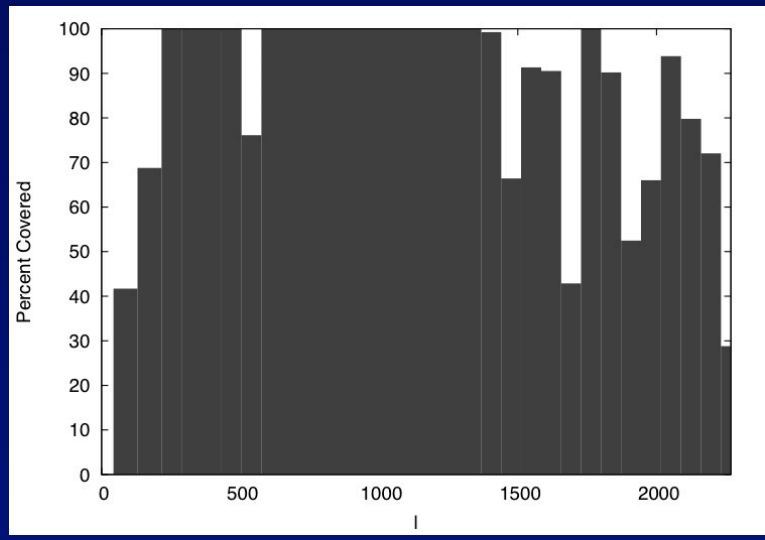
$$\mathbf{J}_i^{(j)} = \begin{pmatrix} 1 + g_1^{(j)} & \epsilon_1^{(j)} \\ \epsilon_2^{(j)} & 1 + g_2^{(j)} \end{pmatrix}.$$

"Beam errors": $\mathrm{A_s}$ not proportional to identity matrix (cross-polarization).
A varies in shape / pointing from one antenna to another (pointing errors, squint, ellipticity, etc.).

Simulations

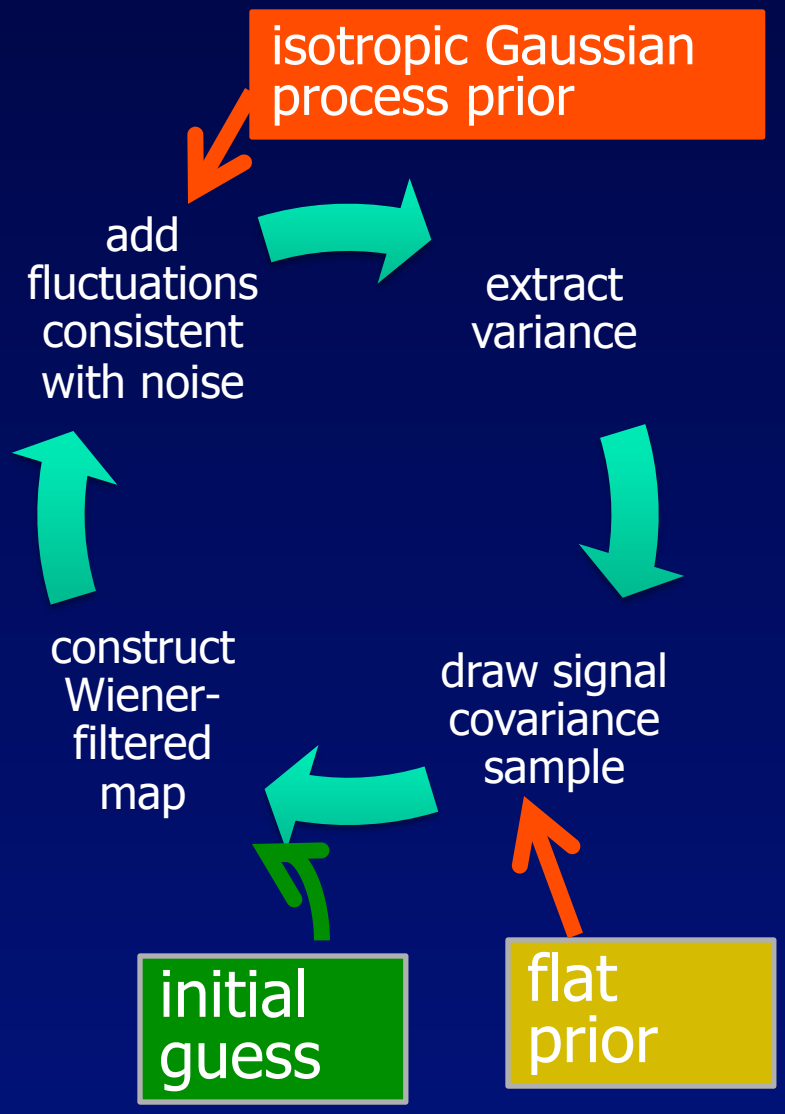
- 20 x 20 close-packed array of horns with $D = 7.89\lambda$.
- Observation at 150 GHz from pole.
- Gaussian beam with $\sigma = 5^{o}$.
- Input maps: ΛCDM with $r = 0.01$.
- Average noise per visibility: 0.015 μK ➔ S/N ~ 5 in Q,U maps.
- "Turn on" various systematic errors, with amplitudes expected via analytical estimates to have 10% effect on B power recovery.



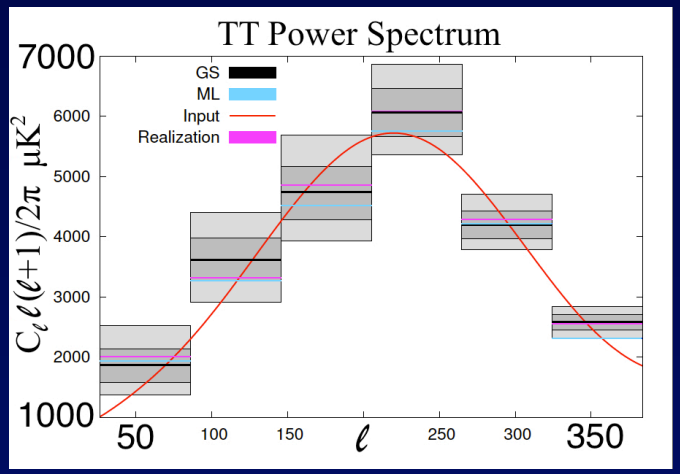


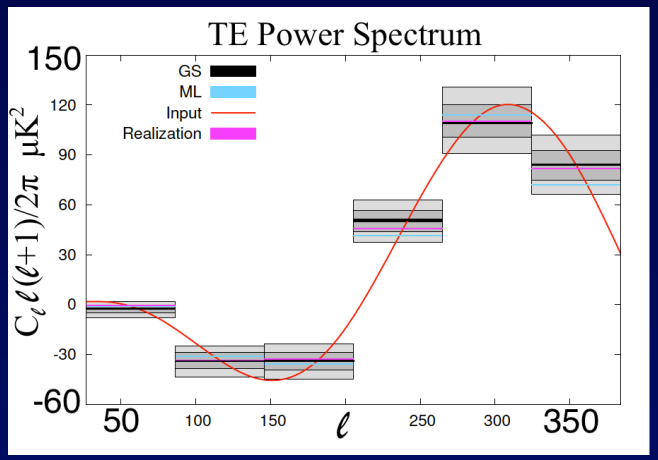
Gibbs sampling

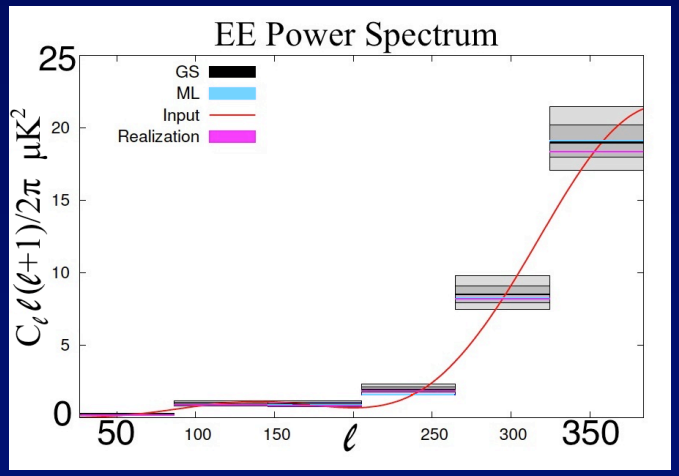
- Sample from joint probability distribution of power spectrum, signal given data $P(C_l, \mathbf{s} \mid \mathbf{d})$.

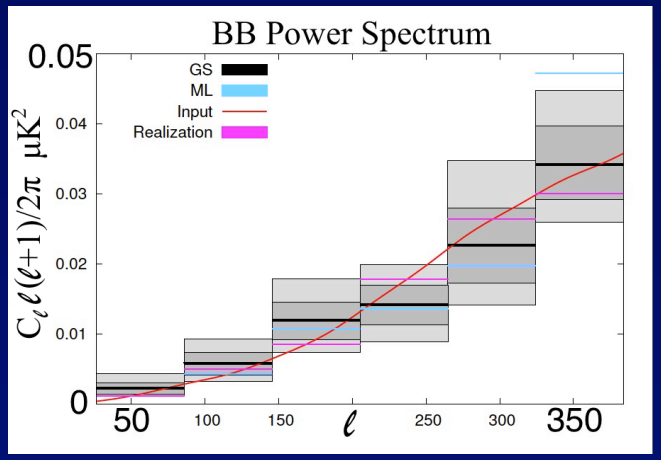


Recovered power spectra

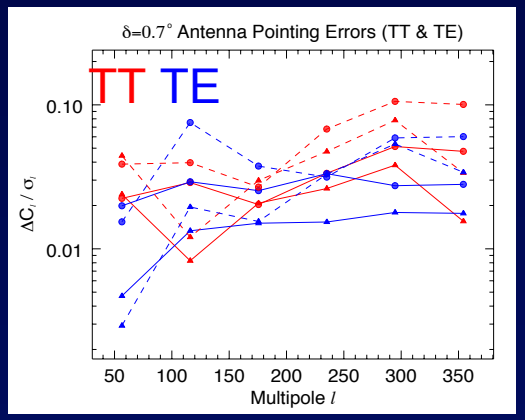


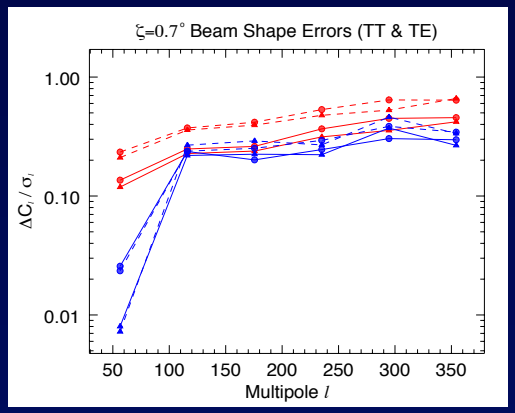


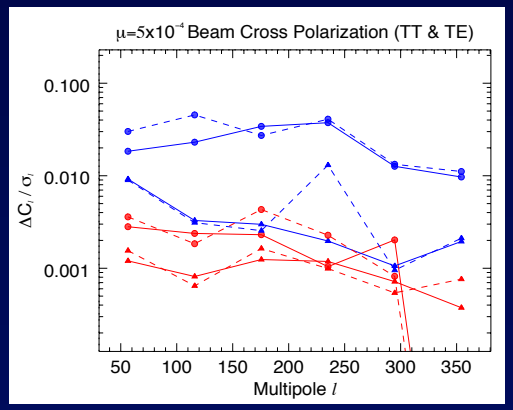




Results: beam errors



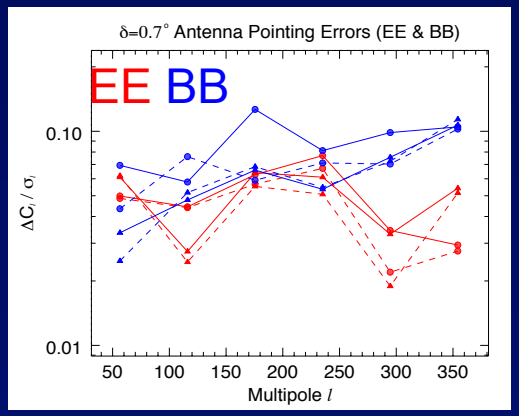


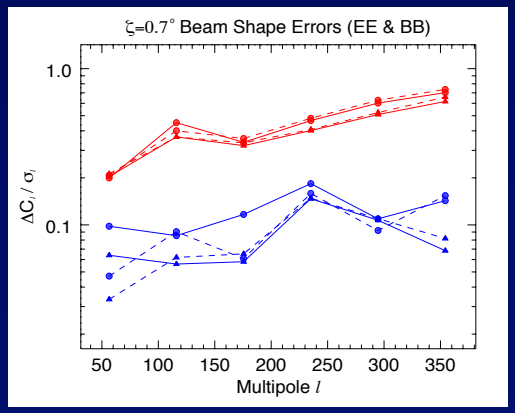


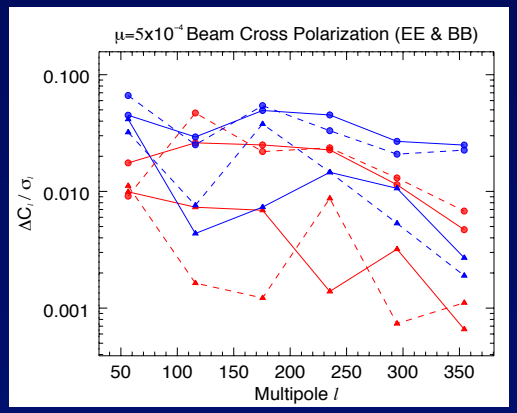
▲ ML
● GS

Linear

Circular





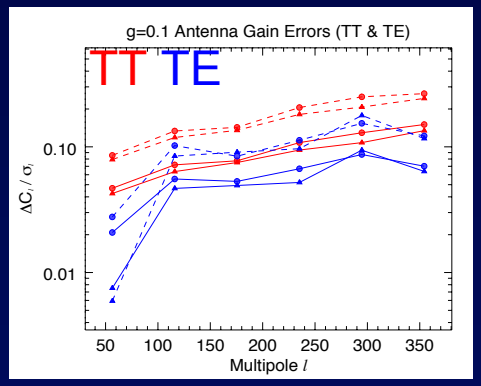


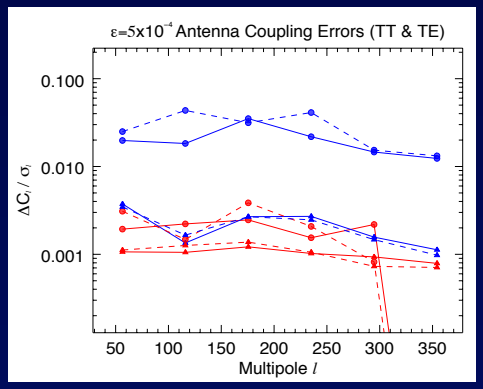
Pointing:
$\delta = (\Delta \mathbf{r}_{\rm center})_{\rm rms}$
E → B

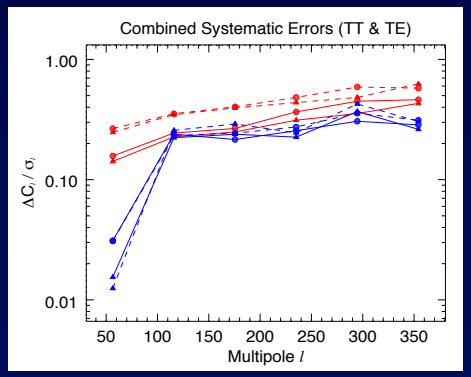
Beam shape:
$\zeta = (\Delta \sigma_{b,xy})_{\rm rms}$
E → B

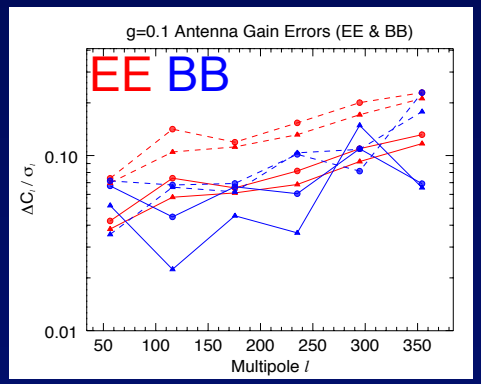
Cross-pol:
$\mu = (A_{\rm off-diag}/A_{\rm diag})_{\rm rms}$
T → B

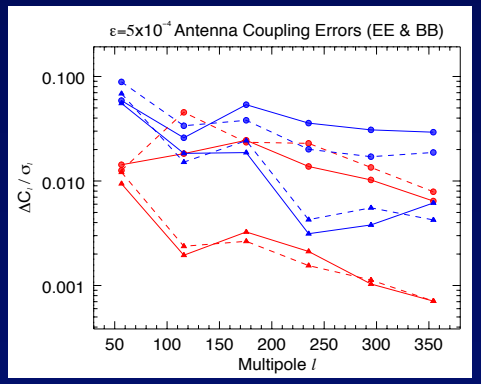
Results: instrument errors

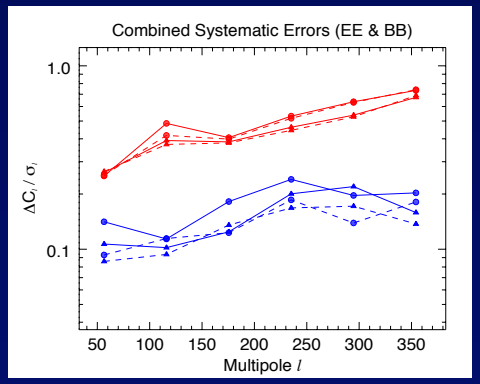












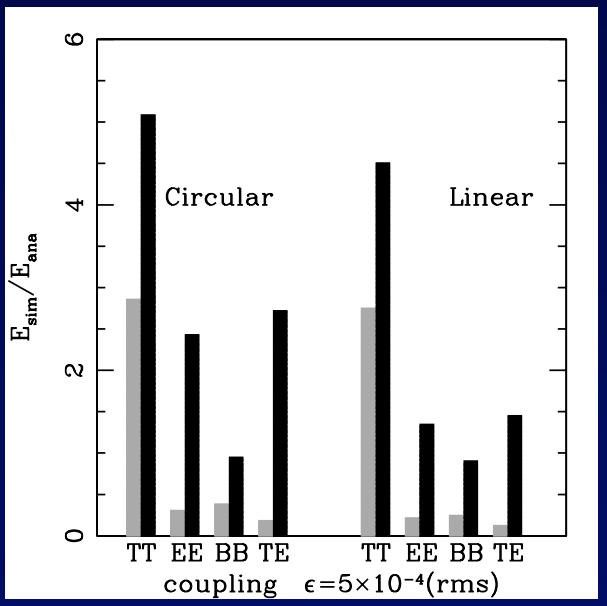
Gain (g):
E → B

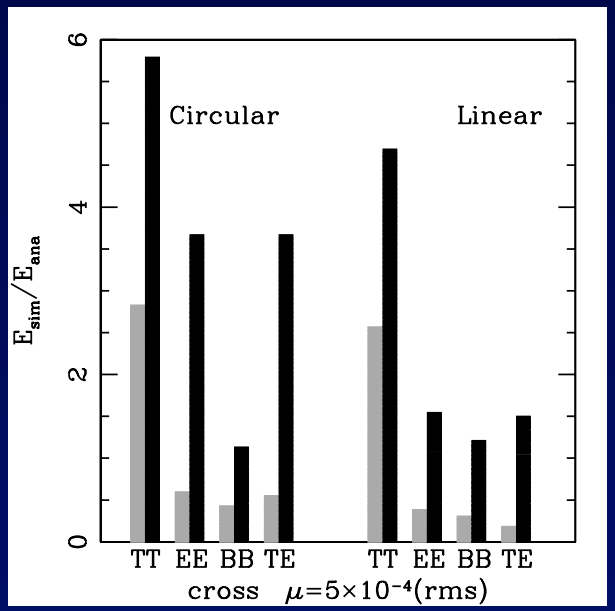
Coupling (ε):
T → B

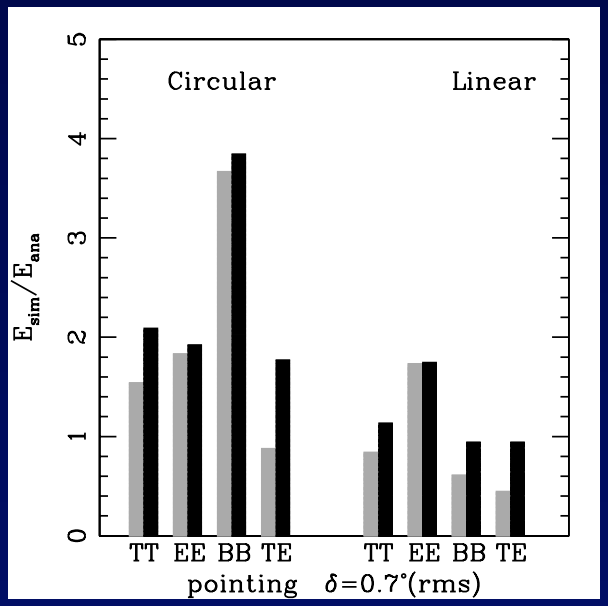
All errors combined

$$\mathbf{J}_i^{(j)} = \begin{pmatrix} 1+g_1^{(j)} & \epsilon_1^{(j)} \\ \epsilon_2^{(j)} & 1+g_2^{(j)} \end{pmatrix}.$$

Simulation vs. analytical estimation





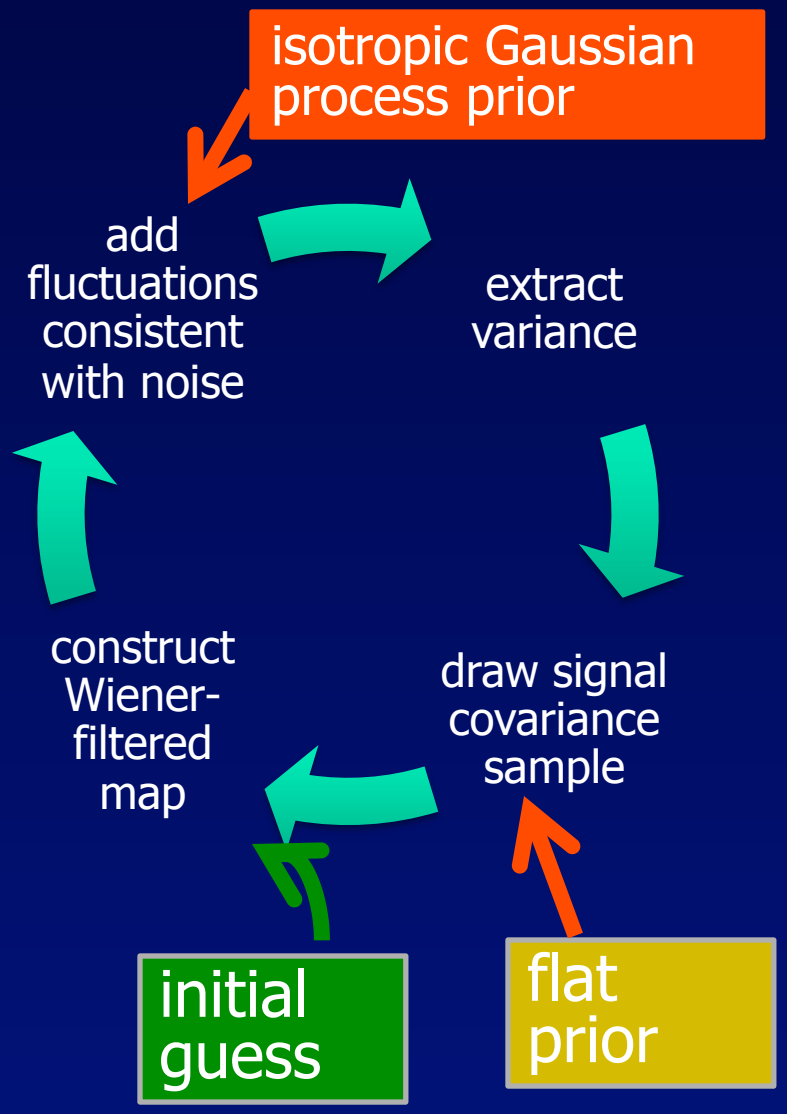


Analytical estimation:
- 1st-order approximation
- Simpleminded approach to: uv-coverage, sky rotation, ...
- Off by factors of ~5 -- Better than you might expect!

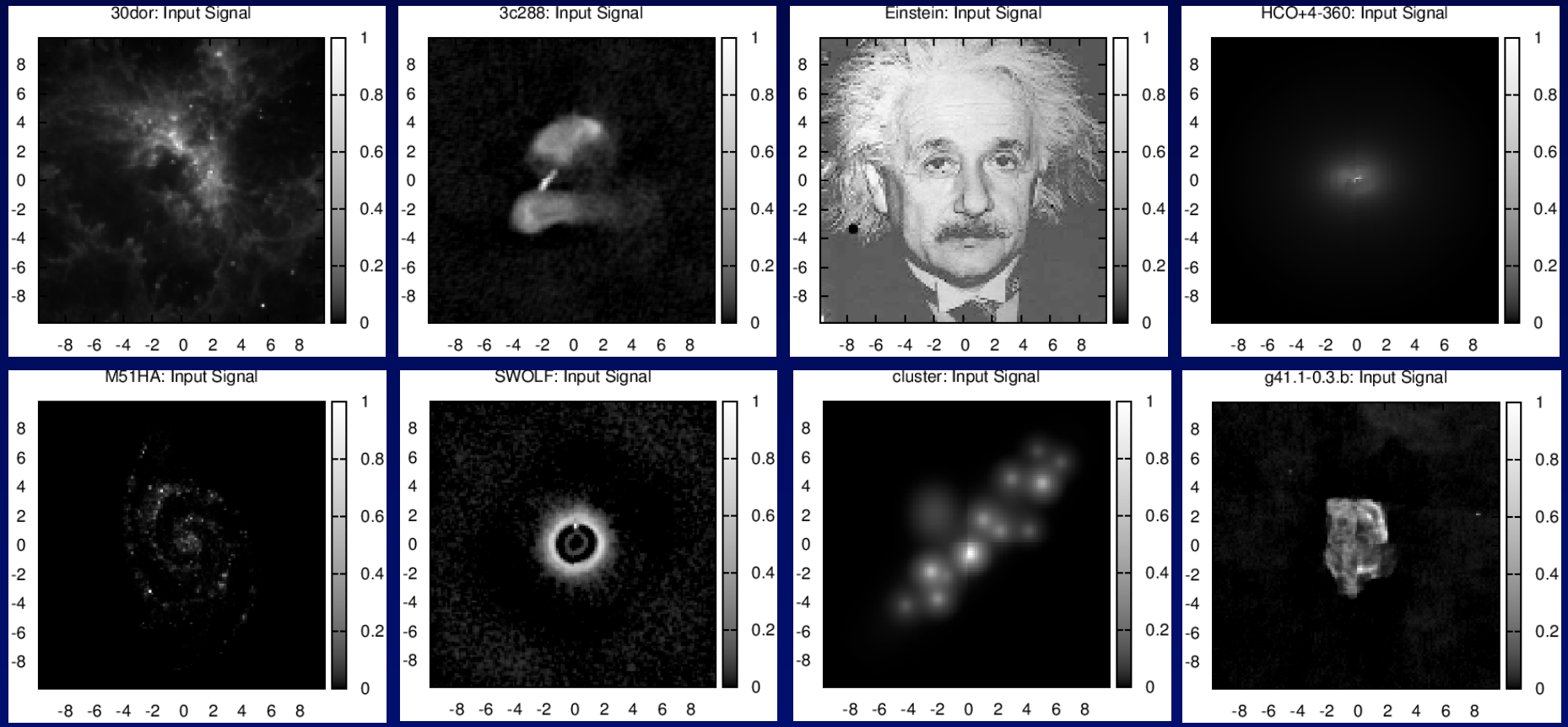
Gibbs sampling – not just for cosmologists?

- Gibbs sampling gives simultaneous estimates of C_l, sky signal.
- Cosmologists care about one; astronomers care about the other.
- Can we use Gibbs sampling to reconstruct astronomical images from interferometric data?

- Assumptions that go into Gibbs sampling are not correct for general astronomical images: things aren't Gaussian or isotropic.
- What if we don't worry about that?
- Gaussian prior is minimally informative / maximum entropy, so this can be thought of as a data-driven approach.

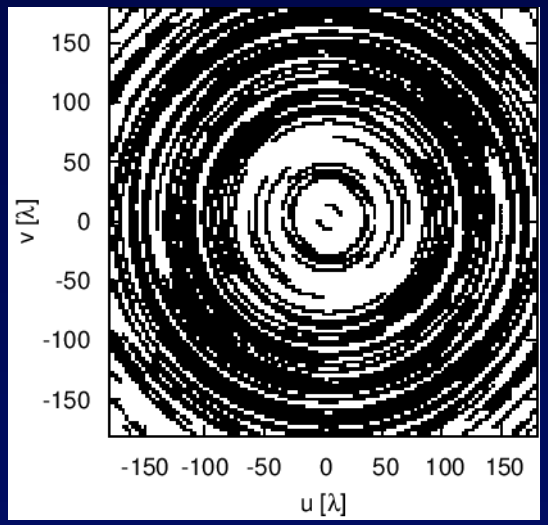


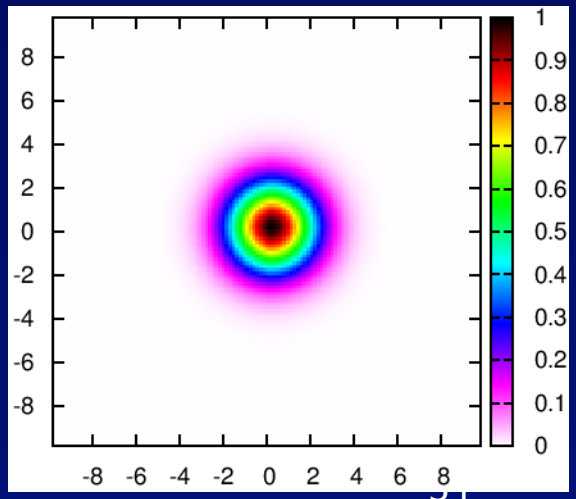
Test images


30dor: Input Signal
3c288: Input Signal
Einstein: Input Signal
HCO+4-360: Input Signal
M51HA: Input Signal
SWOLF: Input Signal
cluster: Input Signal
g41.1-0.3.b: Input Signal
8 6 4 2 0 -2 -4 -6 -8
-8 -6 -4 -2 0 2 4 6 8
1 0.8 0.6 0.4 0.2 0

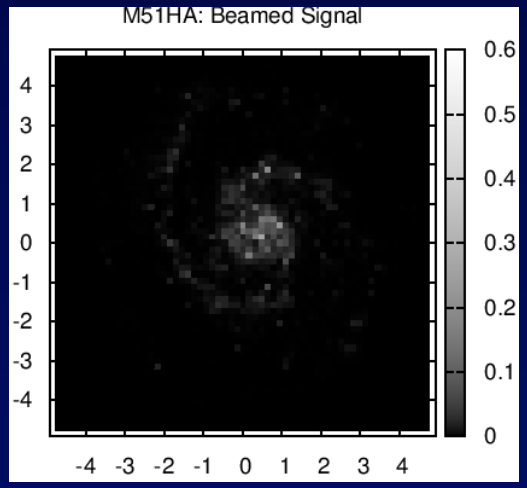
Mock observations

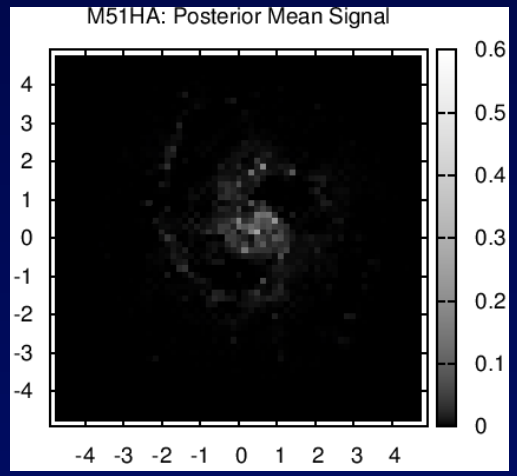
- Gaussian beam width
  σ_b = 0.075 x image width.
- 12 randomly-placed antennas.
- 6-hour observation.
- Average signal-to-noise = 10.

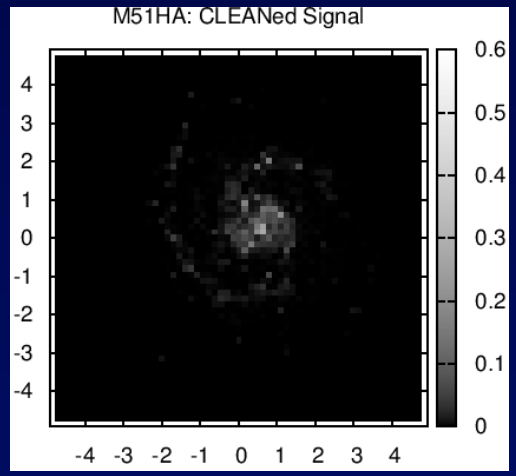


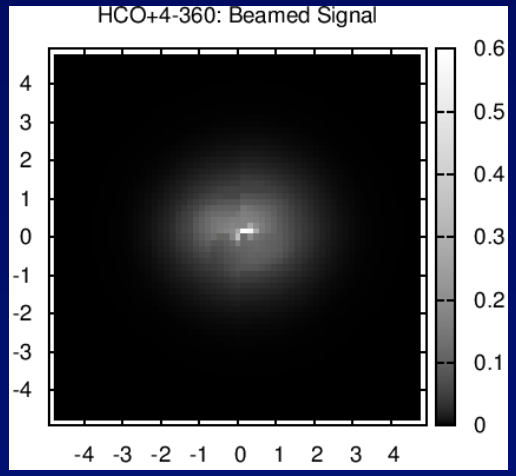


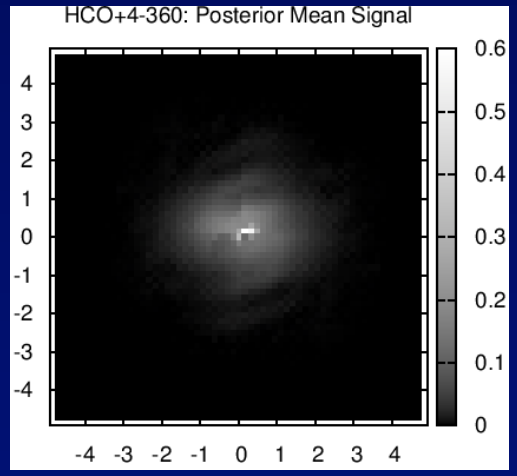
Gibbs vs. CLEAN (1)

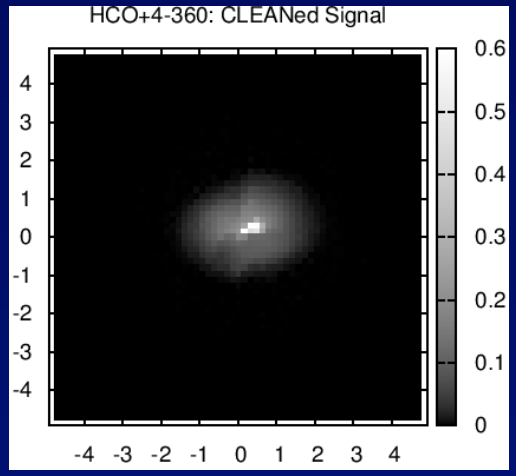




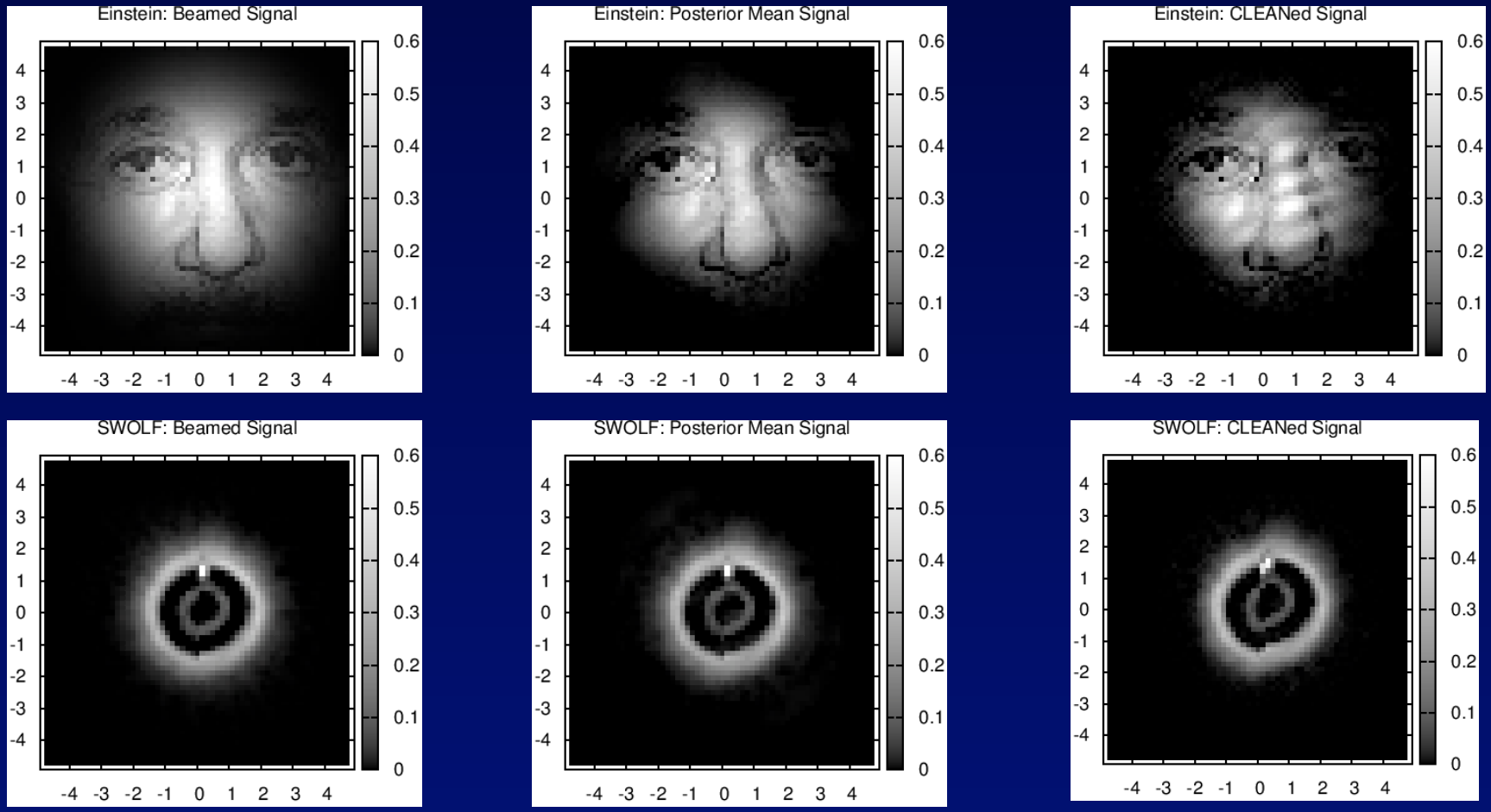




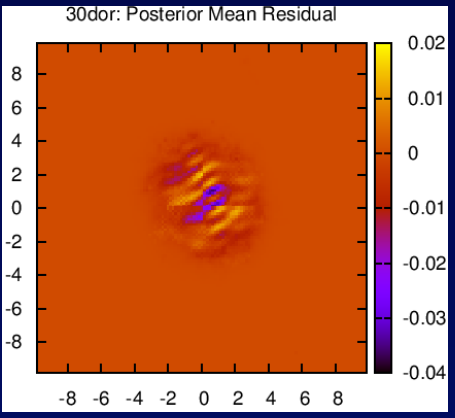


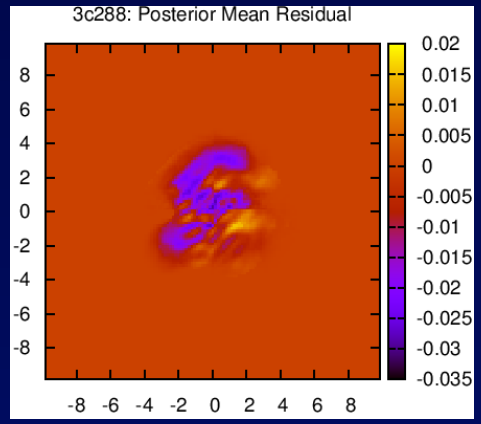


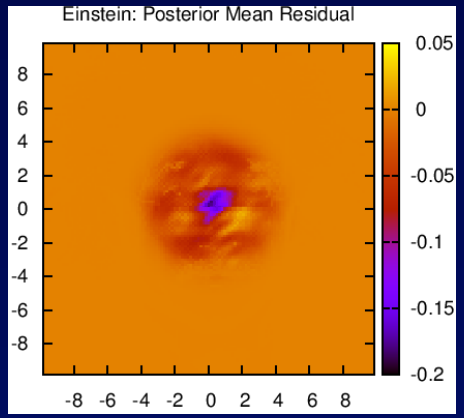
Gibbs vs. CLEAN (2)

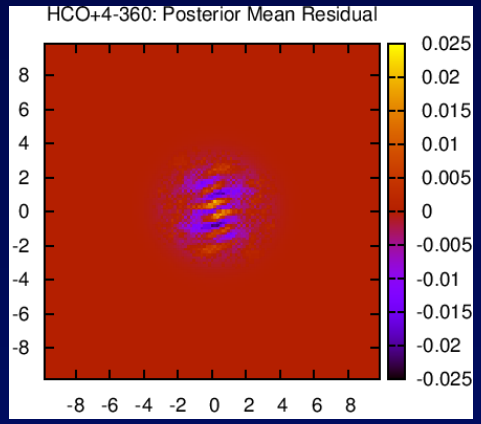


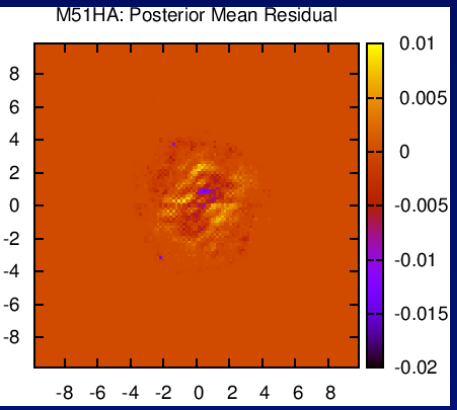
Residuals

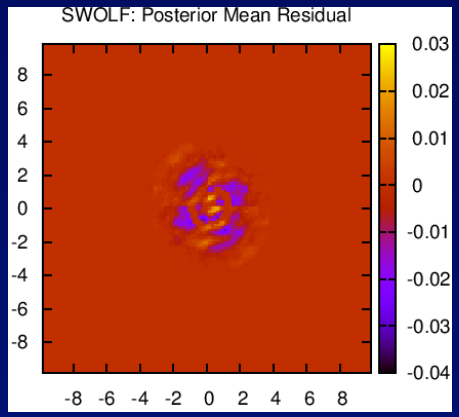

30dor: Posterior Mean Residual
8 6 4 2 0 -2 -4 -6 -8
-8 -6 -4 -2 0 2 4 6 8
0.02 0.01 0 -0.01 -0.02 -0.03 -0.04

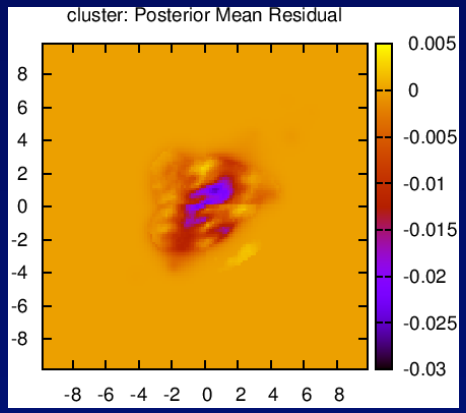

3c288: Posterior Mean Residual
8 6 4 2 0 -2 -4 -6 -8
-8 -6 -4 -2 0 2 4 6 8
0.02 0.015 0.01 0.005 0 -0.005 -0.01 -0.015 -0.02 -0.025 -0.03 -0.035

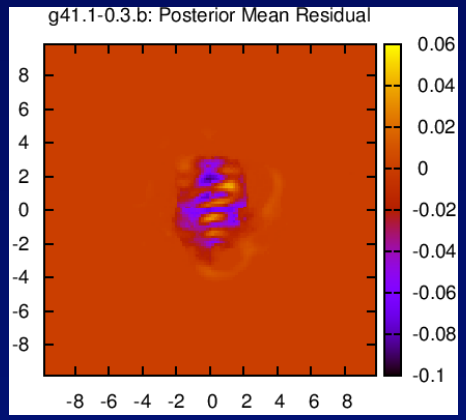

Einstein: Posterior Mean Residual
8 6 4 2 0 -2 -4 -6 -8
-8 -6 -4 -2 0 2 4 6 8
0.05 0 -0.05 -0.1 -0.15 -0.2


HCO+4-360: Posterior Mean Residual
8 6 4 2 0 -2 -4 -6 -8
-8 -6 -4 -2 0 2 4 6 8
0.025 0.02 0.015 0.01 0.005 0 -0.005 -0.01 -0.015 -0.02 -0.025

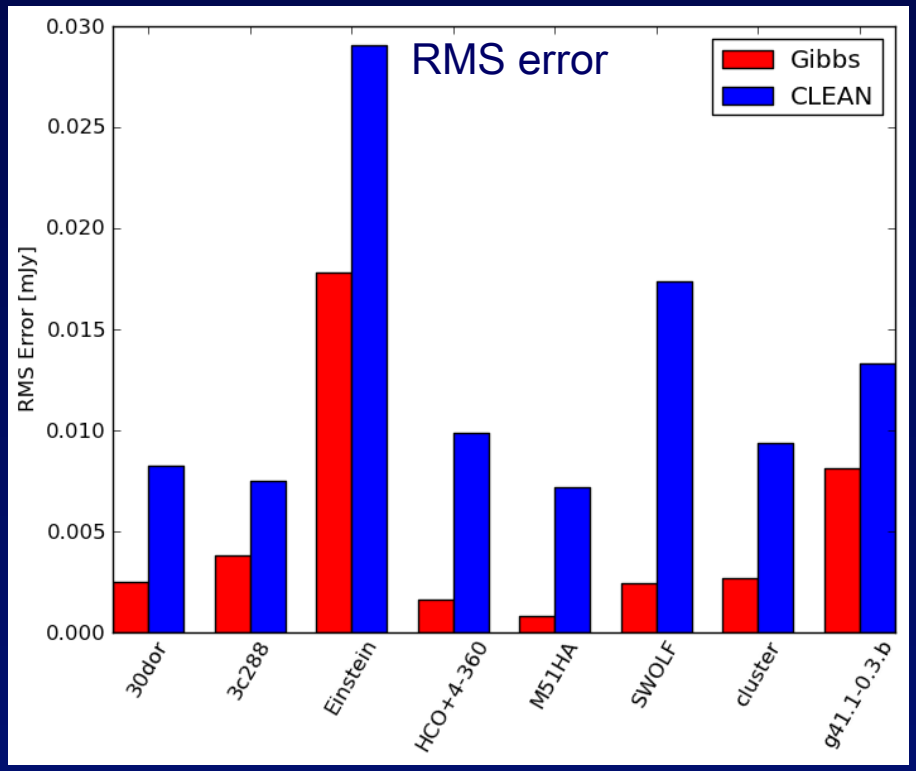

M51HA: Posterior Mean Residual
8 6 4 2 0 -2 -4 -6 -8
-8 -6 -4 -2 0 2 4 6 8
0.01 0.005 0 -0.005 -0.01 -0.015 -0.02

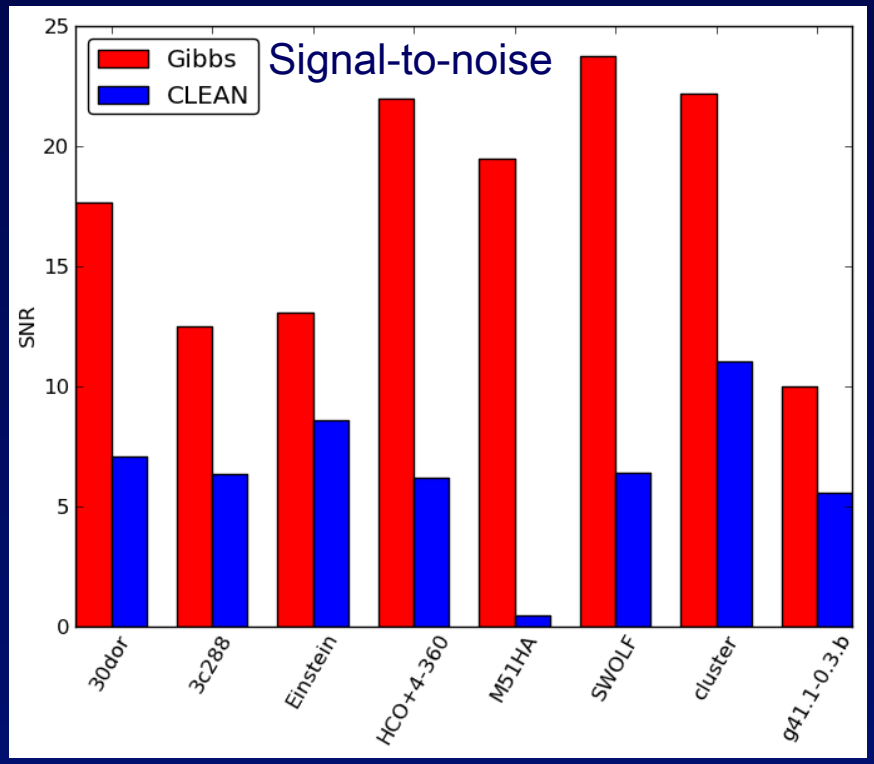

SWOLF: Posterior Mean Residual
8 6 4 2 0 -2 -4 -6 -8
-8 -6 -4 -2 0 2 4 6 8
0.03 0.02 0.01 0 -0.01 -0.02 -0.03 -0.04


cluster: Posterior Mean Residual
8 6 4 2 0 -2 -4 -6 -8
-8 -6 -4 -2 0 2 4 6 8
0.005 0 -0.005 -0.01 -0.015 -0.02 -0.025 -0.03


g41.1-0.3.b: Posterior Mean Residual
8 6 4 2 0 -2 -4 -6 -8
-8 -6 -4 -2 0 2 4 6 8
0.06 0.04 0.02 0 -0.02 -0.04 -0.06 -0.08 -0.1

Error comparison





$$\mathrm{SNR} \equiv 20 \log_{10} \left(\frac{\sigma_s}{\sigma_{\mathrm{resid}}} \right)$$

Advantages of Gibbs sampling

- Data-driven: minimal input assumptions.
- Automatic: No user input required.
- Accurate reconstruction in a variety of test cases.
- Outperforms CLEAN all test cases.
- Scalable (slowest step ~ $N_p \log N_p$).
- Gives quantitative error information, based on a well-motivated probabilistic model.

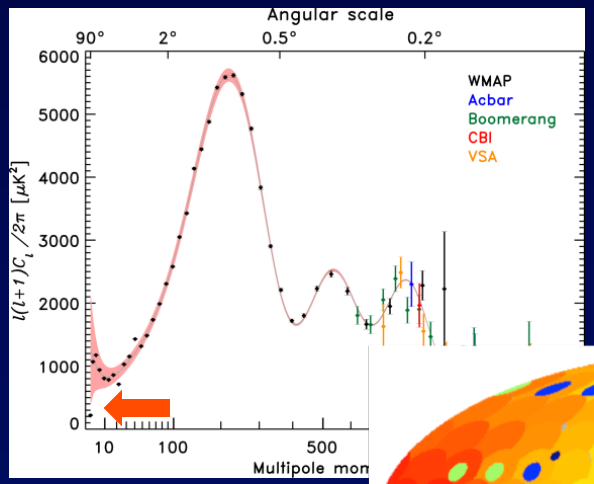
Summary

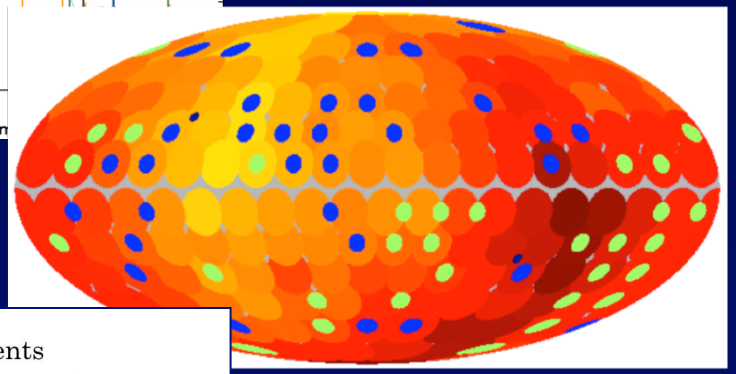
- CMB polarization is an exciting cosmological frontier.
- Interferometric approaches *may* have advantages in E/B separation, systematic error control.
- QUBIC, a bolometric interferometer (of sorts) is moving toward deployment in 2017.
- We have developed an analytic framework and a simulation pipeline for CMB interferometric polarimetry.
- Gibbs sampling provides a way of estimating both power spectrum and sky signal, which may be useful for interferometric image reconstruction in other areas of astrophysics.

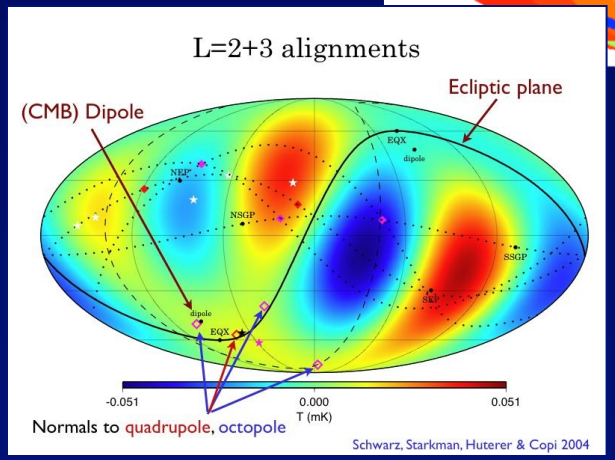
Extra stuff

WMAP anomalies

- Large-scale power deficit
- North-south asymmetry
- Alignment of multipoles ("axis of evil")







Is there anything to explain?

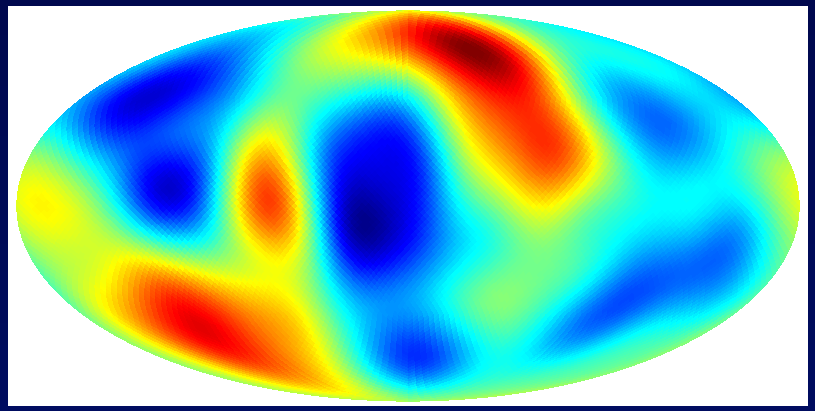
- How not to do statistics:
  1. Notice something strange in your data.
  2. Devise a statistic *a posteriori* to quantify how strange it is.
  3. Take seriously the p-values derived from that statistic.
- *All* claimed anomalies suffer from this problem to some extent
  ➔ Argument for ignoring the whole subject.

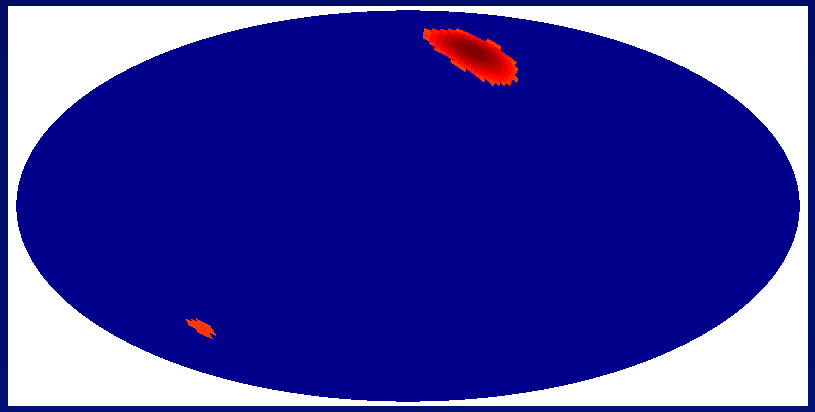
But…

- We use "invalid" statistical methods all the time, relying on intuition and further tests to keep us honest.
- If these anomalies are real, they're important!
- My unjustified opinion: We should proceed, but with skepticism.

Example

- A simulated CMB map, made in the "usual" way (Gaussian, statistically isotropic) and smoothed to show only large-scale features:
- The two most extreme hot spots are almost perfectly antipodal and cause positive skewness.
- Easy to devise statistics to show this is unlikely at > 99% confidence.





Should we care about the large-angle anomalies?

My (unjustified) opinions:

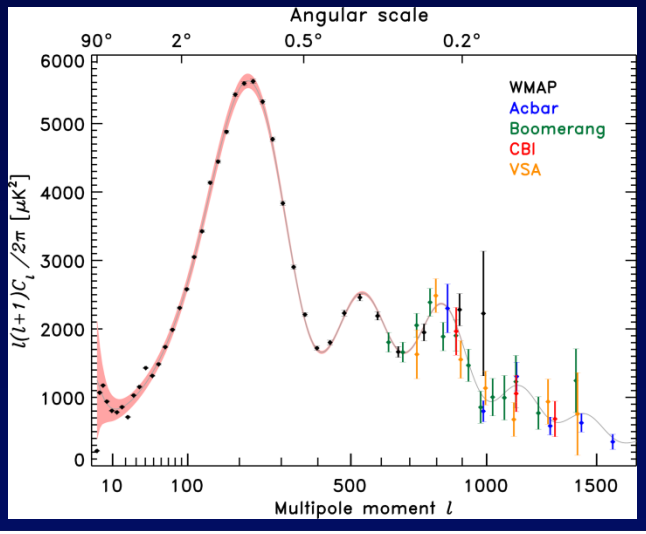
- The problem of *a posteriori* statistics is a reason to tread carefully, but not a reason to dismiss the whole subject outright.
  - » We use "invalid" statistical methods all the time in science, and rely on our intuition to decide whether it's OK.
- The possibility that the CMB is telling us something nonstandard about the largest-scale features in the Universe is very exciting – worth looking into.

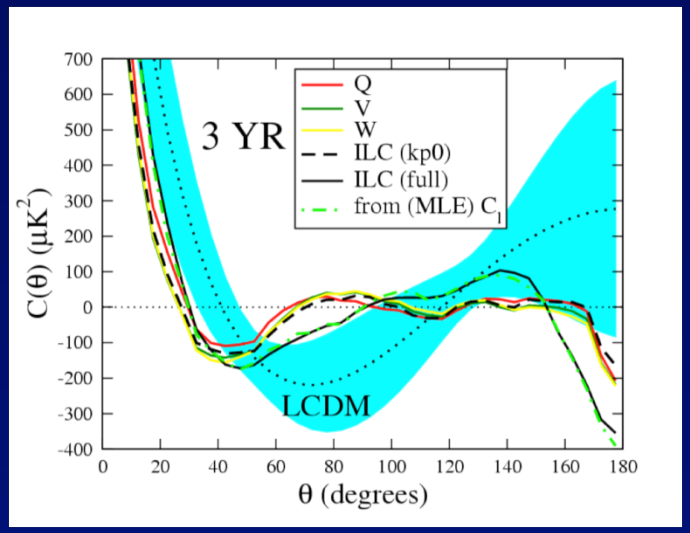
Lack of large-scale power

- Low quadrupole C_2
  - » Large cosmic variance + need to mask ➔ statistical significance is not strong.
- Discrepancy looks much more striking when phrased in terms of the real-space correlation function, instead of the spherical harmonic coefficients.

$$S_{1/2} = \int_{-1}^{1/2} [C(\theta)]^2 d\cos\theta$$

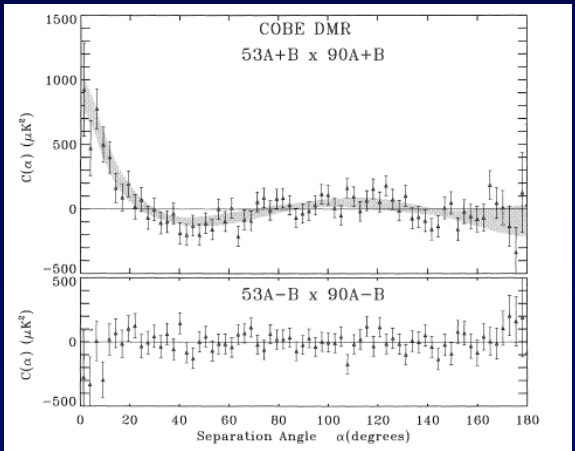
is low compared to standard model at 99.8% confidence (Copi et al. 2006).





How *a posteriori* is this?

- Specific statistic $S_{1/2}$: Very.
- Looking at the correlation function instead of the power spectrum: Not at all.
  - » $C(\theta)$ was *the* standard tool in the old days, especially on small scales.
- My opinion: The fact that $C(\theta)$ is essentially zero for large angles is intriguing.



Smoot et al. 1992

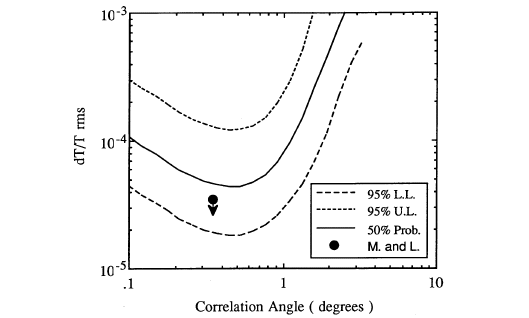


FIG. 9.—Derived limits on Gaussian-correlated CMB anisotropies. 95% upper and lower limits and the most probable value, 50%, are plotted vs. the correlation angle of the anisotropies. The filled circle represents the upper limit of Meinhold & Lubin (1991).

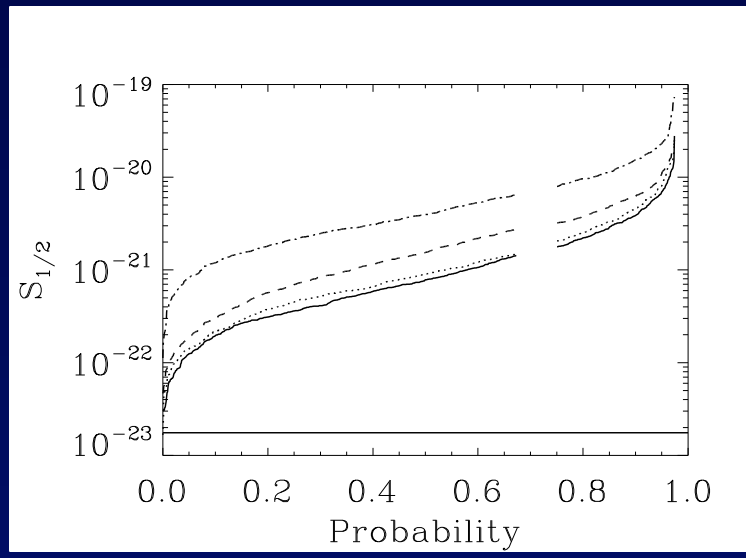
Alsop et al. 1991

What could it mean?

- Possibilities:
  1. Statistical fluke
  2. ~~Foreground~~
  3. ~~Systematic error~~
  4. ~~New physics (topology, …)~~
- Theorem: No independent additive contaminant to the standard model can explain this anomaly:

P(with contaminant) < P(without contaminant)

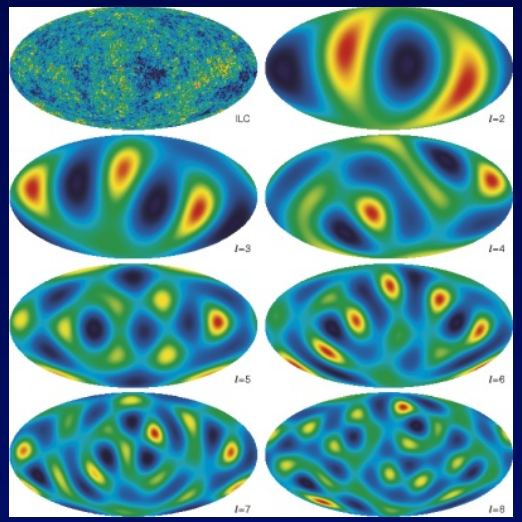
(Copi et al. 06, Bunn & Bourdon 08)



If a contaminant is invoked to explain any other problem, it worsens this one!

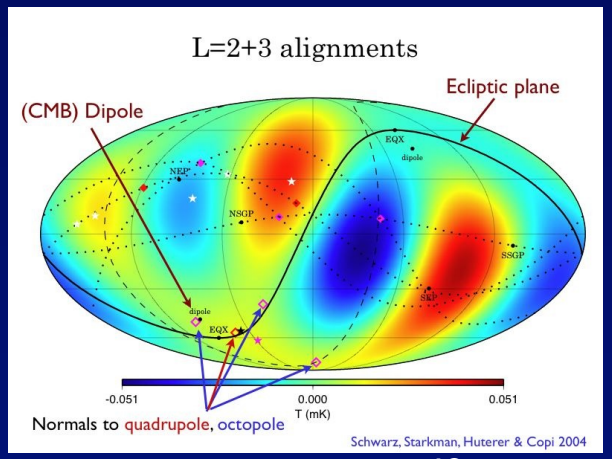
Alignment of multipoles

- Maps with l=2,3 pick out approximately the same plane on the sky.
- l=3 map is more planar than expected.
- Normal to plane lies near ecliptic, CMB dipole.



Hinshaw et al. 2007

- Depending on which of these "surprises" you include, formal statistical significance can be $p \sim 0.001$.
- My unjustified opinion: This is the sort of pattern that humans are good at spotting, whether or not it's there.



Possible explanations?

- Each anomaly could be due to a fluke, systematic error, foreground, or new physics.
- Sample possibilities for new physics:
  - » Spontaneous isotropy breaking (Gordon et al.): Radiation field couples to some other field, with only large-scale fluctuations.

$$\Delta T_{\rm obs}(\hat{\mathbf{r}}) = f(\hat{\mathbf{r}})\Delta T_{\rm iso}(\hat{\mathbf{r}})$$

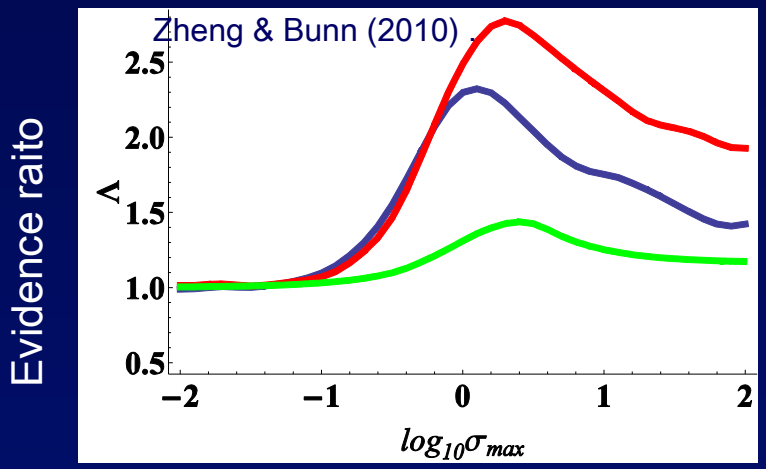
  - » Preferred direction picked out during inflation (Ackerman et al.).

$$P(\mathbf{k}) = P_{\rm iso}(k)(1 + g(\hat{\mathbf{k}} \cdot \hat{\mathbf{n}})^2)$$

Do the data justify a more complicated model?

- Bayesian evidence: One way of deciding whether to prefer a more complicated theory:
  - » *Evidence ratio* = Factor by which posterior probability ratio changes as a result of the data.
  - » Complicated theories automatically disfavored.
- In all cases examined so far, BE provides only weak to moderate support for the nonstandard model.

Hoftuft et al. 2009

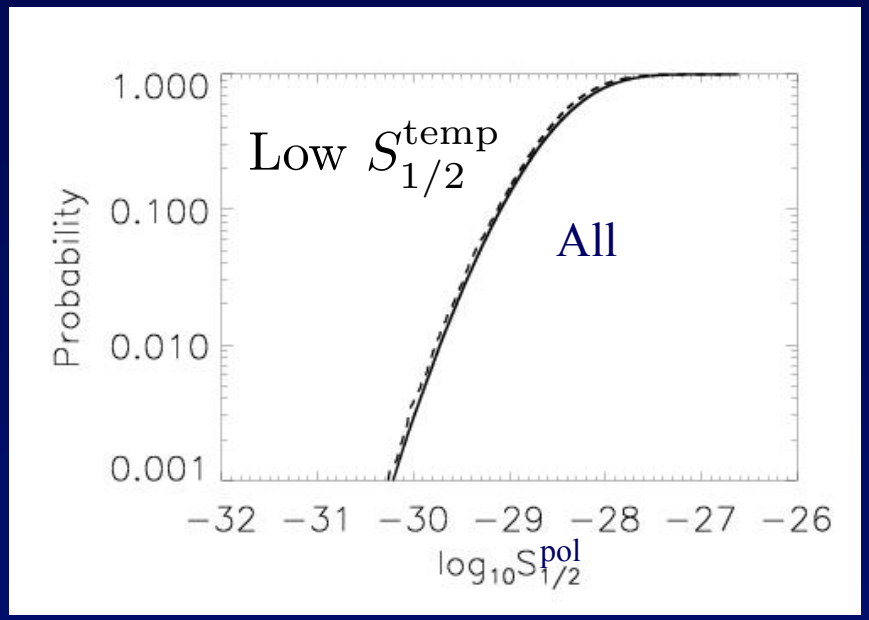


So what should we do?

- Hard to know how seriously to take any of this!
- Solution to the problem of *a posteriori* statistics: Get a new data set, for which the tests are *a priori*.
- Need a data set that probes other perturbation modes on ultralarge scales:
  - » CMB polarization
  - » Large-scale structure / 21 cm tomography?
  - » Kamionkowski-Loeb remote quadrupole measurements?

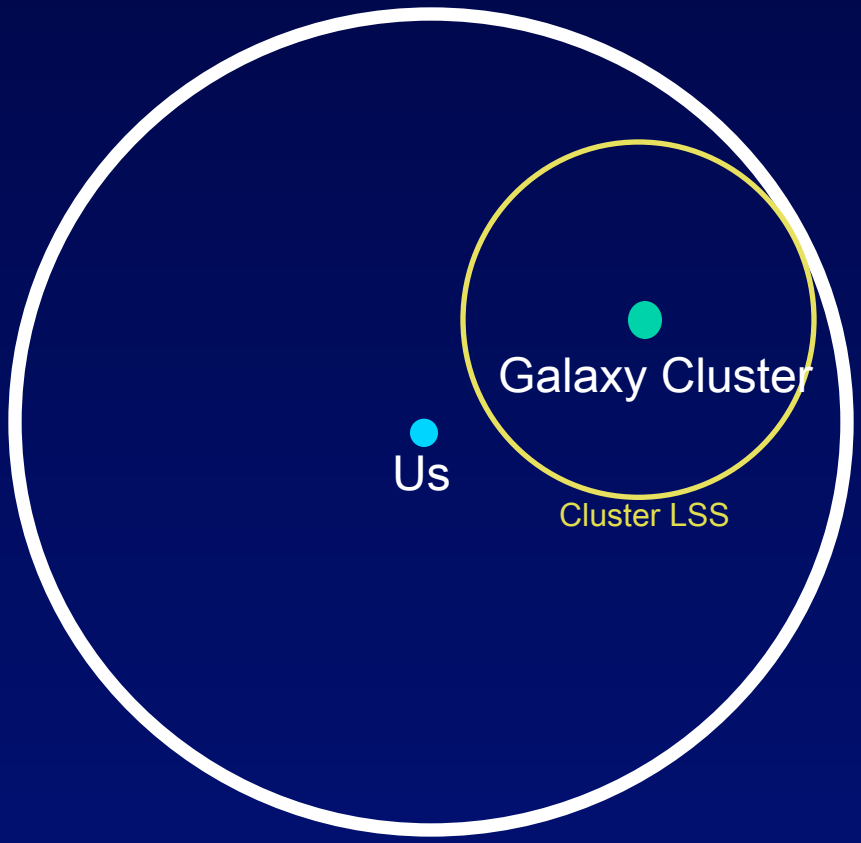
Large-scale power deficit in CMB polarization

- Define a statistic precisely analogous to $S_{1/2}$ for polarization maps.
- $S_{1/2}^{\rm pol}$ is (nearly) uncorrelated with $S_{1/2}^{\rm temp}$. ➔ it can be regarded as an *a priori* statistic.
- If $S_{1/2}^{\rm pol}$ is anomalously small, then something interesting is happening.



Remote quadrupole measurements

- CMB photons scatter in a galaxy cluster.
- Induced polarization tells us the CMB quadrupole at the cluster's location & look-back time.
- Strength of signal < 1 μK. Hard but not impossible.
- Lets us measure new modes of the radiation field on the scales corresponding to $l \sim 5\text{-}10$.



(Kamionkowski & Loeb 1997; Bunn 2006)

Conclusions

- Large-angle CMB anomalies don't "prove" that anything nonstandard is going on, but may provide hints of places to look for interesting phenomena on large scales.
- Important to make predictions and choose statistics in advance for new data sets, to avoid a posteriori statistics.