References:

- **Mohammed & Seljak 2014 (1407.0060)**
- **Mohammed et al. 2014 (1410.6826)**

Towards precision cosmology

The halo model and necessary modifications

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Challenges on its way…

- Accurate description of clustering of matter: Non-linear scales.
- Precise description of clustering of matter: baryonic effects at small scales.
- Full covariance matrix of matter power spectrum P(k,z).
- Many more…

Outline

- Introduction - matter power spectrum and evolution
- The halo model and necessary modifications
  - Non-linear clustering of dark-matter
  - Covariance matrix
  - Baryonic effects - I (marginalising)
  - Baryonic effects - II (distributing baryons)
- Cosmological implications with baryons for Euclid.

Introduction: 2-point statistics

- Cosmology is a statistical science as most of its observables are statistical in nature.
- The simplest statistic is the 2-point correlation function or the power spectrum of the matter density field.

$$\langle \tilde{\delta}(\mathbf{k})\tilde{\delta}(\mathbf{k}')\rangle = (2\pi)^3 \delta_D^3(\mathbf{k}+\mathbf{k}')P(k)$$

- It underlies observables like galaxy clustering, weak lensing, Baryon Acoustic Oscillations (BAO), Lyman-alpha forest etc.
- Various estimators: perturbation theories, fitting formulae from simulations, **<u>the halo model</u>** etc.

The Halo Model

$$P(k) = P_{1h}(k) + P_{2h}(k),$$

$$P_{1h}(k) = \int d\nu f(\nu) \frac{M}{\bar{\rho}} |u(k|M)|^2,$$

$$P_{2h}(k) = \left[\int d\nu f(\nu) b(\nu) u(k|M) \right]^2 P_{\rm L}(k),$$

$$u(k|M) = \frac{4\pi}{M} \int_0^{R_{\rm vir}} {\rm d}r \; r^2 \; \rho(r|M) \; \frac{\sin(kr)}{kr}$$

$$\int_0^\infty f(\nu) d\nu = 1 \qquad \int_0^\infty f(\nu) b(\nu) d\nu = 1$$

- Ingredients:
  - Mass function: f,
  - Halo bias: b,
  - Average density profile of halos:
  - Linear Power Spectrum: $P_L(k)$

Ref: McClelland & Silk 1977; Seljak 2000; Ma & Fry 2000; Peacock & Smith 2000; Cooray & Sheth 2002

The modified Halo model - I

$$P(k,z) = P_{\rm zel}(k,z) + P_{\rm 1h}(k,z)$$

$$P_{\rm 1h}(k,z) = (A_0 - A_2 k^2 + A_4 k^4) F(k)$$

$$A_0 = 1529.87\sigma_8^{3.9} \times (1 + [-0.22 n_{\rm eff} - 0.4]),$$

$$A_2 = 1299.75\sigma_8^{3.0} \times (1 + [-1.58 n_{\rm eff} - 2.8]),$$

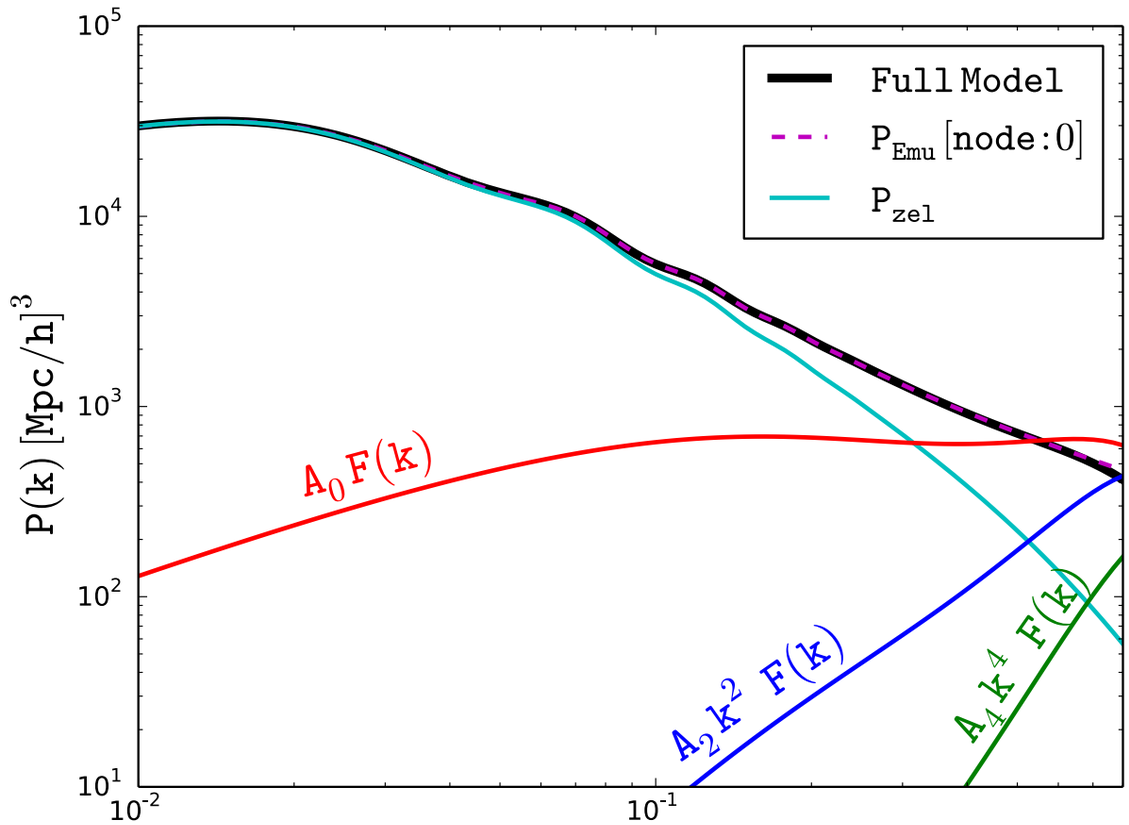
$$A_4 = 758.31\sigma_8^{2.2} \times (1 + [-2.27 n_{\rm eff} - 4.2]),$$

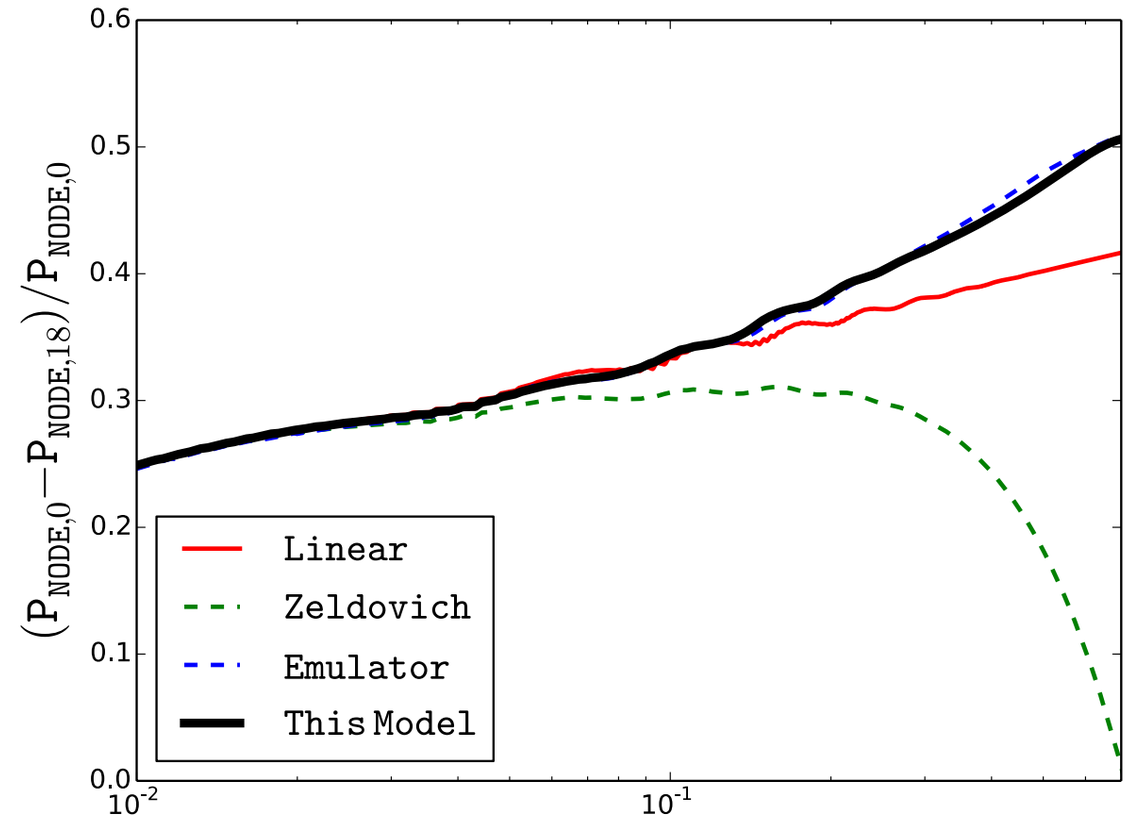
$$F(k) = \sum_{n=0}^{10} a_n k^n$$

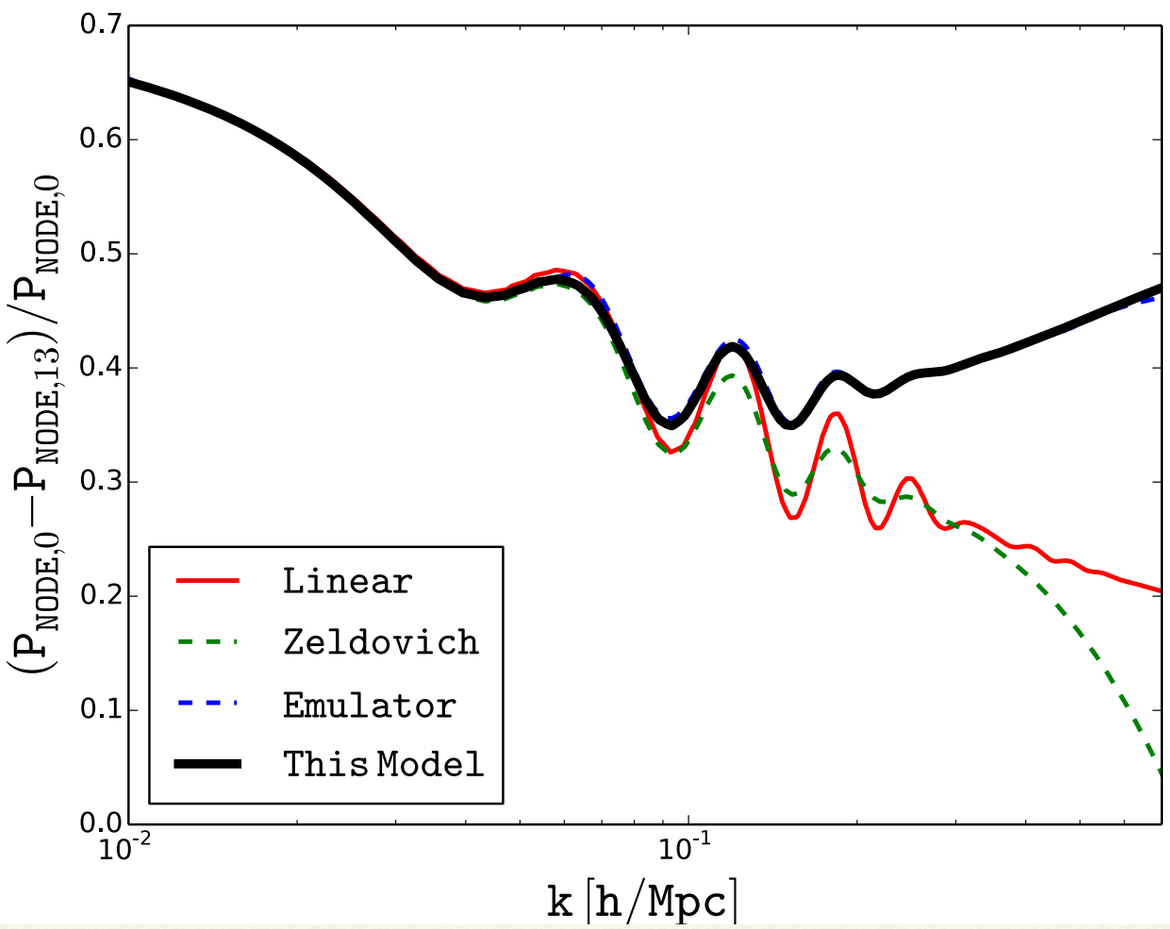
$$\sigma^2(M) = \frac{1}{2\pi^2} \int {\rm d}k\, k^2\, P_{\rm L}(k)\, |\bar{W}(kR)|^2$$

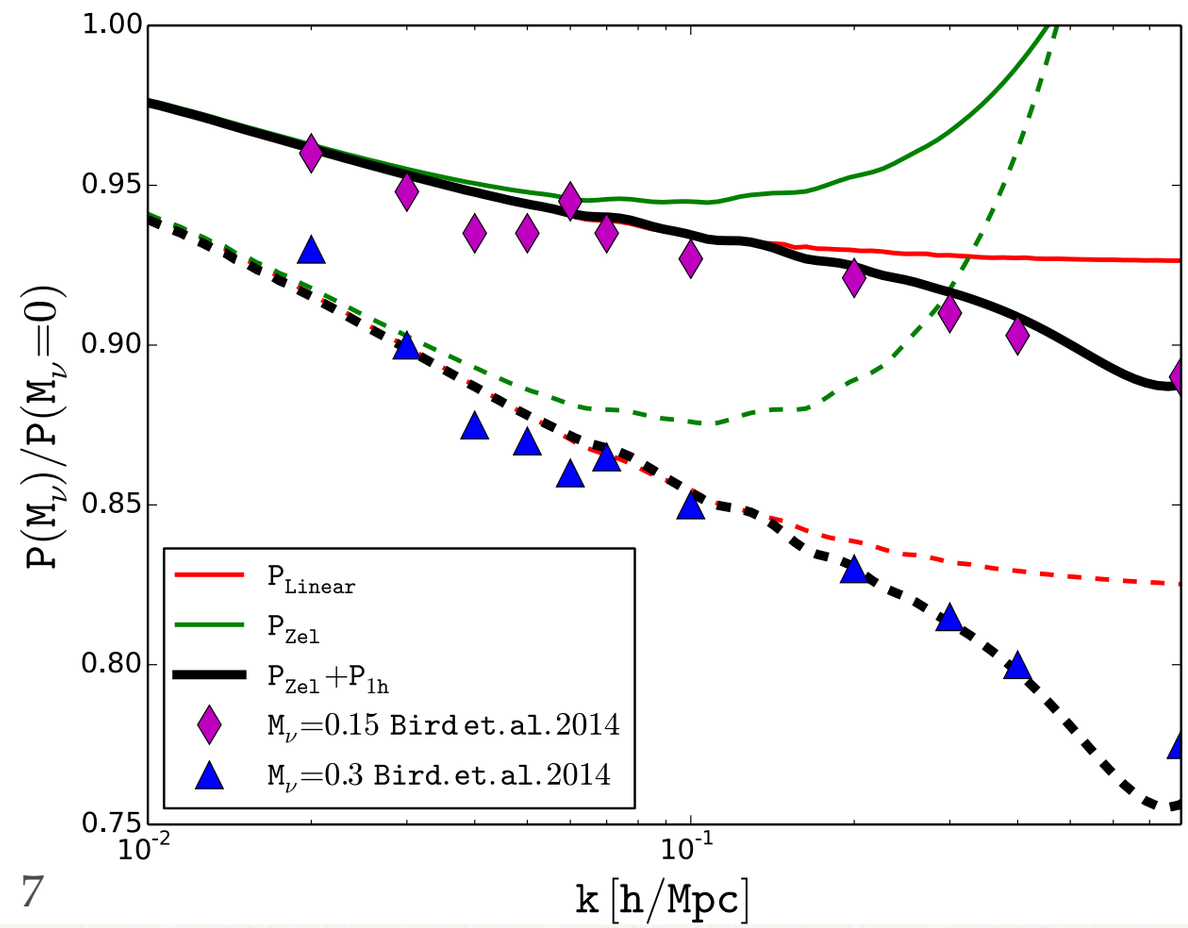
$n_{\rm eff}$ is the slope of $P_L(k)$ at BAO scale (~0.2 h/Mpc)

- Ingredient:
  - Linear Power Spectrum: $P_L(k)$
- Estimating coefficients:
  - Fit to simulations (Emulator),
  - the halo model.


Full Model
P_{Emu} [node:0]
P_{zel}
$A_0 F(k)$
$A_2 k^2 F(k)$
$A_4 k^4 F(k)$
$P(k)$ [Mpc/h]^3


Linear
Zeldovich
Emulator
This Model
$(P_{NODE,0} - P_{NODE,18})/P_{NODE,0}$


Linear
Zeldovich
Emulator
This Model
$(P_{NODE,0} - P_{NODE,13})/P_{NODE,0}$
k [h/Mpc]


P_{Linear}
P_{Zel}
$P_{Zel} + P_{1h}$
M_ν=0.15 Bird et. al. 2014
M_ν=0.3 Bird. et. al. 2014
$P(M_\nu)/P(M_\nu=0)$
k [h/Mpc]

Covariance Matrix - a limit to the measurement

$$\mathrm{Cov}(P(k_i), P(k_j)) = \langle P(k_i)P(k_j) - \langle P(k_i)\rangle\langle P(k_j)\rangle,$$

$$\mathrm{Cov}(P(k_i), P(k_j)) = P(k_i)P(k_j)\left(\frac{2}{N_i}\delta_{ij} + \left(\frac{\sigma_{A_0}}{A_0}\right)^2\right)$$

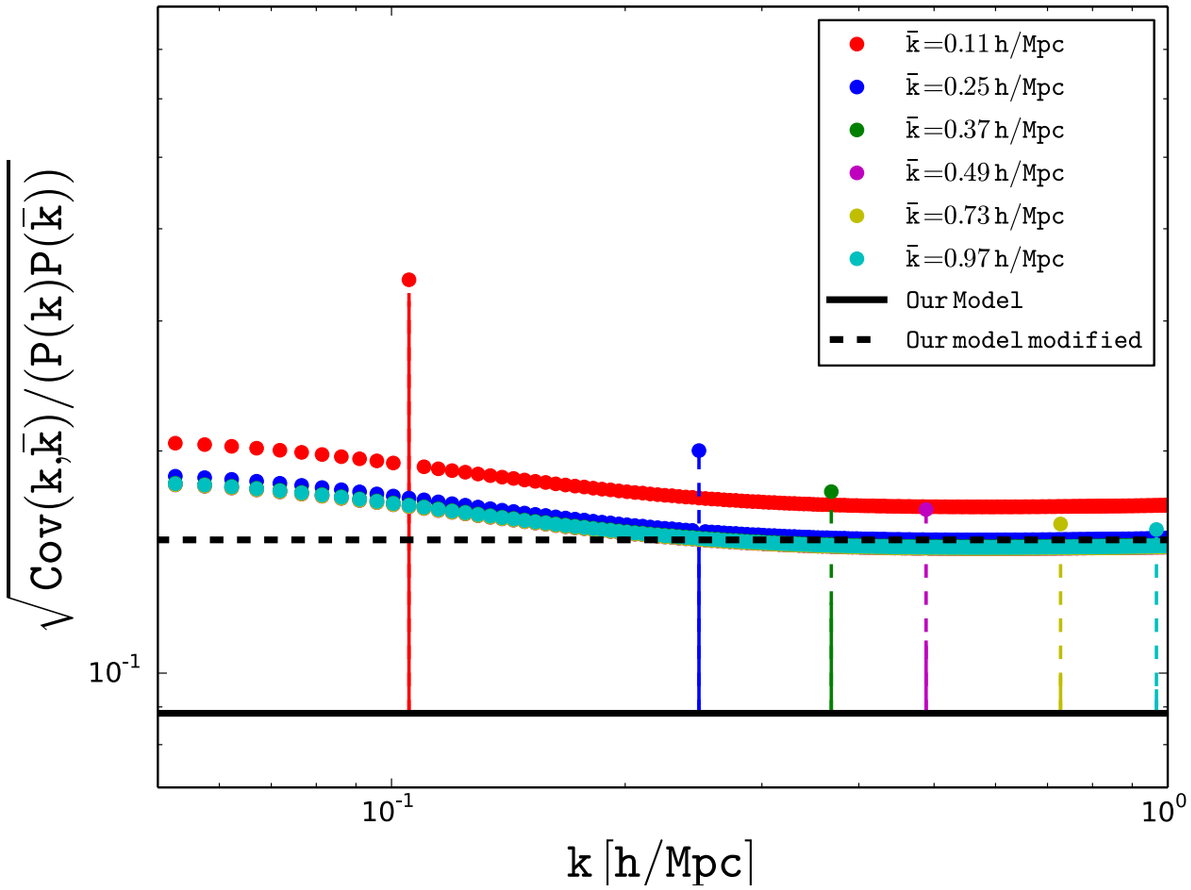
$$\left(\frac{\sigma_{A_0}}{A_0}\right)^2 = \frac{\int f(\nu)d\nu M^3}{[\int f(\nu)d\nu M]^2\bar{\rho}V},$$

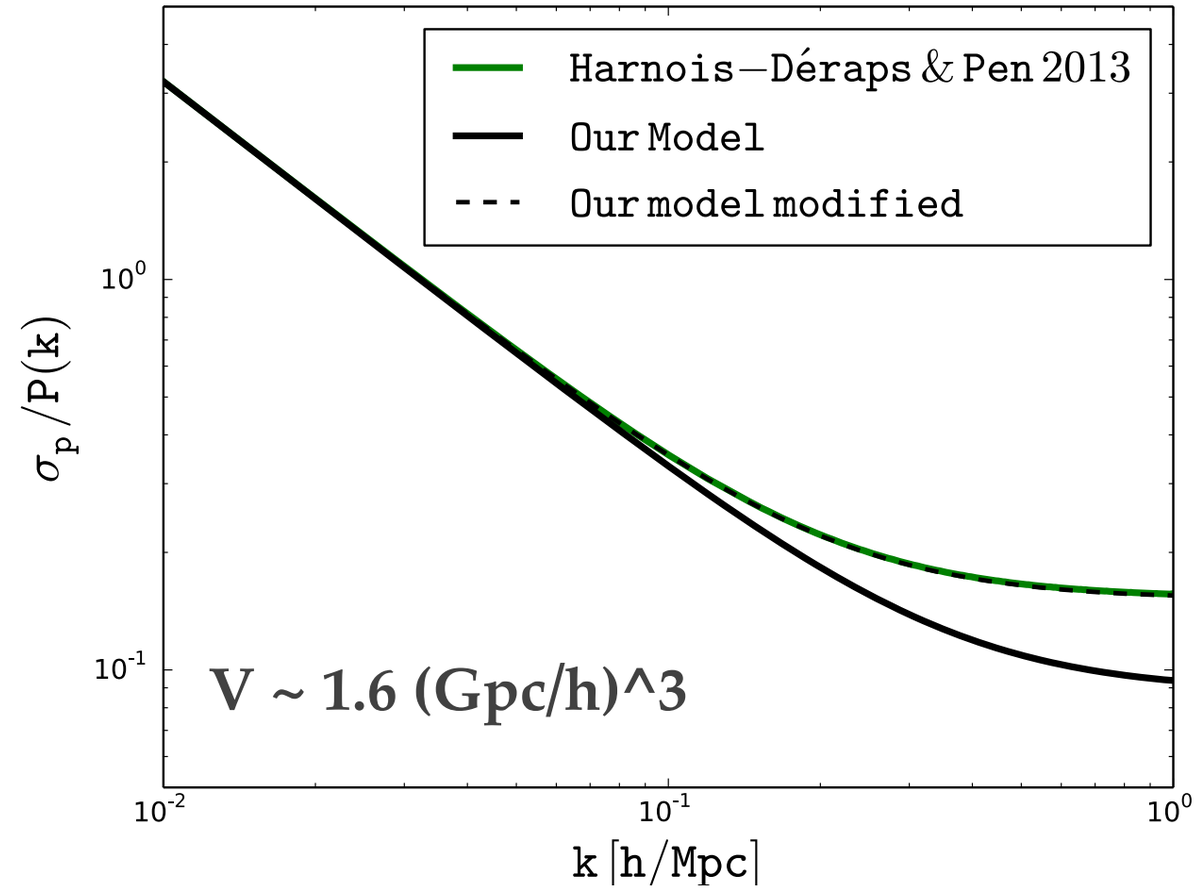
~0.01^2

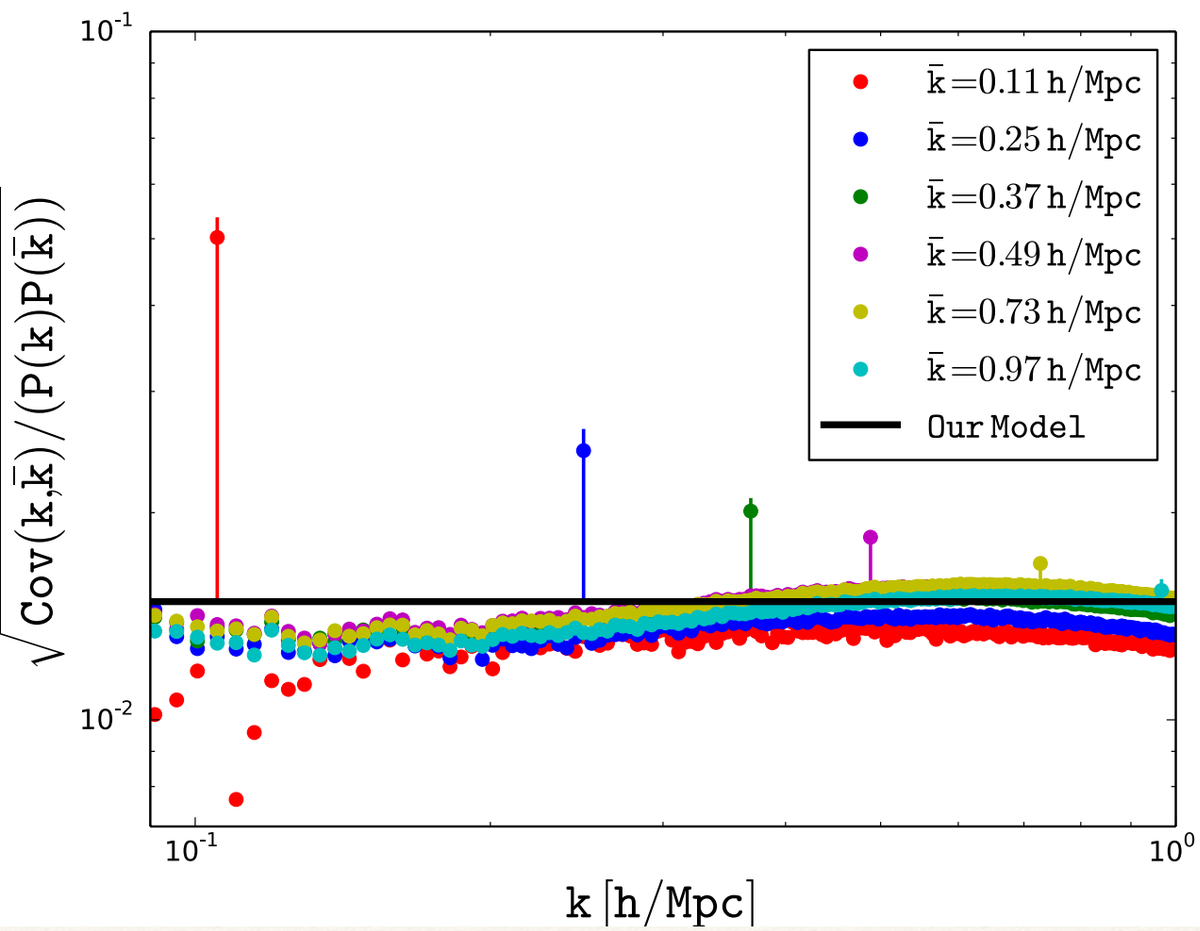
$$\frac{\sigma_{A_0}}{A_0} = \frac{\delta_{A_0}}{[(V/1h^{-1}\mathrm{Gpc})^3]^{1/2}}, \quad \delta_{A_0} = 0.0079(h^{-1}\mathrm{Gpc})^{3/2},$$

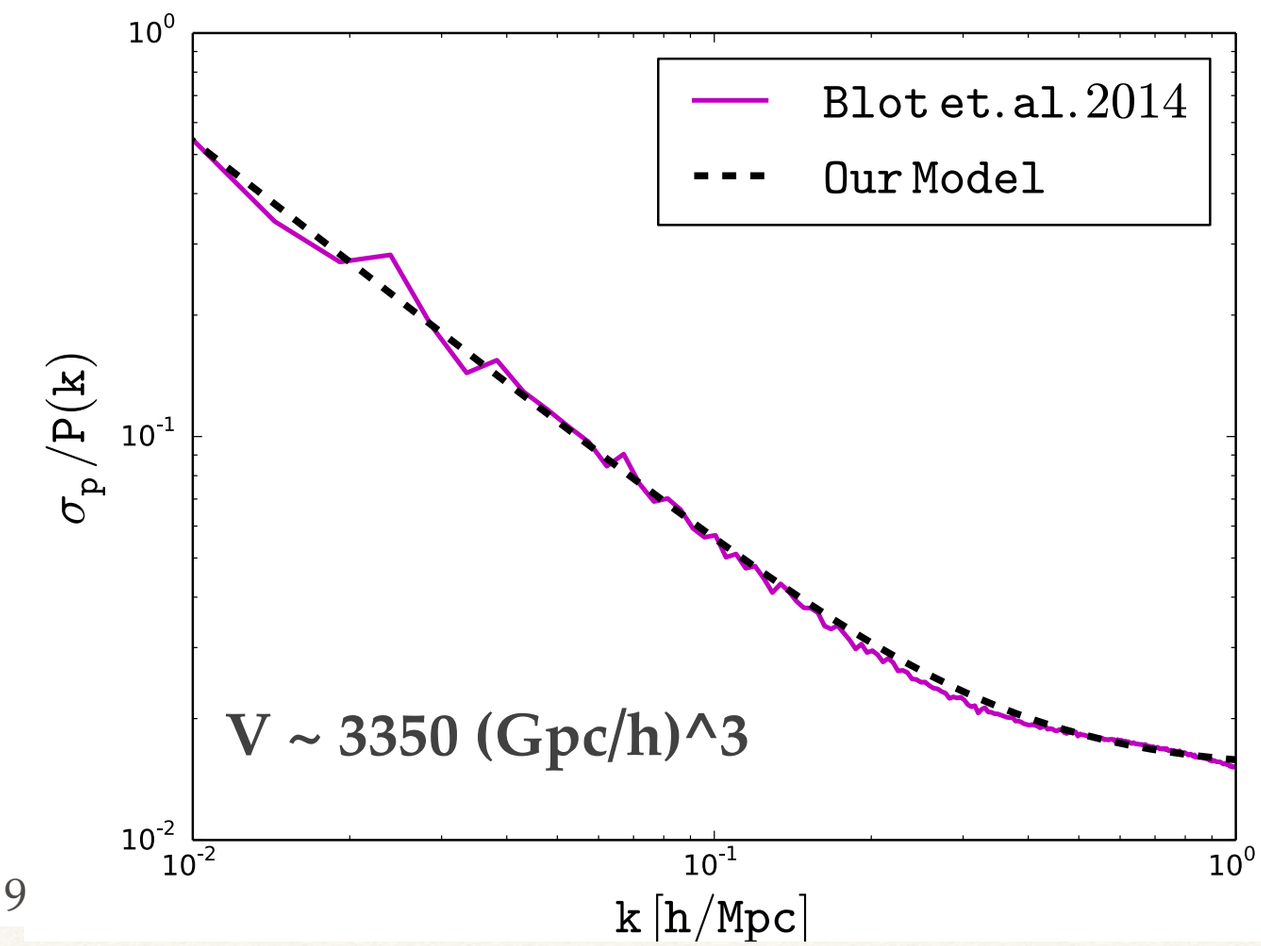
(Fitting with Li et al. 2014)

$$\mathrm{Cov}(P(k_i), P(k_j)) = P(k_i)P(k_j)V^{-1}\left(\frac{4\pi^2}{k_i^2\Delta k}\delta_{ij} + \delta_{A_0}^2\right)$$


$\bar{k}$=0.11 h/Mpc
$\bar{k}$=0.25 h/Mpc
$\bar{k}$=0.37 h/Mpc
$\bar{k}$=0.49 h/Mpc
$\bar{k}$=0.73 h/Mpc
$\bar{k}$=0.97 h/Mpc
Our Model
Our model modified
$\sqrt{Cov(k,\bar{k})/(P(k)P(\bar{k}))}$
10^{-1}
10^{-1}
10^{0}
k [h/Mpc]

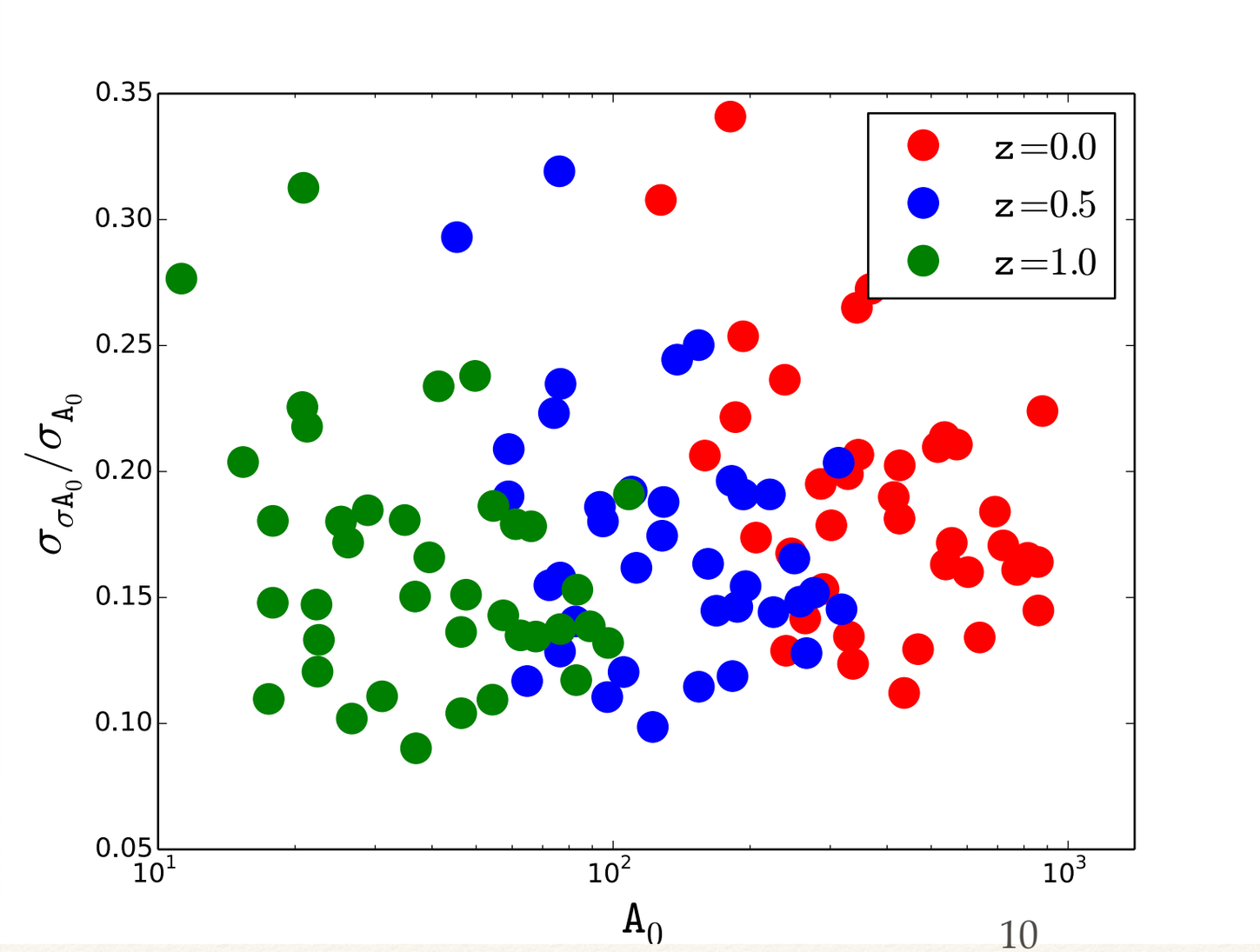

Harnois−Déraps & Pen 2013
Our Model
Our model modified
$\sigma_p/P(k)$
10^{0}
10^{-1}
V ~ 1.6 (Gpc/h)^3
10^{-2}
10^{-1}
10^{0}
k [h/Mpc]


$\bar{k}$=0.11 h/Mpc
$\bar{k}$=0.25 h/Mpc
$\bar{k}$=0.37 h/Mpc
$\bar{k}$=0.49 h/Mpc
$\bar{k}$=0.73 h/Mpc
$\bar{k}$=0.97 h/Mpc
Our Model
$\sqrt{Cov(k,\bar{k})/(P(k)P(\bar{k}))}$
10^{-1}
10^{-2}
10^{-1}
10^{0}
k [h/Mpc]


Blot et. al. 2014
Our Model
$\sigma_p/P(k)$
10^{0}
10^{-1}
V ~ 3350 (Gpc/h)^3
10^{-2}
10^{-2}
10^{-1}
10^{0}
k [h/Mpc]

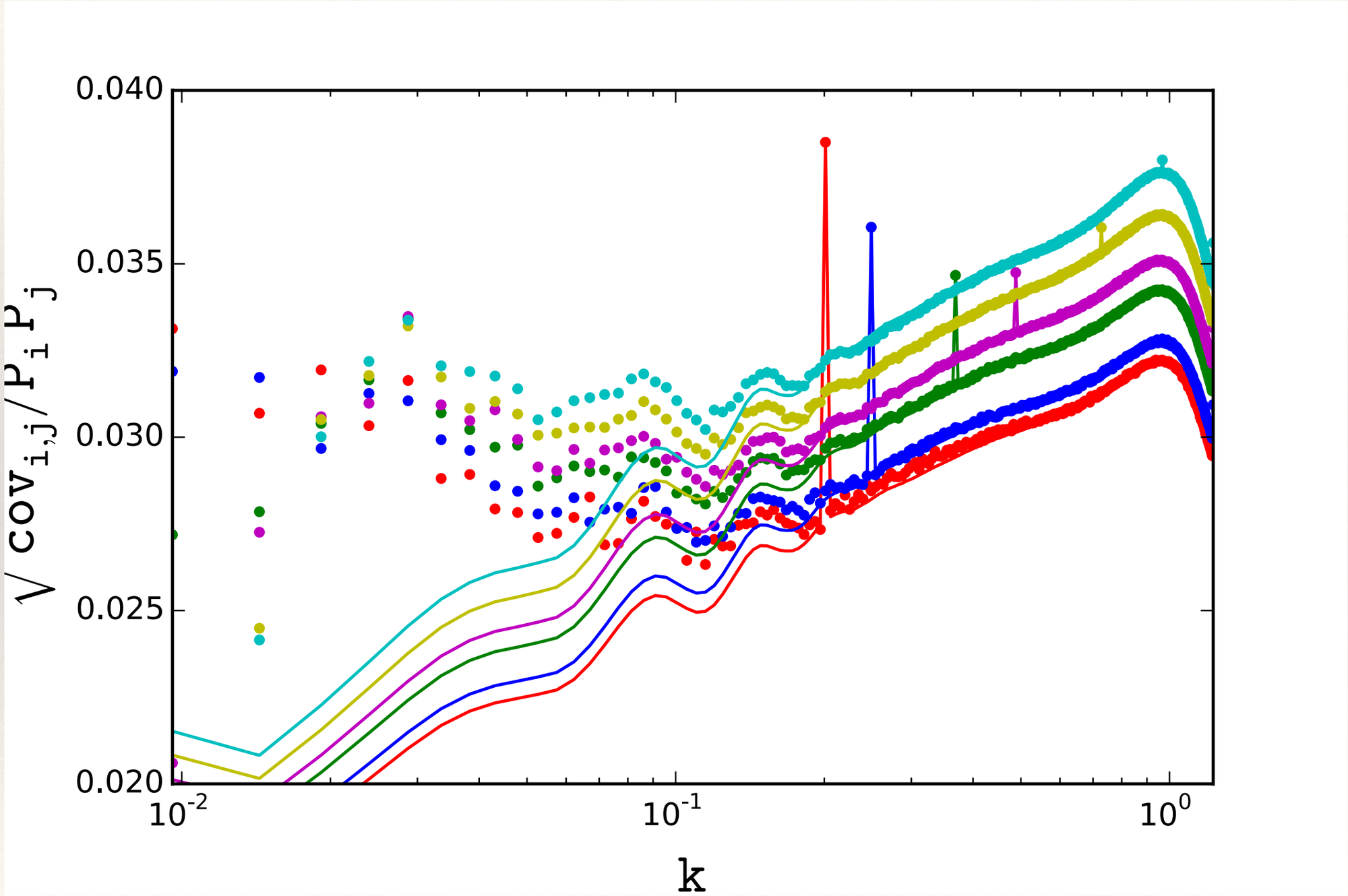
Variance of Covariance Matrix

$$\left(\frac{\sigma(\sigma_{A_0})}{\sigma_{A_0}}\right)^2 = \frac{\int f(\nu)d\nu M^7}{[(\int f(\nu)d\nu M^3]^2\bar{\rho}V},$$



Need a volume of ~ 500-5000 (Gpc/h)³ to converge to 1%

In preparation

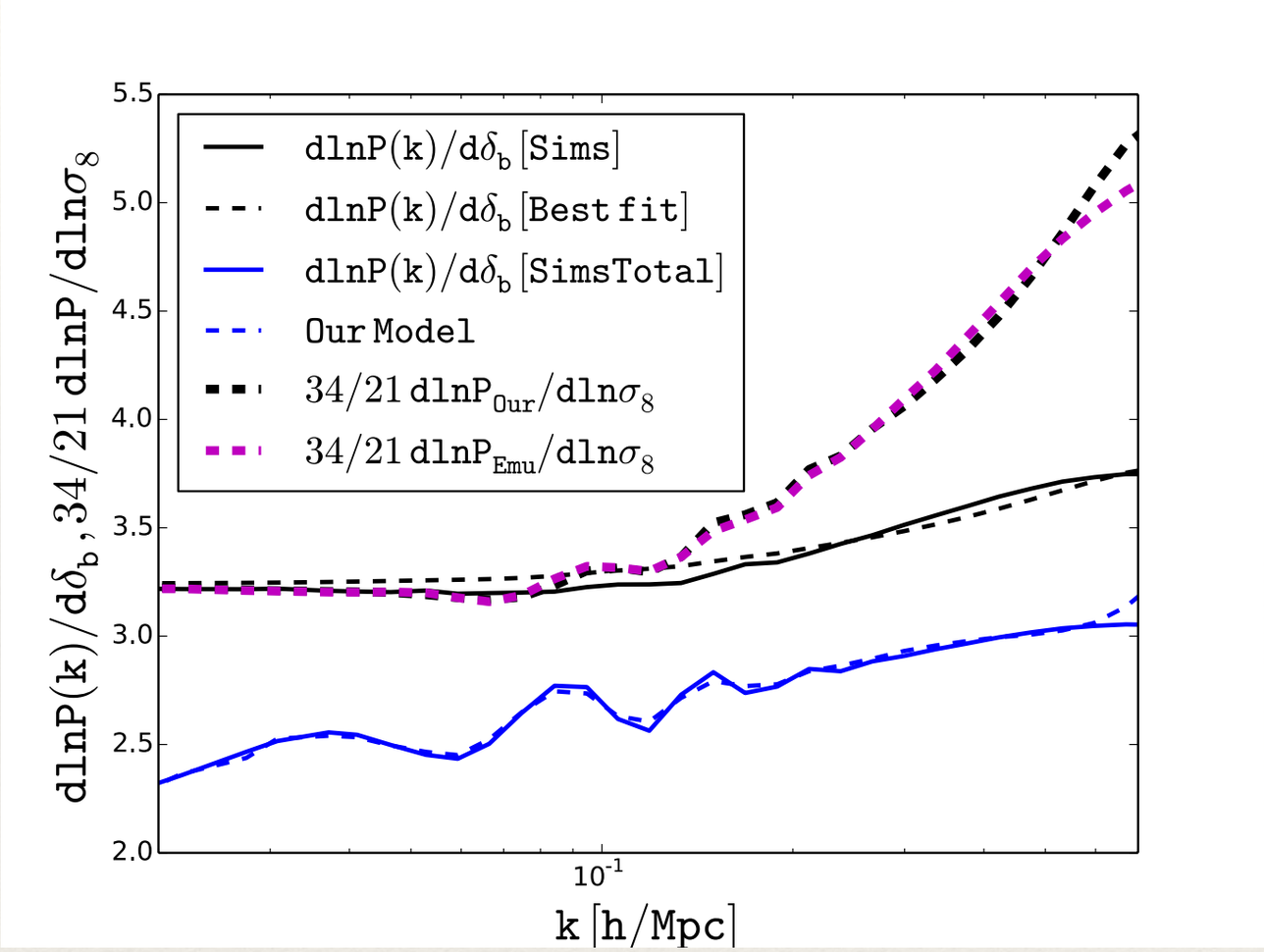


Super-Sample Variance

$$C_{ij} = C_{ij}^{\mathrm{G}} + C_{ij}^{\mathrm{T0}} + \sigma_b^2 \frac{\partial P(k_i)}{\partial \delta_b} \frac{\partial P(k_j)}{\partial \delta_b}$$

$$\frac{\partial P(k)}{\partial \delta_b} \approx \left(\frac{68}{21} - \frac{1}{3} \frac{d \ln k^3 P_{2h}(k)}{d \ln k} \right) P_{2h}(k) + I_2^1(k,k)$$

Li et al. 2014



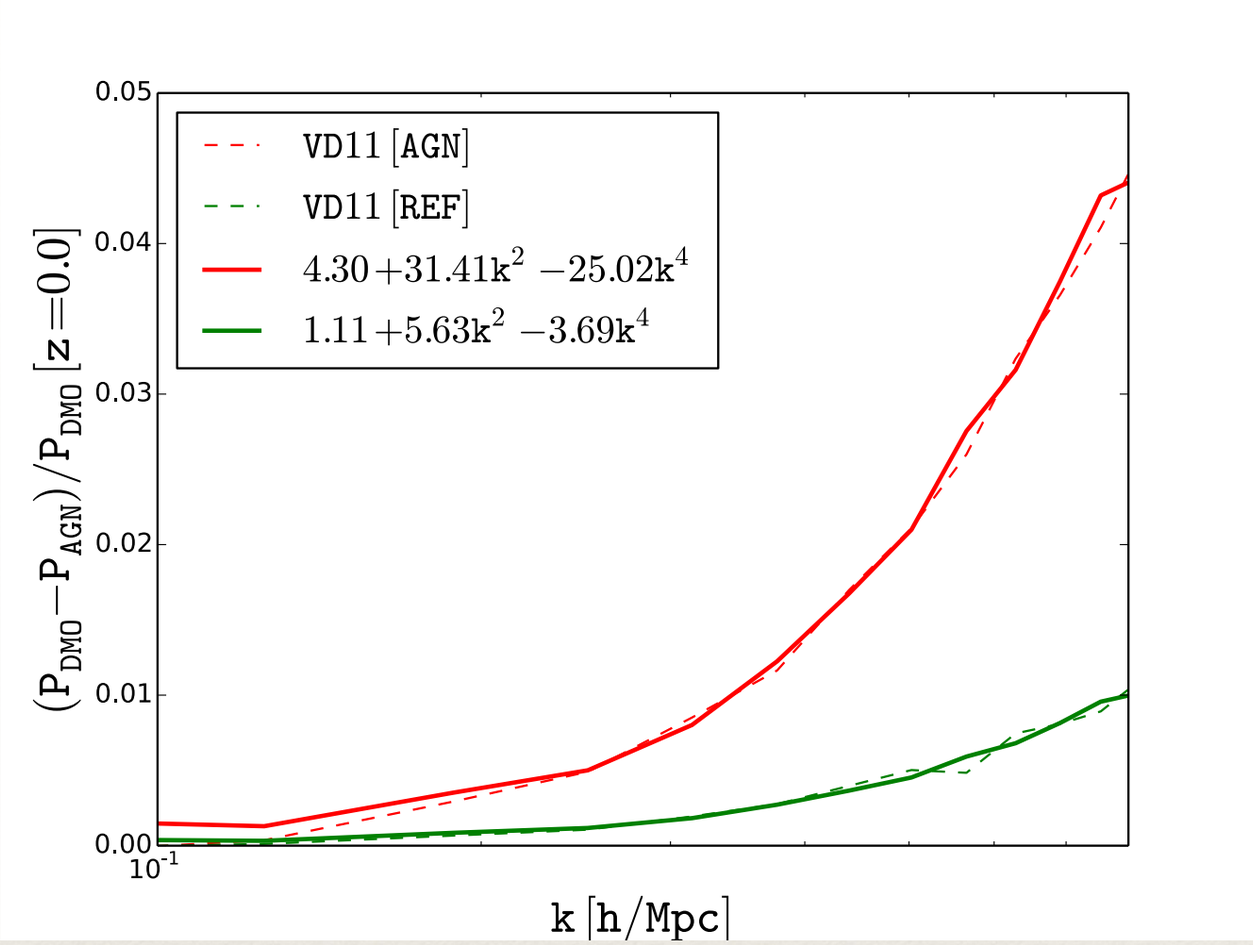
Baryonic effects

- Approach 1: Observe the baryonic effects in A_{2n} coefficients.
- Approach 2: Quantify the baryonic effects by distributing baryonic components.

Approach 1: Observe the baryonic effects on A_{2n} coefficients.

- Because baryons redistributing matter inside haloes all the coefficients A_{2n} for $n > 0$ are strongly affected by baryonic effects, while A_0 remains almost unchanged, a consequence of halo mass conservation.

- Observations such as weak lensing power spectrum can be effectively marginalised over the baryonic effects, while still preserving the bulk of the cosmological information contained in A_0 and Zeldovich terms.



$$\delta A_0 + \delta A_2 k^2 + \delta A_4 k^4$$

Approach 2: Quantifying by distributing baryonic components (The modified Halo model - II)

- Stellar component dominated by central galaxy (Abundance matching).
- Hot intra cluster plasma in hydrostatic equilibrium (X-ray observations).
- Adiabatically contracted dark-matter.
- AGN feedback that regulates the amount of gas inside the halo as a function of halo mass (X-ray observations).
- Only one free parameter M_{crit}.

Approach 2: Quantifying by distributing baryonic components (The modified Halo model - II)

$$\rho_{\rm BAR}(r|M) = f_{\rm dm}\rho^{\rm AC}_{\rm NFW}(r) + \rho_{\rm BCG}(r) + f_{\rm gas}(M)\rho_{\rm gas}(r),$$

$$f_{\rm gas}(M_{\rm halo}) = \frac{\Omega_b/\Omega_m}{1+\left(\frac{M_{\rm crit}}{M_{\rm halo}}\right)^{\beta}}$$

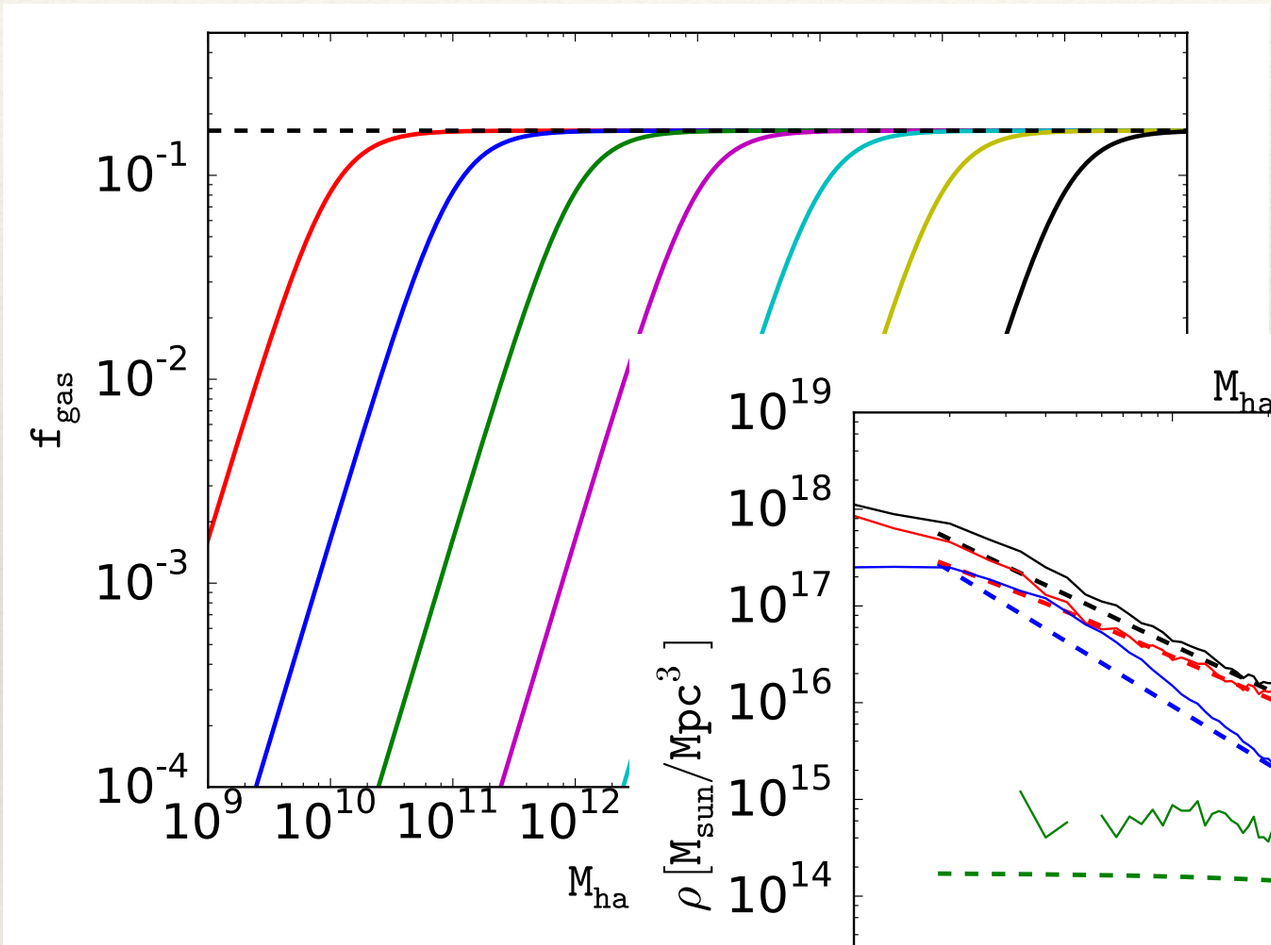
$$f_0 + \int_{\nu_{\rm min}}^{\infty} (f_{dm} + f_{gas}(\nu))f(\nu)d\nu = 1$$

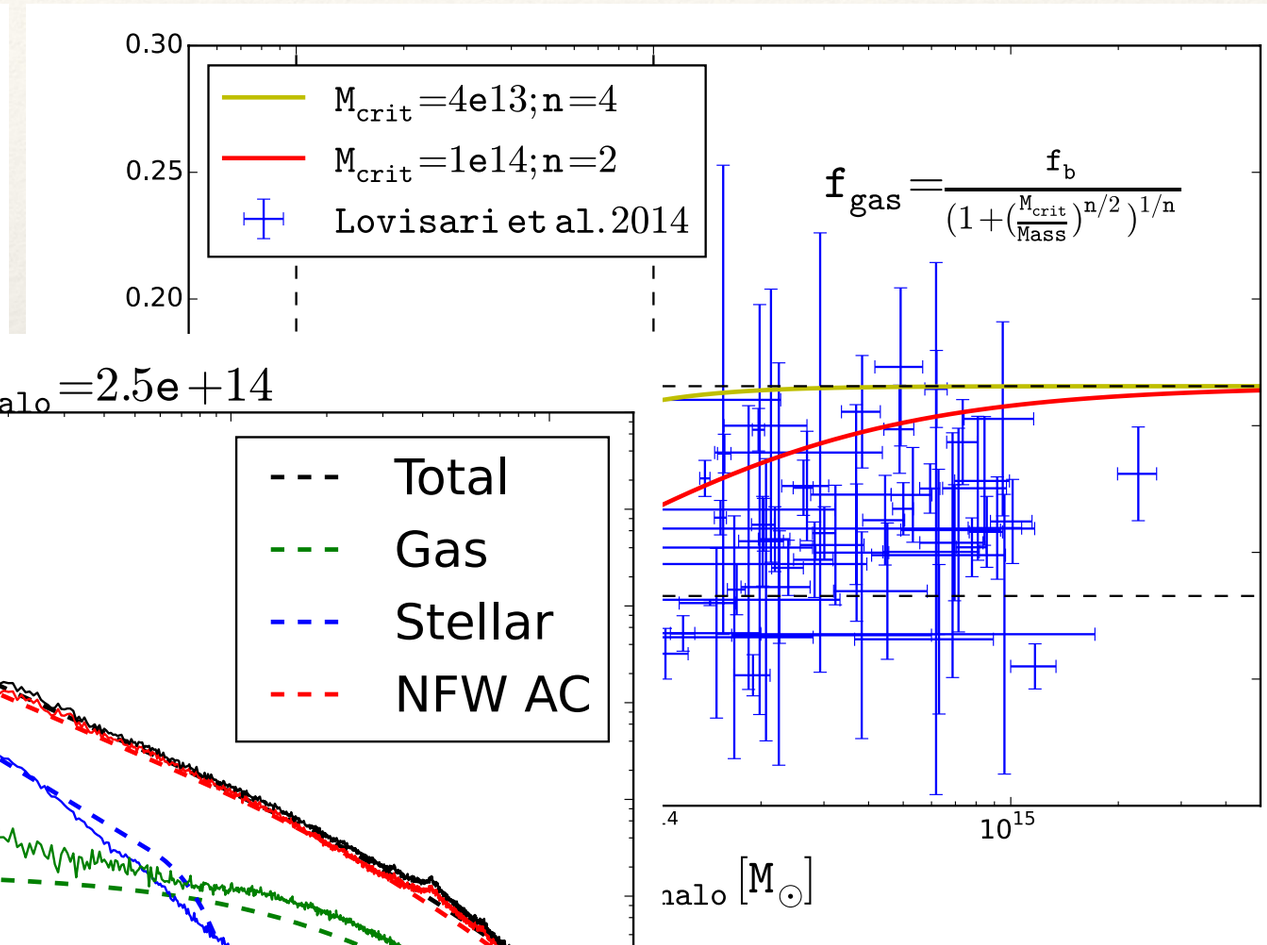
$$f_0 b_0 + \int_{\nu_{\rm min}}^{\infty} (f_{dm} + f_{gas}(\nu))f(\nu)b(\nu)d\nu = 1$$

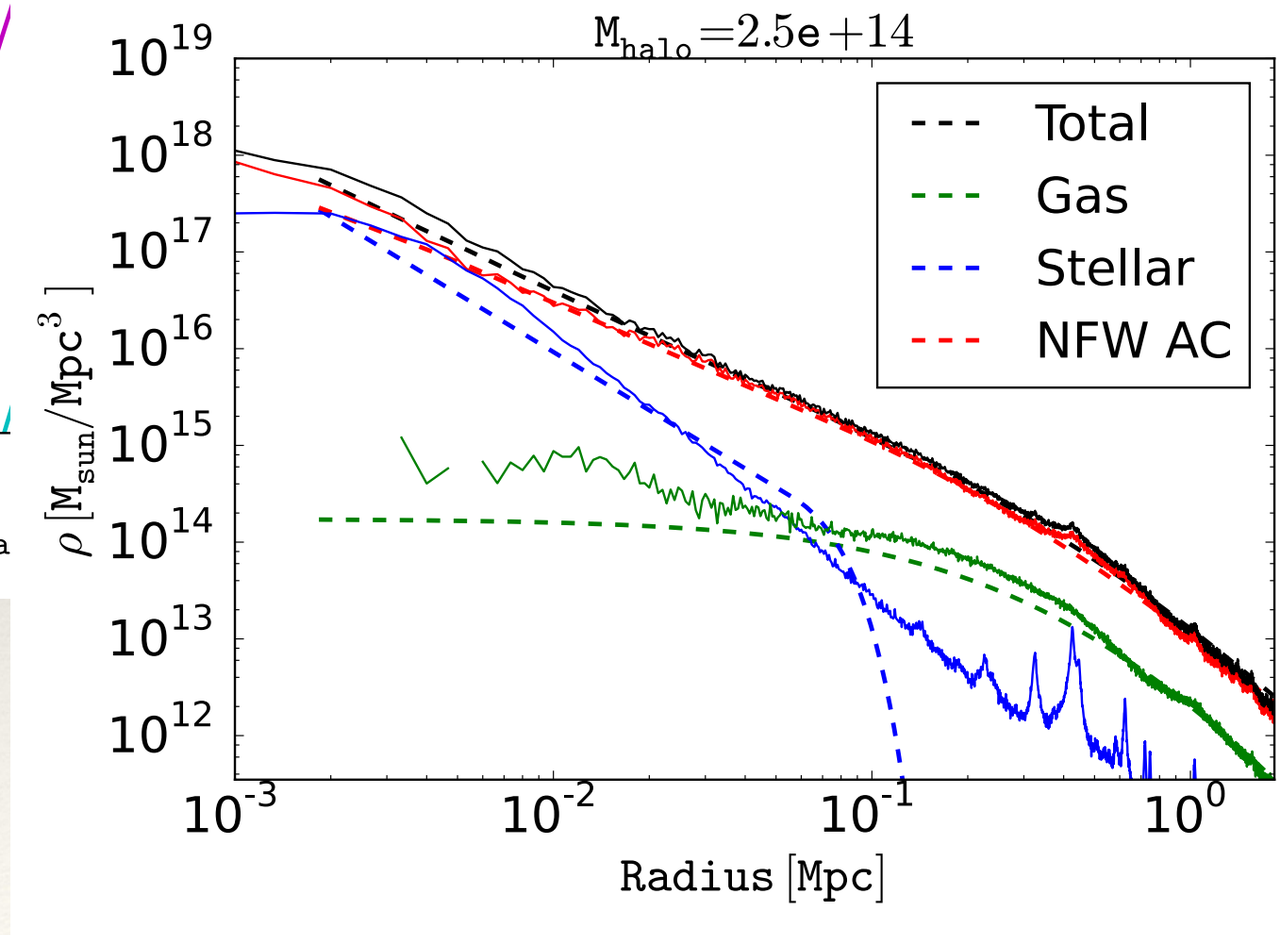
$$P_{2h} = \left(f_0 b_0 + \int d\nu (f_{dm} + f_{gas}(\nu))f(\nu)u(k|\nu)b(\nu)\right)^2 P_{\rm lin}(k),$$

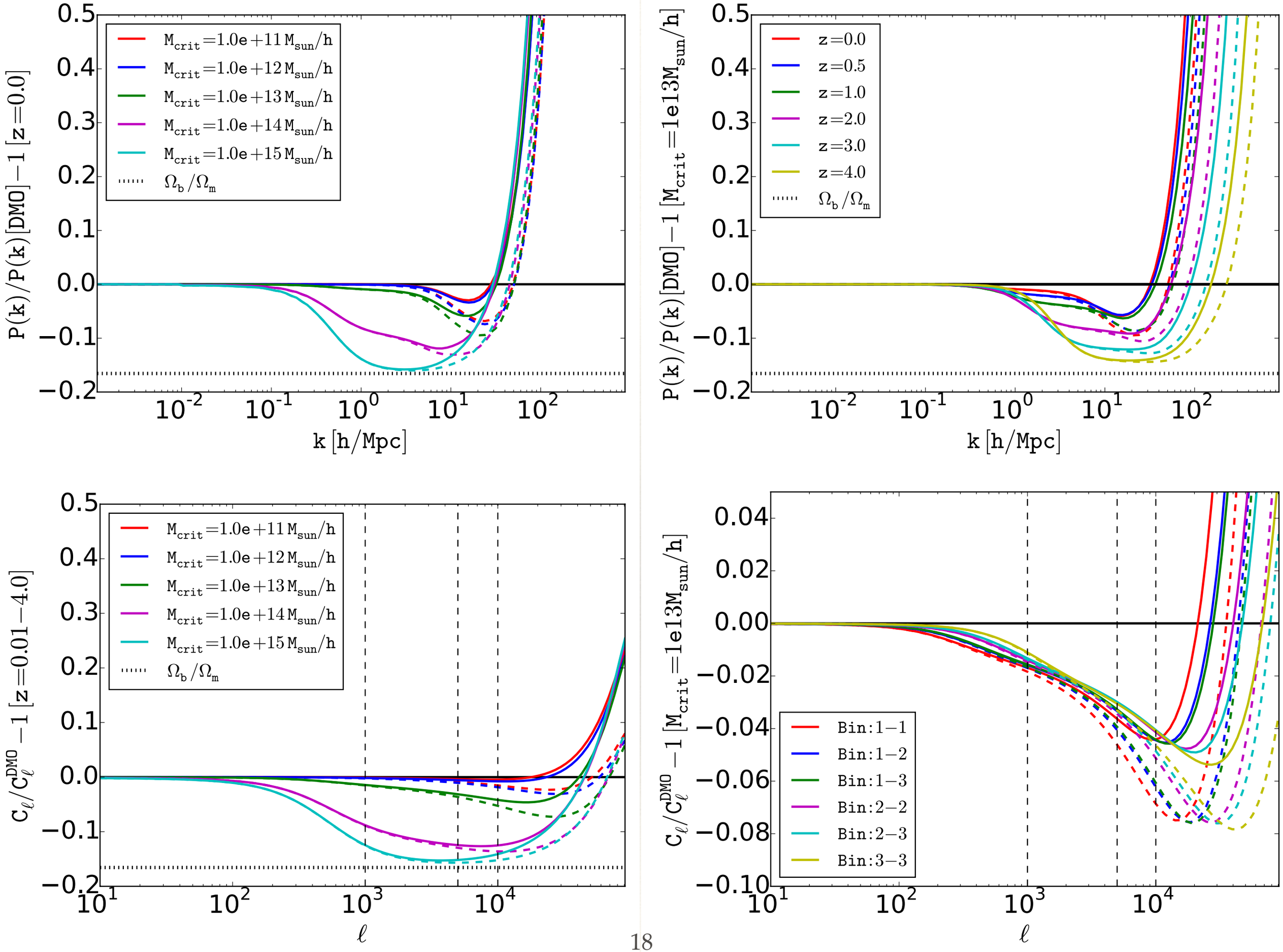
$$P_{1h} = \int d\nu (f_{dm} + f_{gas}(\nu))f(\nu)\frac{M}{\rho}|u(k|\nu)|^2$$

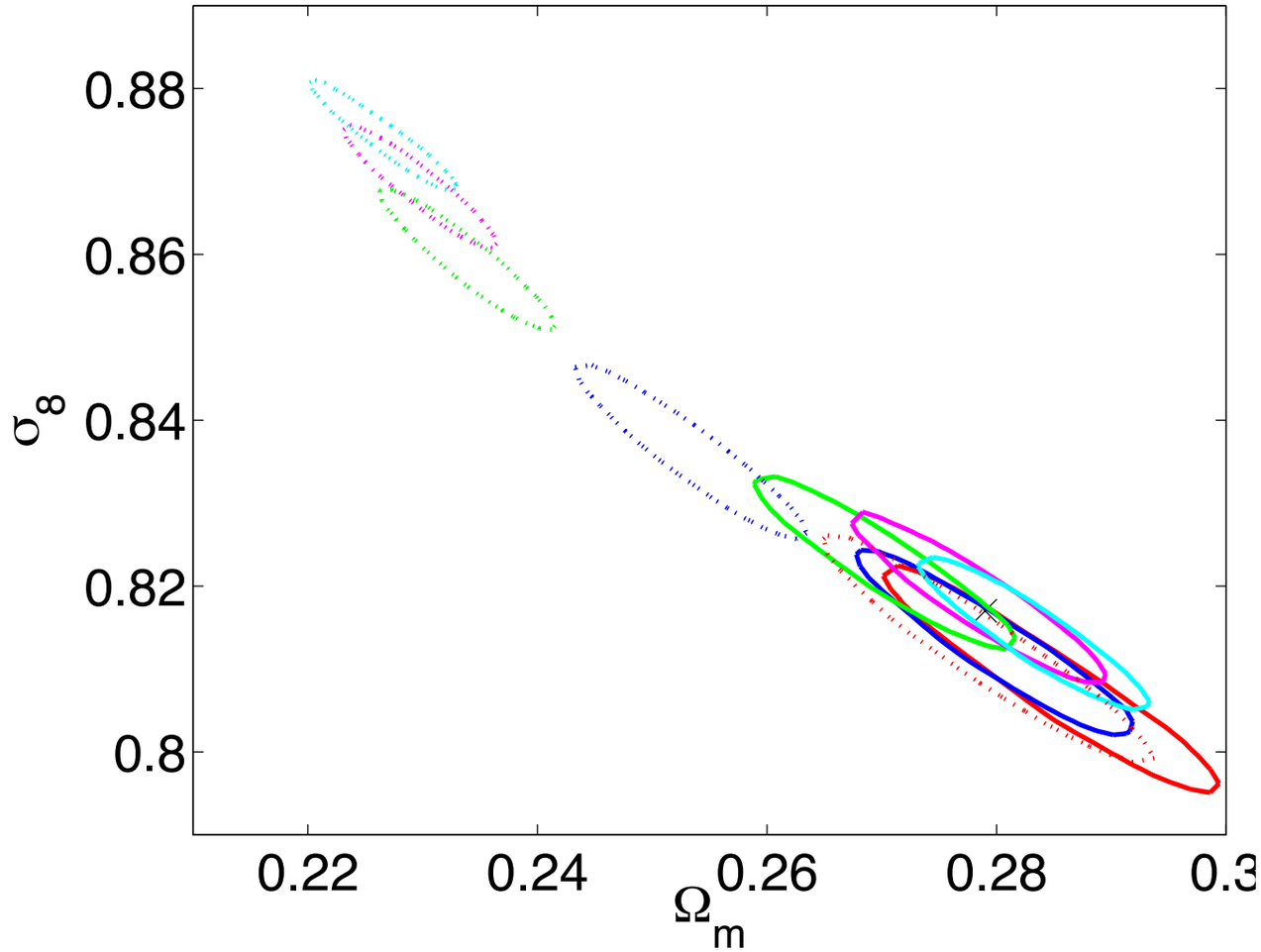
Model and Comparison

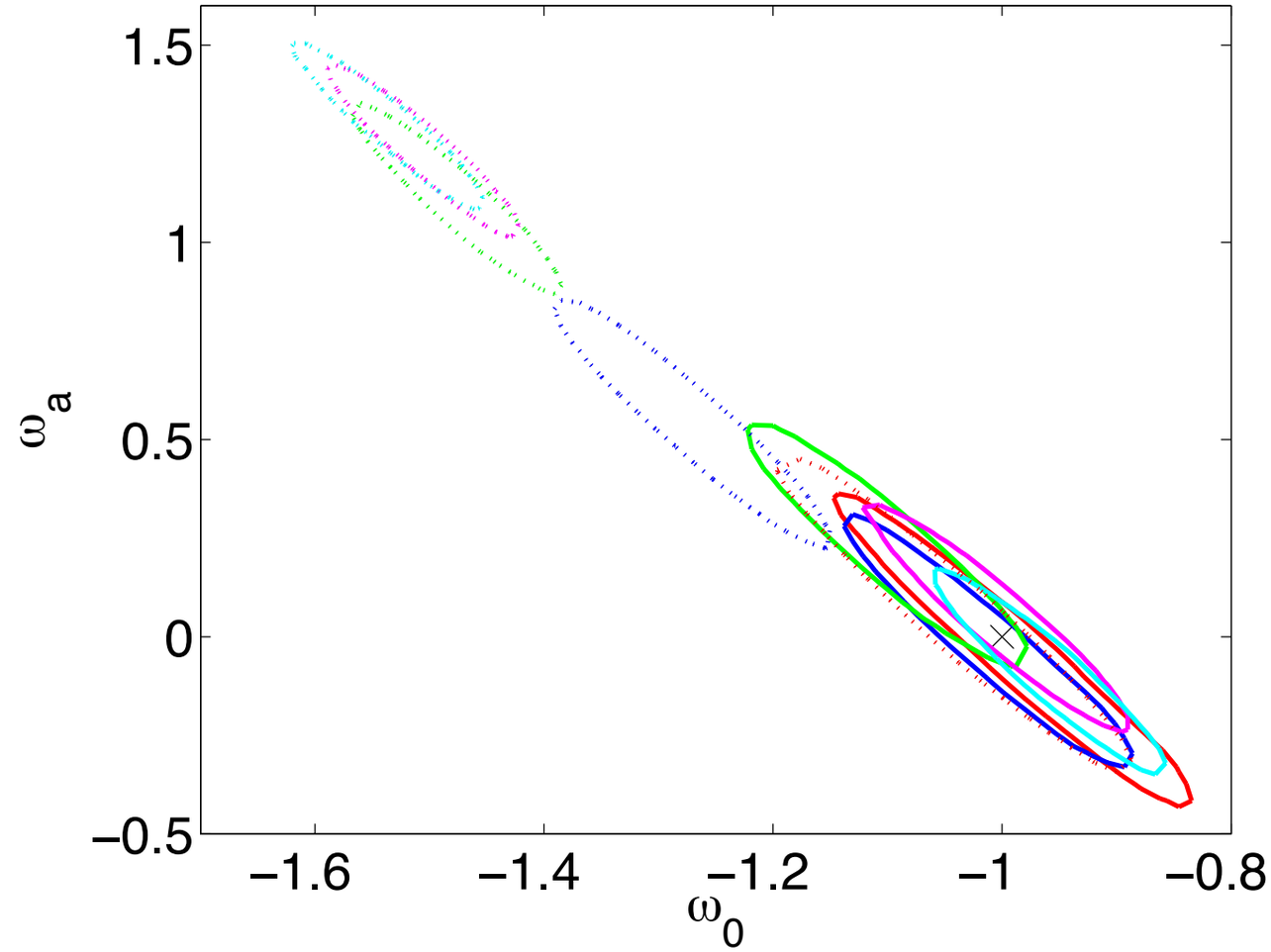


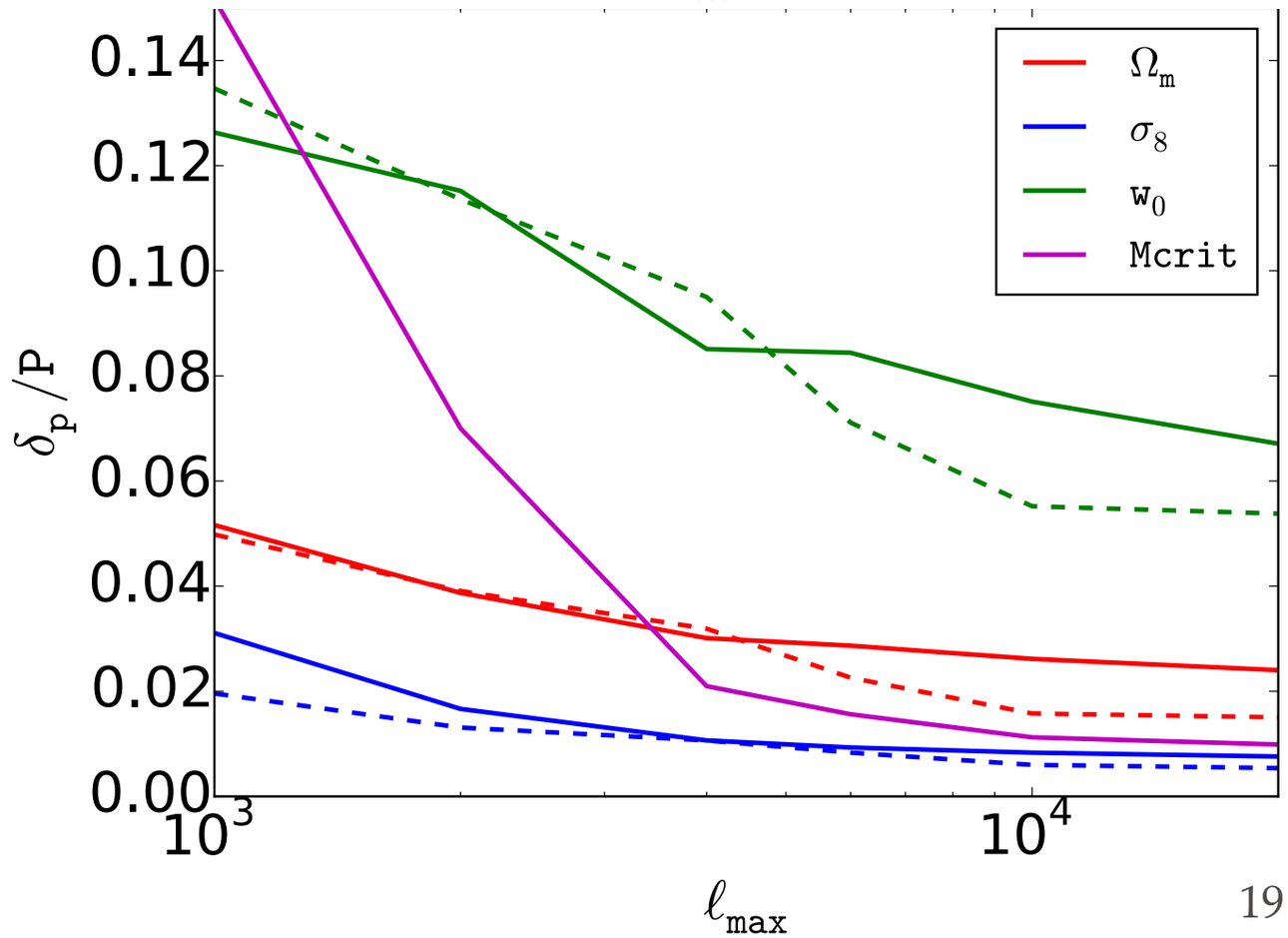


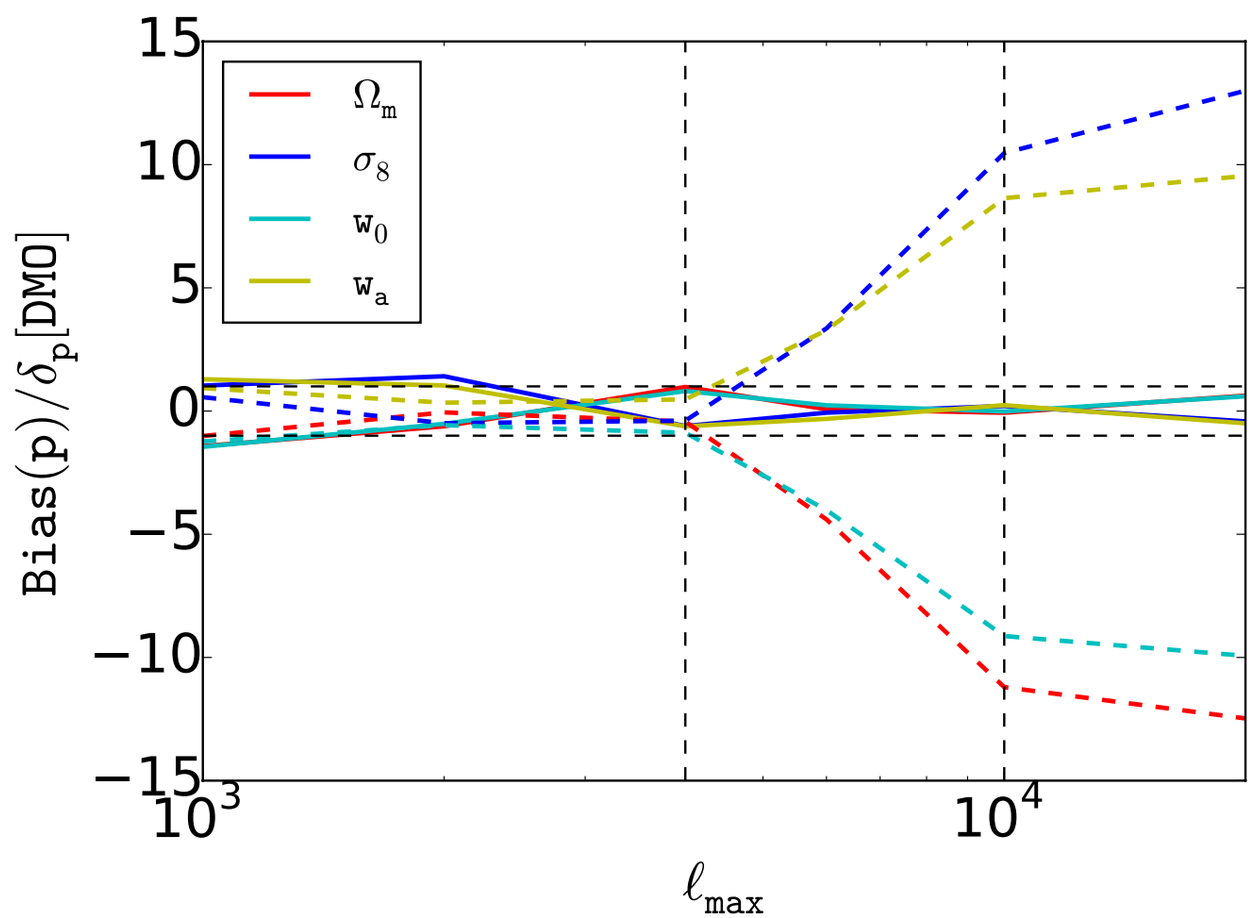



M_{crit}=1.0e+11 M_{sun}/h
M_{crit}=1.0e+12 M_{sun}/h
M_{crit}=1.0e+13 M_{sun}/h
M_{crit}=1.0e+14 M_{sun}/h
M_{crit}=1.0e+15 M_{sun}/h
Ω_b/Ω_m
P(k)/P(k)[DMO]−1 [z=0.0]
k [h/Mpc]
z=0.0
z=0.5
z=1.0
z=2.0
z=3.0
z=4.0
P(k)/P(k)[DMO]−1 [M_{crit}=1e13M_{sun}/h]
C_ℓ/C_ℓ^{DMO}−1 [z=0.01−4.0]
ℓ
Bin:1−1
Bin:1−2
Bin:1−3
Bin:2−2
Bin:2−3
Bin:3−3
C_ℓ/C_ℓ^{DMO}−1 [M_{crit}=1e13M_{sun}/h]

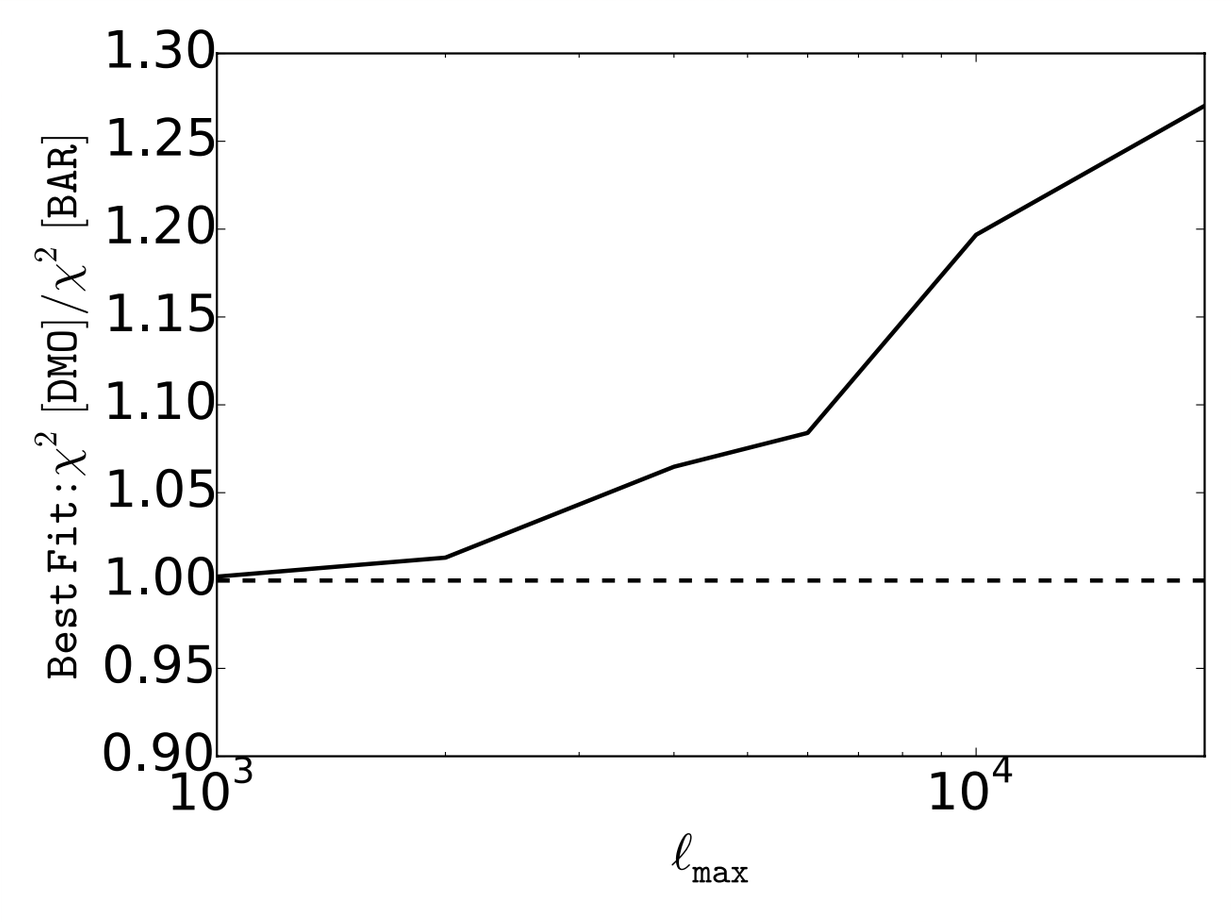

0.88
0.86
0.84
0.82
0.8
σ_8
0.22
0.24
0.26
0.28
0.3
Ω_m

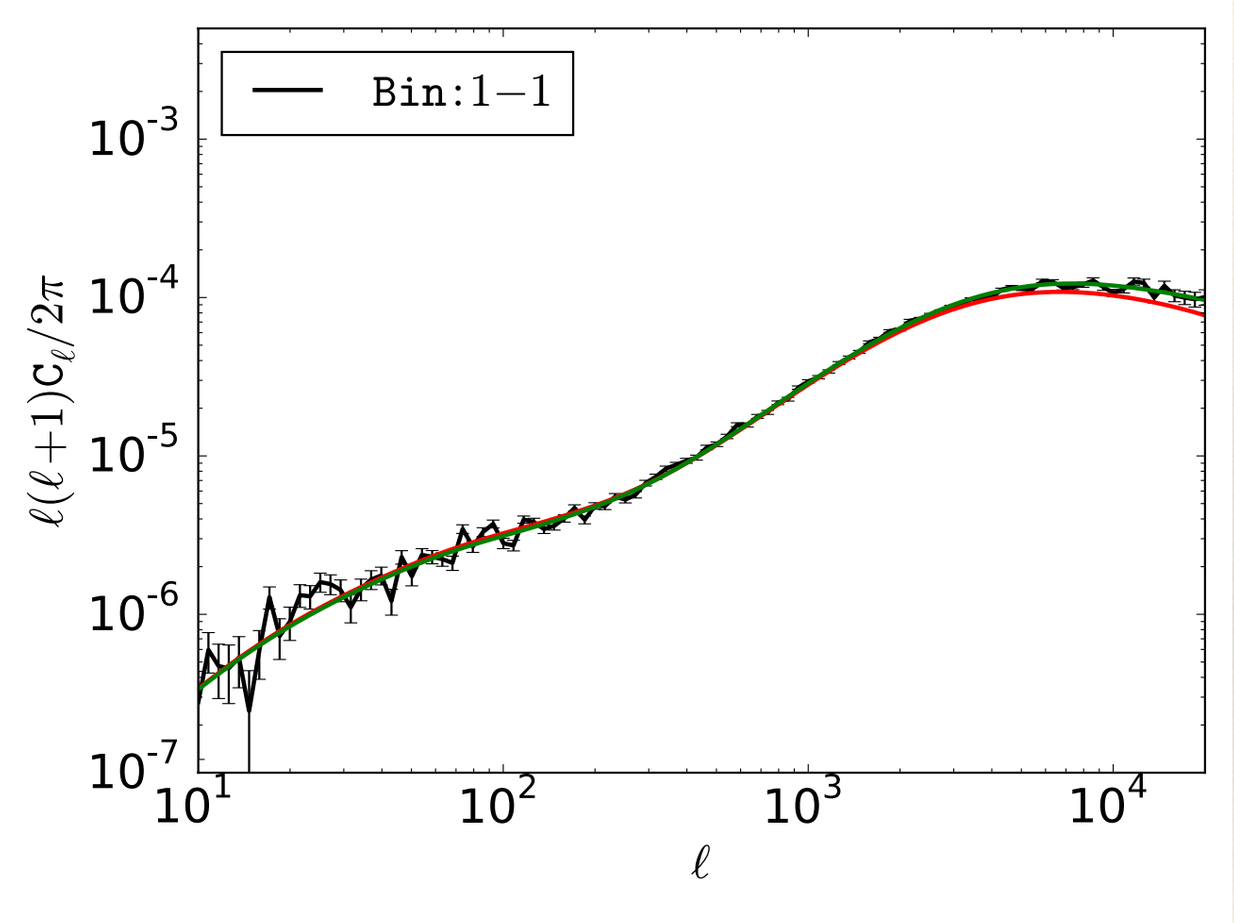

1.5
1
0.5
0
−0.5
ω_a
−1.6
−1.4
−1.2
−1
−0.8
ω_0


0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
δ_p/P
10^3
10^4
ℓ_{max}
Ω_m
σ_8
w_0
Mcrit


15
10
5
0
−5
−10
−15
Bias(p)/δ_p[DMO]
10^3
10^4
ℓ_{max}
Ω_m
σ_8
w_0
w_a

Goodness-of-fit insufficiency





Conclusions

- The halo model can be modified in different ways to achieve accuracy and precision in cosmological models for clustering of matter, both dark-matter and baryons.
- In the first modification, we account for dark-matter clustering,
- percent level accuracy is achieved up to k~0.7 h/Mpc,
- simple form of Covariance matrix, remarkable agreement with simulations,
- baryonic effects can be marginalised.
- In second modification, we propose model for distribution of baryons,
- baryons modify the clustering significantly at small scales, should be taken into account, avoiding baryons can mislead the cosmological interpretations.
- If precision cosmology is the goal, all these effects must be taken into account.

Thank you for your attention…